BOOK REVIEWS

Stability theory of differential equations. By Richard Bellman. New York, McGraw-Hill, 1953. 13+166 pp. $5.50.

This is the first book in the English language on the topic. In fact, while many aspects of stability theory have been summarized, notably by Russian authors, no mathematically satisfactory treatise on the subject has appeared since the classical work of Liapounoff. It is the reviewer's feeling, therefore, that any new book on stability must, at least partially, be judged on how it summarizes and organizes the highlights of the work done in this field since Liapounoff.

Bellman's book is organized in seven chapters. The first and third chapters are introductions to the theory of linear and nonlinear differential equations, respectively. Matrix and vector notation is used throughout these two well-written chapters. Chapter two deals with questions of stability, boundedness, and asymptotic behavior of linear differential equations, many of them originally investigated by Bellman himself. Chapter four is the heart of the book. Three different proofs are given for the stability theorem for the case where the linearized system has constant coefficients. The proofs are clear. A clever counterexample is provided to show that one cannot hope to extend the theorem without modification to the case where the linearized system does not have constant coefficients. In chapter five the asymptotic behavior of certain first order nonlinear equations is considered. Chapter six deals with linear second order equations. After an introduction to the more elementary properties of linear equations the author proves certain theorems about the boundedness in norm of solutions using various norms. Chapter six closes with a study of the oscillatory and asymptotic behavior of the solutions. Finally, chapter seven discusses the linear second order equation,

$$\frac{d}{dt}\left(t^{\rho}\frac{du}{dt}\right) \pm t^{\sigma}u^{\mu} = 0.$$

It is shown that certain conclusions with regard to the nature of solutions can be drawn from a knowledge of ρ, σ and μ.

The book as a whole is well-written and quite readable. There are many exercises to supplement the text. However, it seems unfortunate that the author excluded certain aspects of stability theory which, in a book dedicated to this field, seem more important than chapters five and seven. There is hardly anything in the book, for instance, on the stability of periodic solutions, or in the sixth chapter on the second order linear equation with periodic coefficients. There is comparatively little reference to work done in the last ten years either in this country or abroad. Aside from these omissions, however, Bellman's book is a pleasant and interesting contribution to the theory of differential equations.

F. HAAS

Discontinuous automatic control. By I. Flügge-Lotz. Princeton University Press, 1953. 8+168 pp. $5.00.

Although self-regulating devices have been in operation since the days of the governor on Watt's steam engine, it is only in recent years that the subject of automatic control has assumed a central position in the engineering and industrial world.

From the mathematical side, the control problem leads to systems of nonlinear differential equations in the following way. If we assume that the state of the physical system is specified at time t by the vector $x(t)$, the study of small displacements from equilibrium gives rise, in a system without control, to a linear vector-matrix equation $\dot{x}=Ax$. If we now consider a system with control, where the control is manifested by a forcing term and the magnitude of the control is dependent upon the state of the system, the resulting equation for x has the form $\dot{x}=Ax+f(x)$, and is, in general, nonlinear.

The term "continuous control" will be used to describe situations in which $f(x)$ is a continuous function of x. In many cases, it was found that continuous control was far too expensive to use. In place then of control devices which gave rise to forcing terms of continuous type, it was far cheaper to design control devices yielding forcing terms whose components are step-functions of x. The simplest version of this type of control system is one with a simple on-and-off control mechanism. This type of control is called "discontinuous automatic control."

A simple example of the mathematical equations which result is the following second order equation, $\ddot{u}+a\dot{u}+bu=c \operatorname{sgn}(\dot{u}+ku)$, where u is now a scalar function. This equation has the form $\ddot{u}+a\dot{u}+bu=c$, over the region of phase space described by $\dot{u}+ku>0$, and the form $\ddot{u}+a\dot{u}+bu=-c$, over the region of phase space described by $\dot{u}+ku<0$. If $\dot{u}+ku=0$, the forcing term is taken to be zero.

We observe then the interesting fact that while the equation itself is nonlinear, over the regions $\dot{u}+ku\gtrless 0$, u may be determined as a solution of a linear equation, albeit a different linear equation over different regions.