THE FEBRUARY MEETING IN NEW YORK

The five hundredth meeting of the American Mathematical Society was held at Hunter College in New York City on Saturday, February 27, 1954. The meeting was attended by about 180 persons, including the following 161 members of the Society:

C. R. Adams, Linda Allegri, R. L. Anderson, R. G. Archibald, Richard Arens, P. N. Armstrong, W. G. Bade, F. E. Baker, J. H. Barrett, L. K. Barrett, Anatole Beck, E. G. Begle, Lipman Bers, Armand Borel, Samuel Borofsky, S. G. Bourne, E. H. Boyle, J. W. Brace, A. D. Bradley, F. E. Browder, A. B. Brown, F. H. Brownell, C. W. Burrill, J. H. Bushey, Jewell H. Bushey, Sarvadaman Chowla, Alonzo Church, H. J. Cohen, L. W. Cohen, M. L. Constable, T. F. Cope, A. H. Copeland, Sr., A. H. Copeland, Jr., J. B. Crabtree, J. H. Curtiss, J. M. Danskin, M. D. Darkow, M. D. Davis, R. B. Davis, A. H. Diamond, V. J. Doberly, Jesse Douglas, Nelson Dunford, Aryeh Dvoretzky, H. A. Dye, B. E. Dyer, Jacob Feldman, William Forman, R. M. Foster, Joel Franklin, M. P. Gaffney, G. N. Garrison, H. A. Giddings, Wallace Givens, Sidney Glusman, S. I. Goldberg, Lawrence Goldman, A. J. Goldstein, Laura Guggenbuhl, Felix Haas, G. A. Hedlund, M. H. Heins, Sigurdur Helgason, Alex Heller, Robert Hermann, Abraham Hillman, S. P. Hoffman, Jr., Banesh Hoffmann, C. C. Hsiung, Witold Hurewicz, R. V. Kadison, Shizuo Kakutani, Aida Kalish, M. E. Kellar, L. S. Kennison, J. F. Kiefer, H. S. Kieval, M. S. Klamkin, E. R. Kolchin, Horace Komm, A. G. Kostenbauder, Saul Kravetz, J. B. Kruskal, M. D. Kruskal, M. K. Landers, R. K. Lashof, Marie Lesnick, M. E. Levenson, Eugene Lukacs, Brockway McMillan, L. A. MacColl, H. M. MacNeille, A. J. Maria, M. H. Maria, L. F. Meyers, K. S. Miller, August Newlander, Jr., Albert Nijenhuis, M. A. Oliver, A. F. O'Neill, Steven Orey, E. E. Osborne, L. J. Paige, T. K. Pan, L. E. Payne, E. J. Pellicciaro, Anna Pell-Wheeler, F. P. Peterson, R. P. Peterson, Jr., Hans Rademacher, G. N. Raney, M. S. Rees, B. L. Reinhart, Helene Reschovsky, H. G. Rice, Moses Richardson, Herbert Robbins, M. S. Robertson, Louis Robinson, Robin Robinson, Selby Robinson, P. C. Rosenbloom, A. S. Rosenthal, J. E. Rosenthal, H. D. Ruderman, J. P. Russell, C. W. Saalfrank, H. E. Salzer, Hans Samelson, Arthur Sard, R. D. Schafer, Samuel Schecter, E. V. Schenkman, Seymour Schuster, Abraham Schwartz, Sol Schwartzman, I. E. Segal, G. B. Seligman, I. M. Sheffer, J. L. Snell, J. J. Sopka, J. J. Stoker, R. L. Taylor, P. M. Treuenfels, A. W. Tucker, Annita Tuller, D. H. Wagner, H. F. Weinberger, Alexander Weinstein, Morris Weisfeld, Louis Weisner, Bernard Weitzer, M. E. White, A. L. Whiteman, Albert Wilansky, Jacob Wolfowitz, Arthur Wouk, Hidehiko Yamabe, Karl Zeller, J. A. Zilber, Leo Zippin.

Dr. Armand Borel of the Institute for Advanced Study delivered an address entitled *Topology of Lie groups and characteristic classes* at a general session presided over by Professor Witold Hurewicz, by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings. Sessions for contributed papers were held in the morning, presided over by Professor A. B. Brown, and in the afternoon, presided over by Professor Sarvadaman Chowla.

Abstracts of the papers presented follow. Those having the letter "*t*" after their numbers were read by title. Where a paper has more than one author, that author whose name is followed by "(p)" presented it. Mr. Norman Shapiro was introduced by Professor Alonzo Church and Dr. J. H. Williamson by Dr. I.T.A.C. Adamson.

ALGEBRA AND THEORY OF NUMBERS

336t. H. W. Becker: *Eight-parameter conjugates of rational cuboids.*

Generalize O'Riordan's forms, Dickson's *History* II, p. 504, to $ac\pm bd=E_\pm$, $ad\mp bc=F_\pm$, $eg\pm fh=G_\pm$, $eh\mp fg=H_\pm$. Let $x^2+y^2=u^2$, $y^2+z^2=v^2$, $x^2+z^2=w^2$. Then $v, z=(FH\pm EG)_-$; $u, x=(FG\pm EH)_-$; $w=(FG-EH)_+$, $y=(FH+EG)_+$. The resulting quadratics are $g/h=[(e^2-f^2)(2F_-E_- - F_+E_+)+ef(F^2-E^2)_+\pm 2\Delta_g^{1/2}]/[(fF-eE)_+^2-4ef(EF)_-]$, $\Delta_g=2(EF)_-(ae+bf)(af-be)\{(cf+de)^2-(ce-df)^2\}$, and the same, under: $a, b, c, d\leftrightarrow g, h, e, f$ or else e, f, g, h; and $a, b, g, h\leftrightarrow c, d, e, f$. Thus a cuboid of the kind has 4 conjugates, under change of sign of the 4 discriminants, each in turn having 3 new conjs., etc. ad inf. Expressions satisfying $\Delta=\square$ are deduced from the Euler[3], Lenhart[12], and Cunliffe[9] cuboids (the former, Euler[3] transforms of the Saunderson[2] and Rignaux[24c], ibid.): $a, b; c, d; e, f; g, h=2s(r+s)$, $-r^2+2rs+s^2$; $2s(r-s)$, $r^2+2rs-s^2$; d, $2r(r+s)$; $2r(r-s)$, b; and $r+s$, $2s$; $s(2r-s)(r+s)$, $r^3+3r^2s-4rs^2-2s^3$; $r-s$, $2r$; $r(r+2s)(r-s)$, $2r^3-4r^2s-3rs^2+s^3$; and r, $r-s$; $(r-s)(4r^2-2rs+s^2)$, $rs(4r-3s)$; $(2r-s)(r^2-s^2)$, $2r^3-5r^2s+rs^2+s^3$; e, $s(3r^2-3rs+s^2)$, respectively. A different 4-p.pr. cuboid is typified by no. 7 of Kraitchik's table, Scripta Math. vol. 11 (1945) p. 326: $v, y=\pm F_-H_\mp+E_-G_\mp$; $w, x=F_-G_\pm\mp E_-H_\pm$; $u=-F_+G_-+E_+H_-$, $z=F_+H_-+E_+G_-$. Put $ce+df, cf-de=C, D$, then $\Delta_a=2gh(C^2-D^2)(gC-hD)(gD+hC)$ isomorphic with the 3-p.pr. Δ of the type, and Δ_g is the same under $g, h\leftrightarrow b, a$. $\Delta_e=2abgh(c^2+d^2)^2\{(ah+bg)^2-(ag-bh)^2\}$, and Δ_c is the same under $a, b, c, d \leftrightarrow g, h, e, f$. Such cuboids have 5 conjs., one via a 3-p.pr. Δ_C. (Received January 14, 1954.)

337t. H. W. Becker: *Eight-parameter O'Riordan vectors.*

Dickson's *History* II, p. 502, Euler[32], first solution of $4\boxed{S}$, the sum of any $3\boxed{S}=\square$, is covered by O'Riordan's forms, p. 504, only if they are extended to 8 parameters. Generalize Euler's forms to 2 parameters: $c=a$, $d=b$, $e=(a^2+b^2)^2$, $f=4ab(a^2-b^2)$, $g=e(e^2+f^2)$, $h=f(e^2-f^2)$, and interchange B and C. Then, in the notation of the companion abstract: $A=E_+H_- - F_+G_-=2e^3f(a^2+b^2)$, $\alpha=E_+G_-+F_+H_-=(e^4+f^4)(a^2+b^2)$; B, $\gamma=(EG\mp FH)_-=(e^4+f^4)(a^2-b^2)\mp 4abe^3f$; C, $\beta=(EH\mp FG)_-=2e^3f(a^2-b^2)\mp 2ab(e^4+f^4)$; $D=(EH-FG)_+=-2ef^3(a^2+b^2)$, $\delta=(EG+FH)_+=(e^4+2e^2f^2-f^4)\cdot(a^2+b^2)$. Each O'Riordan form has 4 conj. solutions. Thus $h/g=[2cd(a^2-b^2)(e^2-f^2)\pm\Delta_g^{1/2}]/(e^2E^2+f^2F^2-2efE_-F_-)$, $\Delta_g=4c^2d^2(a^2-b^2)^2(e^2-f^2)^2-s$, $s=(e^4+f^4)E_+^2F_+^2-2ef(e^2-f^2)E_-F_-(E_+^2-F_+^2)+e^2f^2(E^4+F^4-4E_-^2F_-^2)$. In Euler's solution $s=0$, and the conj. $h'=0$. If $g=0$, $h=1$ the O'Riordan forms above reduce to his 3-p.pr. forms. But if $e=1, f=0$ then $\Delta_g=E_-F_-(2E_+F_++E_-F_-)=\square$ (and analogously if $a=1, b=0$) satisfied by $c=a$, $d=b$, $g=(a^2+b^2)^2$, $h=4ab(a^2-b^2)$ as in Euler's second method, where $A=D$, $\alpha=\delta$. But if $c=1$, $d=0$ the vector is imaginary: $A=\beta$, $B=\alpha$, $C^2+D^2=0$. Thus for any e, f, g, h, $a/b=[ef(g^2-h^2)\pm ghi(e^2+f^2)]/(e^2h^2+f^2g^2)$; and the same under $a, b\leftrightarrow e, f$; $i=(-1)^{1/2}$. And for any a, b, e, f, $h/g=\pm(af-be)i/(ae+bf)$, so $A=\beta=(af+be)(ae+bf)+(ae-bf)(af-be)i$, $B=\alpha=a^2e^2-b^2f^2-(a^2f^2-b^2e^2)i$, $C=(ae+bf)\cdot(af-be)(1+i)=D/i$; $\gamma, \delta=(ae+bf)^2\mp(af-be)^2i$. (Received January 14, 1954.)

338t. H. W. Becker: *Equiareal rational triangles and their Euler transforms.*

Basic solutions of $abcd(a^2-b^2)(c^2-d^2)=\square$ are $a, b, c, d=$: (I, II) $2r^2\mp s^2$, $r^2\pm s^2$, $r^2\mp 2s^2$, b (Fermat I & II, 175); (III) $r(3r-s)$, $s(r+s)$, $r(3r+s)$, $s(r-s)$ (Euler-Hillyer triad, 174); (IV) r^4-s^4, $3s^4$, r^4+2s^4, $3r^2s^2$ (Rignaux, 502); (V) $(r^2+s^2)^2$, $4rs(r^2-s^2)$, r, s (Rolle, 447); (VI) $8rs^3$ or $(r^2-2rs+3s^2)(r+s)^2$, $(r^2+s^2)(r^2-3s^2)$ or $2rs(r^2+2rs-s^2)$, $(r^2+2rs+3s^2)(r-s)^2$, $2rs(r^2-2rs-s^2)$ (dual Crussol-Rignaux triad, 502); (VII) $\Pi(r^2-rs\pm s^2)$, $(s\ \&\ r)\cdot(r^3-3r^2s+4rs^2-s^3)$, $s(r^3-2rs^2+2s^3)$; (VIII) $(2rs^2+{}^2)^2$, $4rs(r+s)(2r-s)$, $4r^4+8r^3s+8r^2s^2-4rs^3+s^4$, b (Guibert, 440); (IX) $\Pi(r^3\pm r^2s\pm s^3)$, $3rs^5$, $\Pi(r^3+rs^2\pm s^3)$, $3r^5s$ (dual Gerardin, 647); (X) $2r\Pi(r^3\pm 2r^2s-3rs^2\pm 2s^3)$, $(s\ \&\ r)\cdot(r^2-s^2)(r^4+18r^2s^2+s^4)$, $2s\Pi(2r^3\pm 3r^2s+2rs^2\mp s^3)$ (dual Euler, 646). Pagination is to Dickson's *History*, vol. II, where the above are given explicitly or deducibly. Further solutions are given by the Petrus (446), Euler (474) and other transforms, and their iterates. The transform $a, b, c, b, a\pm b, c\pm b\rightarrow A, B, C, B, A\pm B, C\pm B=(a+b)\cdot\Pi(a-b\pm c)$, $b(a-b)^2+c^2(3a-b)$, $c\{(a-b)(a+2b)+2c^2\}$, B, $(a-b)C$, $a\Pi(a-b\pm 2c)$, $(c\pm b)(a-b\pm c)(a-b\pm 2c)$ is a formula version of Euler's recurrence (a great contribution, but no "general solution," possibility of which is questionable). Solutions (III; IV; VII) are applications of Euler's transform to the degenerate $a, b, c, d=r, s, r, s$; $r^2, 0, s^2, 0$; $s(r\pm s)$, $(r\ \&\ s)(r-s)$. Empirical solutions are found by tabulating $rs(r^2-s^2)/\square$. Maverick solutions fitting none of the above forms or their transforms are: 19, 8, 22, 3; 45, 32, 22, 13; 56, 25, 32, 31; 64, 61, 60, 61. (Received January 14, 1954.)

339t. H. W. Becker: *Six-parameter O'Riordan vectors.*

O'Riordan (Dickson's *History* II, p. 503) gave the general 3-p.pr. criterion for $4\boxed{S}$, the sum of any $3\boxed{S}=\square$. His $(a, b; c, d)$ satisfy $abcd(a^2-b^2)(c^2-d^2)=\square$, and his $e, f=\iota, \theta$ or κ, λ where $(\iota, \theta; \kappa, \lambda)$ is the Petrus transform of $(a, b; c, d)$. Substitution of the Euler-Hillyer forms for $(a, b; c, d)$ gives the Tebay and final Euler forms for O.V.; substitution of the Rolle forms, the second Euler O.V., ibid. line 28. Substitution of the Fermat I forms for $(a, b; c, d)$ gives the O.V.: $A, D; B, C=6(r^2+s^2)(r-s)(r^2-3rs+s^2)\{s(2r^2-s^2),\ r(r^2-2s^2)\}$; $(2r^2-s^2)(r^2-2s^2)(2r^2-3rs+2s^2)\{3rs,\ 2(r^2+s^2)\}$. The dual Fermat II forms give perhaps the next to smallest nonrepetitive O.V.: 792, 385, 840, 1980. Euler's semifinal algorithm is equivalent to O'Riordan's more elegant and convenient procedure: $v, x, y, z=D, B, A, C$; and exhibits their filigree of properties $D/A=[cd(a^2-b^2)/ab(c^2-d^2)]^{1/2}$, $B/C=[(a^2-b^2)(c^2-d^2)/abcd]^{1/2}/2$, $DB/AC=(a^2-b^2)/2ab$, $B=fE_- - eF_-$ and its conjugate $B'=f'E_- - e'F_-=$ (in the Euler-Hillyer case) $eE_+^2-fF_+^2F_-/E_-$. Each 3-p.pr. O.V. has 3 conjugates, under change of sign of the discriminant in the quadratics for a/b, c/d, e/f, wherein is the proof that the index $=$area$/\square$ is unaltered by Petrus transformation of Pythagorean $\triangle$. Conjecture: no general formula for O.V. exists, in a finite number of parameter pairs. (Received January 14, 1954.)

340t. Leonard Carlitz: *A note on generalized Dedekind sums.*

Rademacher has proved (in a paper to appear in vol. 21 of the Duke Math. J.) a three-term relation for Dedekind sums. In the present paper a certain n-fold sum is defined which reduces to the ordinary Dedekind sum for $n=1$. It is proved that this sum satisfies both an $(n+1)$- and an $(n+2)$-term relation, those generalizing both the familiar reciprocity formula and Rademacher's new formula. (Received January 13, 1954.)

341t. Leonard Carlitz: *A note on Euler numbers and polynomials.*

This paper contains various extensions of the known congruence $E_{2m}\equiv 1$

$-(-1)^{(p-1)/2}$ (mod p), where p is an odd prime such that $p-1|2m$. (Received January 13, 1954.)

342t. Leonard Carlitz: *Dedekind sums and Lambert series.*

Apostol (Duke Math. J. vol. 17 (1950) pp. 147–157) has proved a transformation formula for the function $G_p(x)=\sum_{m,n=1}^{\infty} n^{-p}x^{mn}$, where p is an odd integer >1. The writer (Pacific Journal of Mathematics vol. 3 (1953) pp. 513–522) showed that this transformation formula implies a reciprocity relation for the function $f(h, k; \tau) = \sum_{s=0}^{p+1} \binom{p+1}{s}(k\tau-h)^{p-s}c_s(h, k)$, where $c_s(h, k)=\sum_{\mu} \overline{B}_{p+1-s}(\mu/k)\overline{B}_s(h\mu/k)$, the summation extending over a complete residue system (mod k). In the present paper the latter reciprocity relation is proved in an elementary manner using only familiar properties of the Bernoulli functions $\overline{B}_s(x)$ and the representation of $c_s(h, k)$ in terms of "Eulerian" numbers. (Received January 13, 1954).

343. S. I. Goldberg: *Lie algebra extensions and enlargements of modules.*

Let G be a Lie algebra over an arbitrary field F, K a subalgebra of G, and M a G-module. The notions of K-extension and K-enlargement are introduced as follows: A K-extension (E, K, ϕ, u) is an extension (E, ϕ) with a representative function u such that $[u(\gamma), u(\sigma)]=u([\gamma, \sigma])$ for all $\gamma \in G$ and $\sigma \in K$. A K-enlargement (ϵ, K, ψ, μ) is an enlargement (ϵ, ψ) with a representative function μ such that $\sigma \cdot \mu_m = \mu_{\sigma . m}$ for all $\sigma \in K$ and $m \in M$. The following is then proved: (i) the elements of $H^1(G, K, \mathrm{Hom}_G(M, P))$ are in (1-1) correspondence with the equivalence classes of K-enlargements of P by M and (ii) for any given K-extension (E, K, ϕ, u) of G by M with factor set f and K-enlargement (ϵ, K, ψ, μ) of P by M, a necessary and sufficient condition for the existence of a K-extension ϵ^* with $\epsilon^*/P \approx \epsilon$ is that $\Lambda(f)$ be a relative coboundary mod K where Λ: $H^2(G, M) \rightarrow H^3(G, P)$ is the invariant coboundary. (Received October 13, 1953.)

344t. Arno Jaeger: *On derivations and differentiations in algebraic function fields of prime number characteristic.*

Let F be a separably generated algebraic function field in one indeterminate over a constant field f of characteristic $p \neq 0$, and let D be a fixed regular differentiation of F over f in the sense of F. K. Schmidt (cf. Monatshefte für Mathematik vol. 56 (1952) pp. 181–219). For each derivation δ in F over f in the classical sense there exists an element $a_\delta \in F$ such that $\delta = a_\delta D$ holds. A derivation δ belongs to a differentiation if and only if $D^{p-1}a_\delta^{-1}=0$ is satisfied. If δ^n denotes the operator obtained by the n-fold iteration of δ and δ^0 the identity operator, the ring of derivation operators $\sum b_n\delta^n$ $(b_n \in F)$ is isomorphic to the ring of all $p \times p$ matrices with elements in the subfield of F containing all D-constants of F. For the derivation δ^p one finds $\delta^p = c\delta$ where c is a D-constant. The theory of the solutions of the equations $\sum b_n\delta^n y = a$ $(b_n \in F, a \in F)$ can be derived from the theory of differential equations in the sense of F. K. Schmidt. Generalizations of these results are obtained in the case of a separably generated algebraic function field in several (but finitely many) indeterminates where multi-differentiations $\mathfrak{D}$ (cf. J. Reine Angew. Math. vol. 190 (1952) pp. 1–21) take over the role of the differentiations in the one-dimensional case. (Received January 14, 1954.)

345t. K. G. Wolfson: *A class of primitive rings.*

Motivated by the examples of the ring of all linear transformations of an arbitrary vector space over a division ring, and the ring of all bounded operators on a Hilbert space, it is determined when the ring $T(A, B)$ of all continuous linear transformations on the pair of dual vector spaces (A, B) over a division ring D has the property that every right (left) annihilating ideal is a principal right (left) ideal generated by an idempotent element. A necessary and sufficient condition is that every closed subspace S of A (in the sense of Mackey) [Trans. Amer. Math. Soc. vol. 57 (1945) pp. 155–207] possesses a closed complementary subspace Q, such that S and Q form a modular pair in the lattice of closed subspaces of A. In particular, the ring of all bounded operators on a Banach space has the required property, precisely when the lattice of closed subspaces (in the usual norm topology) is complemented. The method is based on the theory of dual spaces of Dieudonné, Jacobson, and Mackey, and previous results of the author [Amer. J. Math. vol. 75 (1953) pp. 358–386]. Applications to the theory of dual rings are made. (Received January 6, 1954.)

ANALYSIS

346t. W. W. Bledsoe, M. J. Norris, and G. F. Rose: *On a differential inequality.*

Using vector-matrix notation and letting the subscripts 1 and 2 indicate the positive and negative part, respectively, of a vector or matrix, the results of this paper are slight generalizations of the one below. If, on the interval $[a, b]$, F and M are finite-valued, F_2 is continuous, D^+F_2 is never $-\infty$ for any component, $F(a)$ has no negative component, M is non-negative almost everywhere off the main diagonal, M is essentially bounded above, and $D^+F \geqq MF$ almost everywhere; then on $[a, b]$, F never has a negative component. Examples are given that show that in a sense the hypotheses are as weak as possible. (Received January 4, 1954.)

347. V. J. Doberly (Dobroliuboff): *Solution of differentiable equations in infinite series.*

The author's own method of solving insoluble by elementary means algebraical equations extends practically to all differentiable equations which can be presented as continuous functions (y) of an unknown and sought for independent variable quantity (x). Although in its final form the root of such an equation always appears as an infinite series, the question of its convergibility usually does not present a problem, since this method permits to effect the desired change in the conditions of convergency through an appropriate choice of the parameter which appears in all terms of the resulting series as a numerical quantity independent of x and y. (Received January 5, 1954.)

348t. Herbert Federer: *An addition theorem for Lebesgue area.*

If X is the space of a 2-dimensional finite Euclidean cell-complex K, f is a continuous function mapping X into Euclidean n-space with $n \geqq 2$, and the restrictions of f to the 1-cells of K are curves of finite length, then the 2-dimensional Lebesgue area of f is the sum of the 2-dimensional Lebesgue areas of the restrictions of f to the 2-cells of K. (Received December 8, 1953.)

349t. R. V. Kadison: *On the general linear group of infinite factors.*

In the paper *Infinite general linear groups* [Trans. Amer. Math. Soc. vol. 76 (1954)], the author determined all the uniformly closed, normal subgroups of the group of all invertible operators in a factor, with the exception of the factors of type

II_∞. The present note is devoted to filling the gap in the information. Theorem 4 of the above mentioned paper is proved again by means of a proof which is valid for both the I_∞ and II_∞ cases. The key lemma of the present note is one which states that an operator in a C^*-algebra lies in the center if and only if each of its inner transforms (by invertible operators in the algebra) is normal. (Received January 14, 1954.)

350*t*. A. J. Lohwater: *On the radial limits of analytic functions*. I.

Nonconstant analytic functions $f(z)$ are studied whose radial limits are constant almost everywhere on an arc A: $[\alpha<\theta<\beta]$ of $|z|=1, z=re^{i\theta}$. It is shown, for example, that every complex number ζ (including ∞) is an asymptotic value on any arc of $|z|=1$ containing any point $e^{i\theta}$ of A, if the points z_k for which $f(z_k)=\zeta$ satisfy the condition $\sum(1-|z_k|)<\infty$. (Received January 11, 1954.)

351*t*. E. R. Lorch: *On certain extensions of the notion of volume*.

The classical notion of volume assigns to a body $\mathfrak{K}$ in n-space a unique number $V(\mathfrak{K})$ which is the volume of $\mathfrak{K}$. It is shown here that the concept of volume depends on a real parameter r, $1\leqq r\leqq\infty$. Thus for each smooth convex body $\mathfrak{K}$ there is defined a function $V_r(\mathfrak{K})$, the volume of order r. The case $r=1$ gives the classical volume. The definition of $V_r(\mathfrak{K})$ follows: Let $\mathfrak{K}$ be defined by $\phi(x)=\phi(x_1, \cdots, x_n)\leqq 1$ where ϕ is positively homogeneous of order 1. Let $r>1$ and let $G(x)=r^{-1}\phi^r(x)$. Let $|G_{ij}(x)|$ be the Hessian of $G(x)$, let Ω be the unit sphere in n-space, let $\mathfrak{K}^*$ be the body adjoint to $\mathfrak{K}$. Then (definition) $V_r(\mathfrak{K}^*)=[n^{-1}(r-1)^{-1}\int_\Omega|G_{ij}(x)|d\Omega]^{1/r}$. The cases $r=\infty$ and $r=2$ receive special attention. It is shown that $V_\infty(\mathfrak{K})=\max_{x\in\Omega}\phi^{-n}(x)$. If $\mathfrak{K}$ is an ellipsoid, its volume of order $r=2$ is shown to be proportional to the product of its principal semi-axes. Thus for the computation of relative volumes of solids which are approximately ellipsoids, volumes of order 2 give approximately the same results as those of order 1. The development is based on previous work of the author (*Differentiable inequalities and the theory of convex bodies*, Trans. Amer. Math. Soc. vol. 71 (1951) pp. 243–266). (Received January 11, 1954.)

352. L. E. Payne and H. F. Weinberger (p): *Bounds for harmonic and biharmonic functions*.

The following identity is proved for the Green's function $G(0, Q)=-(\omega_N r^{N-2})^{-1}+g(0,Q)$ of a closed region in N dimensions, $N\geqq 3$. If h is the function $r\partial r/\partial n$ on the boundary B, then (1) $(N-2)g(0, 0)=\int\int_B h(\partial G/\partial n)^2$. From this and the representation of a harmonic function in terms of its boundary values, it follows that if B is star-shaped with respect to the point 0 so that $h>0$, then (2) $u(0)\leqq(N-2)g(0, 0)\int\int_B h^{-1}u^2$. Putting $u(Q)=g(0, Q)$ results in an upper bound for $g(0, 0)$ and the inequality (3) $u(0)\leqq\int\int_B h^{-1}u^2\int\int_B h^{-1}r^{-2(N-2)}$. In two dimensions the analogue of (1) is (4) $\oint_B h(\partial G/\partial n)^2=(2\pi)^{-1}$ and the second integral in (3) is replaced by $(2\pi)^{-1}$. This inequality gives an error estimate if a harmonic function is chosen to approximate given boundary values on B in L_2 norm. An analogous result can be established for biharmonic functions. In two dimensions, if the clamped plate Green's function $\Gamma=(8\pi)^{-1}r^2\ln r-\gamma$, then $\gamma(0, 0)=(1/2)\oint_B h(\Delta\Gamma)^2$. (Received January 14, 1954.)

353*t*. R. S. Phillips: *A note on the abstract Cauchy problem*.

The notion of an abstract Cauchy problem has recently been introduced by E. Hille. The abstract Cauchy problem (ACP) may be formulated as follows: Given a linear operator U with domain and range in $\mathfrak{X}$ and given an element $y_0\in\mathfrak{X}$, find a function $y(t)=y(t; y_0)$ such that (i) $y(t)\in\mathfrak{D}(U)$ for $t>0$; (ii) $y(t)$ is strongly absolutely continuous and continuously differentiable in each finite subinterval of $[0, \infty)$; (iii) $U[y(t)]=y'(t)$ for $t>0$; and (iv) $\lim_{t\to 0}y(t; y_0)=y_0$. Theorem: Let U be a closed linear operator with dense domain and nonvacuous resolvent set. Suppose that for each $y_0\in\mathfrak{D}(U)$ there is a unique solution to ACP. Then U generates a semi-group $S(t)$ strongly continuous for $t\geqq 0$ with $S(0)=I$ and such that $S(t)y_0=y(t; y_0)$ for all $y_0\in\mathfrak{D}(U)$. If condition (ii) is relaxed so that it is valid only for subintervals of $(0, \infty)$, then a similar result can be proved for a more general class of semi-groups. (Received January 13, 1954.)

354t. R. S. Phillips: *Groups of positivity preserving operators.* Preliminary report.

Let $\mathfrak{S}$ be a locally compact Hausdorff space and let $C(\mathfrak{S})$ denote the Banach space of all complex-valued continuous functions on $\mathfrak{S}$ which vanish at infinity, with $\|f\|=\sup|f(s)|$. The purpose of this paper is to characterize all strongly continuous one-parameter groups $[S(t); -\infty<t<\infty]$ of positivity preserving linear bounded operators on $C(\mathfrak{S})$. In fact, $[S(t)]$ is such a group if and only if it can be represented in the form $[S(t)f](s)=\theta(t, s)f[\phi(t, s)]$ where (1) $\phi(t, s)$ is a group of homeomorphisms of $\mathfrak{S}$ continuous on $E_1\times\mathfrak{S}$ to $\mathfrak{S}$ such that $\phi(t_1+t_2, s)=\phi(t_1, \phi(t_2, s))$ where $\phi(0, s)=s$, and (2) $\theta(t, s)$ is a positive real-valued function continuous on $E_1\times\mathfrak{S}$ to E_1 such that (a) $\theta(t_1+t_2, s)=\theta(t_1, s)\cdot\theta(t_2, \phi(t_1, s))$ where $\theta(0, s)\equiv 1$ and (b) there exist constants $M>0$ and $\omega\geqq 0$ such that $[M \exp(\omega|t|)]^{-1}\leqq\theta(t, s)\leqq M\exp(\omega|t|)$. Suppose next that $\mathfrak{S}$ is an n-dimensional manifold of class C^∞ and let $\mathfrak{D}$ be the set of all infinitely differentiable functions with compact carriers. If $\mathfrak{D}$ is contained in the domain of the infinitesimal generator A, then both $\theta(t, s)$ and $\phi(t, s)$ are continuously differentiable with respect to t; $\partial\theta(t, s)/\partial t|_{t=0}=\beta(s)$ is real-valued and $\partial\phi(t, s)/\partial t|_{t=0}=\alpha(s)$ is a vector lying in the tangent space at s. Finally if $f\in\mathfrak{D}$, then $Af=\beta f+\alpha\cdot\nabla f$ where ∇ is the gradient operator. In particular, this shows that the diffusion equation can never lead to a reversible Markoff process. (Received January 13, 1954.)

355t. V. L. Shapiro: *Logarithmic capacity of sets and double trigonometric series.*

Given a double trigonometric series $T=\sum a_Me^{iMX}$, T is said to be a series of class (U') if $\sum_{M\neq 0}a_M|M|^{-2}e^{iMX}$ is the Fourier series of a continuous periodic function. If this latter series is the Fourier series of a bounded function, then T is said to be a series of class (B'). Let Z be a closed set contained in the interior of the fundamental square Ω. It is shown in this paper that a necessary and sufficient condition that Z be a set of uniqueness for series of class (U') under circular $(C, 1)$ summability is that Z be a set of logarithmic capacity zero. It is further shown that a necessary and sufficient condition that Z be a set of uniqueness for series of class (B') under local uniform circular $(C, 1)$ summability is that Z be a set of logarithmic capacity zero. Using Fourier-Stieltjes series, necessary and sufficient conditions that Z be a set of positive logarithmic capacity are obtained in terms of double trigonometric series of classes (U') and (B'). (Received January 13, 1954.)

356. Alexander Weinstein: *The general solution for a class of composite equations.*

Let $v=v(x_1, x_2, \cdots, x_m, y)$ and let $L_\alpha v$ denote either the operator $v_{yy}+\alpha y^{-1}v_y+\Delta_x v$ (occurring in generalized axially symmetric potential theory) or the hyperbolic Darboux operator $v_{yy}+\alpha y^{-1}v_y-\Delta_x v$. Let $u^{(\alpha)}$ denote in both cases any solution of $L_\alpha v=0$.

It is proved that the general solution of $L_\beta v = u^{(\alpha)}$, $\alpha \neq \beta+2$, is $v = u^{(\alpha-2)} + u^{(\beta)}$. Applications: (A). The general solution of the *polyharmonic equation* of order n, $L_0^n v = 0$, is given by the new formula $v = u^{(0)} + u^{(-2)} + \cdots + u^{(-2n-2)}$. (B). The general solution of the equation $L_{2n}^{n+1} v = 0$ is $v = u^{(0)} + u^{(2)} + \cdots + u^{(2n)}$. In the case of a Darboux operator L_{2n}, the fact that the equation $L_{2n}^{n+1} v = 0$ admits the particular solution $v = u^{(0)}$ was formulated in a special case as a theorem of Friedrichs in the book of Courant-Hilbert, vol. 2, pp. 416 ff. See also A. Weinstein, Bull. Amer. Math. Soc. Abstract 59-4-395. (Received January 14, 1954.)

357t. J. G. Wendel: *Haar measure and the semigroup of measures on a compact group.*

Let S be a compact semigroup. Results of Numakura, Wallace, and Peck show that (i) S contains idempotents, and (ii) if S is not a group but contains an identity then it contains additional idempotents. We show that these facts imply the existence of Haar measure on any compact group G. The S which intervenes consists of the regular normalized nonnegative Borel measures on G, with convolution as multiplication, topologized by pointwise convergence as functionals on $C(G)$. G is embedded in S via the unit masses at the points of G, and $e \in G$ is the identity of S; no measure in $S-G$ has an inverse relative to e, so that S is not a group if $G \neq \{e\}$. If it exists, the Haar measure on a compact subgroup H of G defines an idempotent in S. Conversely, any idempotent in S arises in this way. Hence by (ii) some nontrivial subgroups have Haar measure. Using (i) it is shown that the set of idempotents is directed in the partial ordering $\mu \leqq \nu$ in case $\mu\nu = \nu\mu = \nu$. Then S contains a greatest idempotent, which annihilates all the elements of S, and which therefore is the Haar measure on G. (Received January 6, 1954.)

358t. J. H. Williamson: *On topologising the field $C(t)$.*

It is possible to topologise the field $C(t)$ of rational functions of the indeterminate t, with complex coefficients, so that (i) $C(t)$ is a linear topological space over the complex field, and (ii) multiplication is continuous. If local convexity is not required, there is a known metrisable topology with the required properties; regard $C(t)$ as a subspace of the set of almost everywhere finite Lebesgue measurable functions on (0, 1), with the topology of convergence in measure. It is also possible to produce a locally convex metrisable topology satisfying (i) and (ii). It can be shown that any such topology must be neither too fine nor too coarse. In particular, multiplication is not continuous in the finest locally convex topology compatible with the linear space structure of $C(t)$. (Received December 3, 1953.)

359. Karl Zeller: *Proofs of completeness.*

A general method for proofs of completeness is presented, and exhibited in a special case. The space of the functions $x(t)$, measurable in (0, 1), is complete in the known Fréchet-metric. The functional $p(x) = \int_0^1 |x(t)| dt \leqq \infty$ is lower semicontinuous in this space. It is concluded that the space (L^1) is complete with the norm $p(x)$. (Received January 18, 1954.)

360. Karl Zeller and Albert Wilansky (p): *Inverses of matrices and matrix-transformations.*

Given a matrix A with range R (a set of sequences), one may investigate the existence of right (A'), left $('A)$, and two-sided (A^{-1}) inverses for A, and for T, the transformation with matrix A, and connections between them. Typical results: A' exists iff $R \supset E = (x_n = 0$ for almost all $n)$, is unique iff T is 1-1. Either one of $'A$, $'T$ may exist without the other. If a row-finite $'A$ exists, $'T$ exists. If A is row-finite: if A' exists, there exists a row-finite A'; there exists row-finite $'A$ iff T is 1-1, if A is reversible, it has row-finite A^{-1}. If A is column finite: A' may exist and no column finite A'. MacPhail has shown that $\{C_n\}$, Banach, p. 50, may be unbounded. If A is regular, or even co-regular, it is shown that $C_n \equiv 0$. (Received January 18, 1954.)

Applied Mathematics

361. F. H. Brownell: *A lemma on rearrangements and its application to certain delay differential equations.*

Consider the nonlinear delay differential equation (1) $x'(t) = -(c/\theta)\int_0^\theta(\theta-h) \cdot \{f(x(t-h))-1\}dh$, $c>0$, $\theta>0$, over $t \geqq 0$, where $f(x)>0$ is assumed to possess a continuous derivative $f'(x)>0$ for all real x with $f(0)=1$. By using a simple modification of a theorem of Hardy, Littlewood, and Pólya on rearrangements, it is possible to prove that any solution of (1) approaches in a certain sense as $t \rightarrow +\infty$ the collection of periodic solutions of (1), and that every nonzero periodic solution of (1) must have its period $T=\theta/k$ for some integer $k \geqq 1$ and also satisfy the ordinary nonlinear differential equation (2) $\gamma = F(x(t)) - x(t) + (1/2c)[x'(t)]^2$ for some constant $\gamma > 1$, where $F(x) = 1 + \int_0^x f(y)dy$. Using the resulting formula for T as a function of γ for these periodic solutions of (2), it becomes clear that if $F(x)$ is analytic, then (1) can have only a finite number of essentially distinct periodic solutions, and in this case any solution of (1) must approach just one of these as $t \rightarrow +\infty$. (Received January 14, 1954.)

362t. Aaron Fialkow and Irving Gerst: *Bounded analytic functions in network synthesis.*

The transformations $f=(Z-1)/(Z+1)$, $z=(p-1)/(p+1)$ which convert the domain of positive real functions $Z(p)$ into the domain of so-called unit functions $f(z)$ (i.e. $f(z)$ analytic and $|f|<1$ in $|z|<1$) have been used in connection with the network synthesis of $Z(p)$. It is the purpose of this paper to exploit this relationship by considering f as the transform of Z and to investigate those elementary closed operations in the f-domain whose maps in the Z-domain lead to network realizations of Z. Among the results obtained the following are particularly simple and have wide applicability: (here f, f_1, f_2 are unit functions and Z, Z_1, Z_2 are their corresponding inverse transforms) (i) $f=(f_1-a)/(1-af_1)$, $-1<a<1$, a real, corresponds to an impedance level change in Z. (ii) A factorization $f=f_1f_2$ corresponds to the realization of Z as a series-parallel circuit involving Z_1 and Z_2. (iii) A partition $f=(f_1+f_2)/2$ corresponds to the realization of Z as a symmetric bridge network whose arms are Z_1 and Z_2. The results are applied to effect simplifications in the synthesis of positive real functions without mutual inductance. (Received January 18, 1954.)

363t. Aaron Fialkow and Irving Gerst: *Impedance synthesis without mutual inductance.*

The present paper provides a new synthesis procedure for rational positive real functions (p.r.f.) $Z(p)$ which completely eliminates the Brune minimization process. In this synthesis the zeros of the function $(*)$ $Z(p)+Z(-p)$ determine both the network structure and the computational sequence. There is no necessity for the Bott-

Duffin (Journal of Applied Physics vol. 20 (1949) p. 816) algorithm unless ($*$) has some pure imaginary zeros, in which case it need only be applied to a p.r.f. $Z_1(p)$ of reduced degree, for which all the zeros of Re $[Z_1(i\omega)]$ are real. Corresponding to the zeros of ($*$) which are not pure imaginary the method utilizes the theorem that $Z_4(p)=[1-Z_1(p)Z_2(p)]/[Z_1(p)-Z_2(p)]$ is a p.r.f. of lower degree than $Z(p)$ if Z_1 and Z_2 are a certain p.r.f. and a reactance function respectively whose construction depends upon Z and the zeros of ($*$). In this way the synthesis of $Z(p)$ is made to depend upon that of the simpler function $Z_4(p)$. If Re $[Z(i\omega)]\neq 0$ for all real ω, the same will be true of the p.r.f. $Z_4(p)$ of reduced degree at each stage of the synthesis. (Received January 18, 1954.)

364*t*. K. S. Miller: *A remark on stability.*

Let L be a linear differential operator, $H(t, y)$ its impulsive response, and $\{\phi_k(t)\}$ a fundamental set of solutions of $Lu=0$. Then it is shown that (i) $\int_0^\infty |H(t, y)|\,dt<\infty$ for all $y\in I$, and (ii) $\int_0^\infty |\phi_k(t)|\,dt\leqq M<\infty$, $k=1, 2, \cdots, n$, are coextensive. The connection between this result and the stability of linear systems is briefly discussed. (Received December 21, 1953.)

365. L. E. Payne: *On three-dimensional flows which do not possess rotational symmetry.*

The exact solution to the classical incompressible flow problem is known for only a limited number of three-dimensional bodies. A review of the literature reveals that the only such nonaxially symmetric problem, for which the exact solution has been obtained, is the flow about an ellipsoid (See Lamb, *Hydrodynamics*). F. C. Karal (Journal of Applied Physics vol. 24 (1953)) has indicated a method for obtaining the general flow about two equal separated spheres, but has not carried it through to completion. In this paper, solutions are obtained for the flow at any angle of attack about a lens, a spindle, a torus and two separated spheres. These solutions are given in terms of generalized Legendre functions. The components of the virtual mass tensor associated with the hemisphere are calculated explicitly. (Received January 14, 1954.)

Geometry

366. T. K. Pan: *Complementary surfaces for a vector field.*

Let v be a vector field in a surface in an ordinary space. The author defined the curve of v and the asymptotic line of v and proved that, at a point P on a nondevelopable surface S, they are curves along each of which the straight lines on v form generators of a developable surface [Amer. J. Math. vol. 74 (1952) pp. 955–966]. Define the orthocenter of associate curvature of v at P as a point on the straight line on v at a distance $-{}_vr_g \cos\theta$ from P, where ${}_vr_g$ is the radius of associate curvature of v along the asymptotic line of v and θ the angle at P between the asymptotic line of v and the associate curvature vector of v along the asymptotic line of v. It is found that the locus of the orthocenter as P varies is a surface which is the locus of the edges of regression of the developable surfaces consisting of straight lines on v along the asymptotic lines of v. This surface is called the surface complementary to S for v, a generalization of the surface complementary to S for a geodesic family according to Bianchi. Properties of these surfaces and related curves are studied. (Received January 5, 1954.)

367. J. J. Stoker: *On the embedding of surfaces of negative curvature in three-dimensional Euclidean space.*

Hilbert showed that an abstract two-dimensional Riemannian surface provided with a complete metric having Gauss curvature $K\equiv -1$ can not be embedded without a singularity in three-dimensional Euclidean space. The same statement holds for nonconstant Gauss curvature if $K\leqq -c^2<0$, c=constant, and if the mean curvature is bounded. (Received January 14, 1954.)

Logic and Foundations

368*t*. J. W. Addison: *Some unsolvable problems about cancellation semigroups.*

The problems of classifying the (finite) defining systems for cancellation semigroups (c.s.) according to whether or not the c.s. they define are (1) trivial (problem of triviality), (2) finite, (3) abelian, (4) groups, (5) embeddable in a group, (6) isomorphic (isomorphism problem), or (7) embeddable in one another are unsolvable. More generally so is the problem of classifying the defining systems of c.s. according to whether or not the c.s. they define have property P for any P normal in the following sense: A property P of c.s. is *normal* if P is invariant under isomorphism, if the trivial c.s. has P, and if a defining system S can be given in which no proposition of the form $A\sim AB$ holds for words A, B of S and which defines a c.s. not embeddable in any c.s. with P. The proof consists in showing that if this problem were solvable so would be the word problem (problem of identity) of a particular c.s. shown by Turing (Ann. of Math. vol. 52 (1950) pp. 491–505) to have an unsolvable word problem. The methods also prove that the metaproblem of identity for c.s. (the problem of classify ing their defining systems according to whether or not their word problems are solvable) is unsolvable. (Received January 13, 1954.)

369*t*. S. C. Kleene: *On the forms of the predicates in the theory of constructive ordinals.* II.

Correcting the first paper of this title (Bull. Amer. Math. Soc. Abstract 48-5-215), it is now shown that the predicates $a\in O$ and $a<_O b$ of the system S_3 of notation for ordinal numbers are expressible in the respective forms $(\alpha)(Ey)R(a, \alpha, y)$ and $(\alpha)(Ey)S(a, b, \alpha, y)$ where α is a variable for a 1-place number-theoretic function and R and S are primitive recursive (i.e. as predicates of their number variables they are primitive recursive uniformly in α). Conversely, every predicate of the form $(\alpha)(Ey)R(a, \alpha, y)$ where R is general recursive is expressible in the form $\phi(a)\in O$ where ϕ is primitive recursive. There is a primitive recursive linear ordering of the natural numbers in which there exists an infinite descending sequence but no infinite descending arithmetical sequence. (Received December 4, 1953.)

370. Norman Shapiro: *Recursive sets of real numbers.*

By a dyadic sequence is meant a recursive sequence of zeros and ones which does not ultimately consist of ones. Given a set S of real numbers let $P_S(x)$ mean that the sequence whose Gödel number is x represents in the binary notation an element of S, P_S being a predicate whose domain of definition is the set of Gödel numbers of dyadic sequences. If S does not contain all recursive real numbers and if S contains the rational numbers, then every predicate of the form $(\exists y)R(x, y)$, R recursive, is many-one-reducible to P_S. If S is a set of algebraic real numbers containing the rationals, then every predicate of the form $(\exists y)(z)R(x, y, z)$, R recursive, is many-one-reducible to P_S. Either of the above theorems shows that if we regard real numbers as being presented by algorithms for the generation of dyadic sequences defining them, the decision problem for such S's is recursively unsolvable. By methods similar to these, most standard subsets of the real line have, in this sense, recursively unsolvable decision problems. An example of a set with solvable decision problem is $S=\{x \mid 0 \leqq x<1/2\}$. In fact P_S is potentially partial recursive. (Received January 11, 1954.)

Statistics and Probability

371. R. P. Peterson: *Density unbiased point estimates.*

Let F be an $s+1$-parameter family of probability density functions $p(x, \theta, \sigma)$ where θ and $\sigma=(\sigma_1, \cdots, \sigma_s)$ are unknown parameters. Let $X_n=(x_1, \cdots, x_n)$ be a point in n-dimensional sample space M_n, where $x_1, \cdots, x_n$ are n (not necessarily independent) observed values of x, and let $P(X_n, \theta, \sigma)$ denote the joint probability density function at $X_n \in M_n$. Then the *mean probability density function of F generated by the point estimate $f(X_n)$ relative to θ* is given by $\phi_f(x, \theta, \sigma)=\int_{M_n} p(x, f(X_n), \sigma) P_n(X_n, \theta, \sigma) dX_n$, where $\phi_f(x, \theta, \sigma)$ is a probability density function itself provided $p(x, f(X_n), \sigma)$ is measurable in X_n over M_n. A point estimate $f(X_n)$ of θ is *density unbiased* if $\phi_f(x, \theta, \sigma) \equiv p(x, \theta, \sigma')$, where σ' is any admissible value of σ. If $p(x, \theta, \sigma) \equiv (1/\sigma)p((x-\theta)/\sigma)$, $\sigma>0$, then the characteristic function of $p(x, \theta, \sigma)$ is of the form $e^{i+\theta}h(+\sigma)$ and if $h(+\sigma)$ is closed under multiplication $(h(+\sigma_1)\cdot h(+\sigma_2)=h(+\sigma_3)$ where σ_1, σ_2, and σ_3 are admissible values of $\sigma)$, then any linear homogeneous point estimate $\sum_1^n a_i x_i$, $\sum_1^n a_i=1$ is a density unbiased estimate of θ. It is further shown that the ordinary sample mean $\bar{x}=\sum_1^n x_i$ is, in a certain sense, the "best" density unbiased point estimate for the mean of any normal population. (Received January 15, 1954.)

Topology

372t. Seymour Ginsburg and J. R. Isbell: *Uniformly convergent functions on partially ordered sets.*

A function f from a partially ordered set P to a uniform space uX [J. Tukey, *Convergence and uniformity in topology*] is called *uniformly convergent* (u.c.) if, for each covering $\{U_\alpha\}$ in u, there is a family of residual subsets Q_α of P, whose union is cofinal in P, satisfying $f(Q_\alpha) \subseteq U_\alpha$; if $\{Q_\alpha\}$ can always be chosen finite, f is *essentially bounded* (e.b.). If uX is a locally compact group then each u.c. function satisfies the defining condition for all open coverings of X. Consider the ring $U(P, R)$, and its subring $U(fP, R)$, consisting of all u.c. resp. e.b. real-valued functions on P, modulo functions converging to zero. $U(fP, R)$ is naturally ismorphic with the ring of all continuous real-valued functions on the character space βP [M. Stone, *Applications of Boolean rings to general topology*, Trans. Amer. Math. Soc. vol. 41 (1937) pp. 375–481] of the Boolean algebra of maximal residual subsets of P [S. Ginsburg, *A class of everywhere branching sets*, Duke Math. J. vol. 20 (1953) pp. 521–526]. Some residue class ring (empty in a nontrivial case) of $U(P, R)$ is isomorphic with the ring of continuous real-valued functions on a certain pseudo-discrete space uP [L. Gillman and M. Henriksen, *Concerning rings of continuous functions*, to appear in Trans. Amer. Math. Soc.]. (Received February 12, 1954.)

373t. J. R. Isbell: *Some details concerning function rings.*

Terminology as in the preceding abstract. The ring $U(P, R)$ shares with the ring of continuous functions on a pseudo-discrete space the property that every ideal is closed in the m-topology [L. Gillman and M. Henriksen, op. cit.]. Its residue class fields are all real closed; the proof for continuous functions [J. Isbell, *More on the continuity of the real roots of an algebraic equation*, Proc. Amer. Math. Soc. vol. 5] applies, by use of the result of the preceding abstract, on the locally compact group E^{2n+1}. When such a field is not the real numbers, every countable subset has an upper bound in the field ordering; a proof is given which applies as well for continuous functions, simplifying the proof of Hewitt and Henriksen [E. Hewitt, *Rings of real-valued continuous functions*, I, Trans. Amer. Math. Soc. vol. 64 (1948) pp. 45–99; M. Henriksen, Bull. Amer. Math. Soc. Abstract 60-2-319]. (Received January 15, 1954.)

374. R. K. Lashof: *Lie algebras of locally compact groups.*

An LP-group is the projective limit of Lie groups. Yamabe has shown that every connected locally compact group is an LP-group. If $G=\text{Lim } G_a$, G_a Lie groups, then the (possibly infinite-dimensional) Lie algebra g of G is the projective limit of g_a, g_a the Lie algebra of G_a. The existence and uniqueness of the Lie algebra of an LP-group is shown and also its connection with the group by means of an exponential map. The notion of a universal covering group for connected groups with the same Lie algebra is extended to LP-groups (with a modified definition of covering). Classical theorems connecting homomorphisms of group and algebra are generalized. A 1-1 correspondence is established between "canonical LP-subgroups" of a group and subalgebras of its Lie algebra. The Lie algebra of a locally compact group is shown to have the form $L\times A\times S$, where L is a finite-dimensional Lie algebra, A an (infinite-dimensional) abelian Lie algebra, and S the (infinite) direct product of simple finite-dimensional compact Lie algebras. (Received December 21, 1953.)

375*t*. P. M. Swingle: *m-divisible n-point connexes.*

Among other sets we define: let M be a connexe; then M is said to be an n-point connexe if M is not disconnected by the omission of a subset of power n; an n-point connexe M is an m-divisible n-point connexe if M is the sum of n, but not of a greater number, of mutually exclusive n-point subconnexes. Among other things is proven: if n is an integer $\geqq 2$ and M is a 1-divisible n-point connexe and also a $(4n+1)$-point connexe, then M does not have a finite cut-set; if n is an integer $\geqq 2$, M is a $(2n+1)$-point connexe in the plane with complete minimal cut-set N, and if $M-N=H+K$ strongly separate where H is disconnected and N' is a subset of power $n+1$, then $H+N'$ is an n-point connexe. (Received January 13, 1954.)

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