other two authors. In this sense, his book is the most complete of the three. Unfortunately, too many of his proofs are ungainly and complicated. Essentially, this stems from ineffective organization. In some places (compare his development of topology on the real line with Rudin's, for example) proofs could be simplified by rearranging the theorems. In other places (note the reviewer's comment on his treatment of differentiation of Lebesgue integrals) he repeats an argument several times because he fails to pull out an intermediate result which could be reused. Thielman falls in between. His proofs are not particularly striking, but standard and reasonably efficient and uncomplicated. He does not dig as deeply as Goffman, but more deeply than Rudin. The blight on Thielman's book is the surprising number of inaccurate statements and incomplete proofs. On a first reading the reviewer found ten examples, each of a flaw of one of the following types: informal statements of results that are not true, failure to consider all cases in a proof, use of concepts which have not been defined, use of results either without proof or before they have been proved. One such example was mentioned in the discussion of Fubini's theorem above. To cite one other, Thielman's proof that the union of a denumerable set of denumerable sets is denumerable tacitly assumes that the sets are disjoint.

Each of the three authors writes a very pleasing line of prose; so there is very little choice to be made on that score. Also, each book has an adequate supply of worthwhile exercises. The reviewer is unable to rate one above the other on this point.

There are a few typographical errors in each book. Those worth mentioning involve errors in cross references: Rudin—p. 74, line 22, for 4.1 read 4.2. Thielman—p. 97, line 6, for 3.2.2 read 3.2.1. Thielman—p. 184, line 7 from bottom, for 9.8.1 read 9.11.1.

M. E. MUNROE

Numerical solution of differential equations. By W. E. Milne. New York, Wiley, 1953. 11+275 pp. $6.50.

This book contains the first general treatment, in English, of numerical methods for solving differential equations. The author has been able to cover in the 275 pages only those classes of problems and methods which he considers most important. The methods are presented very clearly, with completely worked numerical examples, and should be easily mastered by the average reader. On the other hand it is evident from the choice of methods, particularly for problems involving latent roots of matrices and elliptic differential equations, that the emphasis is on the use of desk calculators rather than high speed computers. In fact, although the increased activity in the field of numerical analysis is largely due to the availability of high speed computers, these are mentioned only in the preface and in an appendix. Moreover, only one page is devoted to the treatment of round-off errors.

The first half of the book is devoted to methods for solving ordinary differential equations. Nine methods, some with variants, are presented for solving a single first-order equation. These include a method based on the trapezoidal rule, one based on Taylor's series, Picard's method, Adams' method, Moulton's method, Milne's method, and the Runge-Kutta method. Many of these methods involve the use of various numerical quadrature formulas which are derived in a special chapter on "analytic foundations." Additional methods are given for solving systems of first-order equations and initial value problems with equations of higher order. A special method is given for solving second-order equations with the first derivative absent. For each method, error estimates are derived which involve higher derivatives of the solution.

Six methods are presented for solving two-point boundary value problems. These include a method for linear equations where one solves a related initial value problem, a trial and error method, a variant of Picard's method, a method of differential variation, the Ritz method, and Galerkin's method.

The second half of the book is devoted to partial differential equations, and primarily to linear equations of second order. The author first considers problems of "explicit" type involving parabolic or hyperbolic equations where the numerical solution can be calculated step by step from the differential equation and the known initial and boundary conditions. A detailed study is made of difference equation methods for solving the heat and wave equations with two and three independent variables. Error estimates involving higher derivatives of the solution are given together with a discussion of the limitations on the size of the time interval which must be imposed for a given space interval for the sake of stability of the numerical procedure. In the reviewer's opinion the method given for treating curved boundaries is too complicated; the analysis of Collatz, reference 26, indicates that the use of four coefficients, as is usually done in practice, rather than ten, should be sufficiently accurate for most problems.

As a preliminary to the consideration of problems of "implicit" type involving elliptic equations where the unknowns are related by a system of simultaneous equations, the author devotes a chapter to methods for solving linear systems and for finding latent roots of matrices. After a brief review of matrix theory he discusses various methods of successive approximation which are based on an attempt to minimize a certain quadratic form. He considers a method involving changes in one unknown at a time, group changes, relaxation, a gradient method, and other iterative methods including one which uses Tchebycheff polynomials to accelerate the convergence. It might be well to mention that if the matrix of coefficients is positive definite, many of the methods can be simplified by the use of the quadratic form $Q(v)=(v\cdot Av)/2-(v\cdot b)$ rather than the one actually used. Among the methods for determining latent roots are the "escalator method," the use of iterated vectors, iteration and elimination, a method based on orthogonality properties of the latent vectors, and a gradient method. None of the methods given for finding latent roots seem to be suitable for large matrices.

Next, the author treats difference equation methods for solving elliptic equations in two dimensions including Laplace's equation, Poisson's equation, and the biharmonic equation. Both the usual five-point and the nine-point difference representations of the Laplacian operator are used. Special iterative methods are considered for solving the difference equations. Estimates of the accuracy of the solution of the difference equation for various mesh sizes are given involving higher derivatives of the solution. Although these estimates are not applicable in some cases, nothing better is usually available. The chapter is concluded with a nice discussion of the effect of discontinuities on the boundary.

The final chapter is devoted to methods for obtaining "characteristic numbers" both for ordinary and partial differential equations. These methods essentially involve the calculation of the characteristic equation for the associated difference equation and are not suited for use on computing machines. Three short appendices are given: (A) Round-off Errors, (B) Large Scale Computing Machines, (C) The Monte Carlo Method.

Although more emphasis on high speed computer methods would have increased the usefulness of the book, nevertheless the book is to be recommended as providing an excellent background in numerical methods. Examples for the student have been included and these, together with the clear presentation, the excellent format, and the relative freedom from misprints, make the book well adapted for classroom use.

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