THE APRIL MEETING IN NEW YORK

The four hundred ninetieth meeting of the American Mathematical Society was held at New York University in New York City on Thursday through Saturday, April 23–25, 1953. Over 600 persons attended the meeting, including the following 444 members of the Society:

C. R. Adams, L. V. Ahlfors, M. I. Aissen, A. A. Albert, R. G. Albert, F. L. Alt, D. B. Ames, S. A. Amitsur, A. G. Anderson, R. D. Anderson, R. G. Archibald, D. N. Arden, P. N. Armstrong, H. A. Arnold, Emil Artin, M. C. Ayer, George Bachman, W. G. Bade, F. E. Baker, K. Y. Bal, R. P. Bambah, W. E. Barnes, J. H. Barrett, L. K. Barrett, E. H. Batho, J. D. Baum, L. W. Baumhoff, E. G. Begle, F. D. Bennett, Jerome Berkowitz, C. H. Bernstein, D. L. Bernstein, Lipman Bers, Nicholas Bilotta, M. A. Biot, Garrett Birkhoff, D. W. Blackett, Jerome Blackman, W. E. Bleick, I. E. Block, E. K. Blum, H. W. Bode, Paul Boschan, S. G. Bourne, J. W. Bower, C. B. Boyer, A. D. Bradley, C. C. Bramble, Richard Brauer, Eleazer Bromberg, F. E. Browder, A. B. Brown, J. L. Brown, Jr., Lorenzo Calabi, F. P. Callahan, Jr., G. F. Carrier, W. C. Carter, C. R. Cassity, M. N. Chase, Y. W. Chen, Peter Chiarulli, Joshua Chover, Sarvadaman Chowla, S. I. Ciolkowski, F. M. Clarke, E. J. Cogan, Haskell Cohen, L. W. Cohen, R. M. Cohn, H. R. Cooley, G. A. Coon, T. F. Cope, Richard Courant, V. F. Cowling, M. J. Cox, H. S. M. Coxeter, J. B. Crabtree, R. H. Cramer, E. H. Crisler, A. B. Cunningham, P. M. Curran, H. B. Curry, M. L. Curtis, P. C. Curtis, Jr., J. H. Curtiss, M. D. Darkow, D. A. Darling, J. A. Daum, D. R. Davis, Philip Davis, R. B. Davis, R. M. Davis, Karel DeLeeuw, C. R. DePrima, A. H. Diamond, J. B. Diaz, C. E. Diesen, Alexander Dinghas, Avron Douglis, F. G. Dressel, R. J. Duffin, Nelson Dunford, W. L. Duren, Jr., J. S. Dwork, W. F. Eberlein, Albert Edrei, Samuel Eilenberg, C. C. Elgot, Robert Ellis, D. I. Epstein, M. P. Epstein, M. E. Estill, H. J. Ettlinger, Trevor Evans, R. M. Exner, W. H. Fagerstrom, Herbert Federer, W. E. Ferguson, F. A. Ficken, Irwin Fischer, A. D. Fleshler, E. E. Floyd, William Forman, Harold Forstat, R. M. Foster, Phyllis Fox, W. C. G. Fraser, Gerald Freilich, Bernard Friedman, M. B. Friedman, K. O. Friedrichs, Orrin Frink, Jr., R. E. Fullerton, J. W. Gaddum, M. P. Gaffney, A. S. Galbraith, David Gale, H. P. Galliher, Jr., Mariano Garcia, Jr., Boris Garfinkel, G. N. Garrison, F. W. Gehring, Hilda Geiringer, Leonard Geller, B. H. Gere, J. J. Gergen, Murray Gerstenhaber, H. A. Giddings, J. H. Giese, David Gilbarg, B. P. Gill, Wallace Givens, R. D. Glauz, Sidney Glusman, Herbert Goertzel, Karl Goldberg, Samuel Goldberg, S. I. Goldberg, J. K. Goldhaber, Saul Gorn, H. M. Griffin, H. C. Griffith, Emil Grosswald, Laura Guggenbuhl, Felix Haas, V. B. Haas, R. W. Hamming, M. E. Hamstrom, O. G. Harrold, Jr., B. I. Hart, Alvin Hausner, G. A. Hedlund, A. E. Heins, Alex Heller, I. R. Hershner, Jr., J. H. Hett, T. H. Hildebrandt, I. I. Hirschman, Jr., F. E. P. Hirzebruch, S. P. Hoffman, D. L. Holl, Carl Holtom, H. G. Hopkins, Alfred Horn, J. G. Horne, L. A. Hostinsky, A. S. Householder, W. A. Howard, C. C. Hsiung, L. C. Hutchinson, D. H. Hyers, Eugene Isaacson, H. G. Jacob, Jr., Nathan Jacobson, T. J. Jaramillo, W. S. Jardetzky, Fritz John, R. E. Johnson, F. B. Jones, Bjarni Jónsson, M. L. Juncosa, R. V. Kadison, Shizuo Kakutani, Aida Kalish, Wilfred Kaplan, S. N. Karp, W. H. Keen, D. G. Kendall, J. F. Kiefer, A. R. Kirby, M. D. Kirby, J. W. Kitchens, M. S. Klamkin, George Klein, J. R. Kline, Morris Kline, I. I. Kolodner, B. O. Koopman, Saul Kravetz, M. D. Kruskal, R. R. Kuebler, Jr., Serge Lang, R. E. Langer, P. D. Lax, C. Y. Lee, J. R. Lee, Solomon Lefschetz, Joseph Lehner, Marguerite Lehr, Marie Lesnick, M. E. Levenson, D. J. Lewis, J. A. Lewis, J. V. Lewis, B. W. Lindgren, M. A. Lipschutz, E. R. Lorch, Lee Lorch, Mark Lotkin, S. C. Lowell, C. I. Lubin, G. S. S. Ludford, Eugene Lukacs, P. B. McKowen, E. J. McShane, L. A. MacColl, R. W. MacDowell, G. W. Mackey, D. B. MacMillan, H. M. MacNeille, W. G. Madow, Wilhelm Magnus, W. R. Mann, Morris Marden, A. J. Maria, M. H. Maria, M. H. Martin, W. S. Massey, A. P. Mattuck, L. F. Meyers, Joseph Milkman, K. S. Miller, W. H. Mills, R. v. Mises, Don Mittleman, E. E. Moise, Deane Montgomery, Halina Montvila, C. N. Moore, T. W. Moore, W. E. Moore, C. S. Morawetz, Morris Morduchow, G. W. Morgan, William Moser, Wolfe Mostow, B. H. Murdoch, W. L. Murdock, C. H. Murphy, Jr., Bernard Mushinsky, John von Neumann, L. W. Neustadt, D. J. Newman, H. K. Nickerson, Katsumi Nomizu, A. B. Novikoff, F. G. O'Brien, R. E. O'Donnell, E. M. Olson, E. T. Onat, Alex Orden, Robert Osserman, Morris Ostrofsky, R. H. Owens, J. C. Oxtoby, S. V. Parter, Emanuel Parzen, L. E. Payne, L. G. Peck, A. M. Peiser, Anna Pell-Wheeler, A. J. Penico, A. S. Peters, I. D. Peters, F. P. Peterson, B. J. Pettis, C. M. Petty, R. S. Pierce, Edmund Pinney, Everett Pitcher, Morris Plotkin, H. O. Pollak, Hillel Poritsky, Aditya Prakash, C. M. Price, W. W. Proctor, M. H. Protter, Hans Rademacher, Arthur Radin, R. A. Raimi, H. E. Rauch, G. E. Raynor, M. S. Rees, Russell Remage, Moses Richardson, P. R. Rider, P. D. Ritger, Herbert Robbins, J. H. Roberts, R. A. Roberts, M. S. Robertson, G. de B. Robinson, L. V. Robinson, Mark Robinson, Robin Robinson, Selby Robinson, F. V. Rohde, G. F. Rose, I. H. Rose, M. E. Rose, David Rosen, A. S. Rosenthal, Edward Rosenthall, M. F. Rosskopf, J. P. Roth, S. G. Roth, Walter Rudin, J. P. Russell, C. W. Saalfrank, Charles Saltzer, H. E. Salzer, Hans Samelson, J. E. Sammet, Jacob Samoloff, James Sanders, W. C. Sangren, Arthur Sard, W. K. Saunders, A. B. Schacknow, A. T. Schafer, R. D. Schafer, J. A. Schatz, Samuel Schecter, Albert Schild, Abraham Schwartz, G. B. Seligman, D. B. Shaffer, Edna Sheinhart, R. W. Shephard, R. T. Shield, J. R. Shoenfield, S. S. Shu, K. M. Siegel, Robert Simon, David Singer, James Singer, Jerome C. Smith, P. A. Smith, J. J. Sopka, Clifford Spector, S. K. B. Stein, Marvin Stern, R. L. Sternberg, F. M. Stewart, J. J. Stoker, R. R. Stoll, R. L. Swain, Olga Taussky, R. L. Taylor, William Clare Taylor, Feodor Theilheimer, G. L. Thompson, D. L. Thomsen, Jr., R. M. Thrall, D. E. van Tijn, E. W. Titt, John Todd, M. L. Tomber, P. M. Treuenfels, A. W. Tucker, Bryant Tuckerman, R. J. Turyn, S. I. Vrooman, H. E. Wahlert, H. V. Waldinger, J. L. Walsh, C. Y. Wang, Jack Warga, W. R. Wasow, G. C. Webber, M. T. Wechsler, J. V. Wehausen, H. F. Weinberger, J. H. Weiner, Alexander Weinstein, Louis Weisner, Paul Weiss, Bernard Weitzer, David Wellinger, W. J. Wells, Franc Wertheimer, F. J. Weyl, G. N. White, Jr., J. H. White, Jr., A. L. Whiteman, P. M. Whitman, Hassler Whitney, G. T. Whyburn, W. M. Whyburn, H. H. Wicke, Albert Wilansky, Herbert Wilf, J. E. Wilkins, Jr., A. B. Willcox, František Wolf, N. Z. Wolfsohn, Arthur Wouk, Hidehiko Yamabe, Michael Yanowitch, D. M. Young, Jr., Arthur Zeichner, J. A. Zilber, R. E. Zindler, Leo Zippin.

A Symposium in Applied Mathematics, sponsored jointly by the Office of Ordnance Research and the Society, took place on Thursday and Friday.

On Friday, Professor W. S. Massey of Brown University delivered an invited address on *Some new algebraic methods in topology* at a general session, Professor Hans Samelson presiding. On Saturday, Professor Emil Artin of Princeton University delivered an invited address on *Cohomology and class field theory* at a general session, Professor Richard Brauer presiding.

Sessions for contributed papers were held Friday afternoon and Saturday morning and afternoon. Presiding at these sessions were Professors Y. W. Chen, F. A. Ficken, R. E. Langer, Dr. L. A. MacColl, Professors M. H. Martin, C. N. Moore, R. D. Schafer, P. R. Rider, J. H. Roberts, G. de B. Robinson.

The members of the Institute for Mathematics and Mechanics of New York University entertained at tea on Friday at 5:00 P.M.

The Council met on April 24 at 5:00 P.M. in Waverly Building, reconvening after dinner.

The Secretary announced the election of the following fifty-five persons to ordinary membership in the Society:

Mr. James Robert Boyd, San Marcos High School, San Marcos, Texas;
Mr. George Ulrich Brauer, University of Michigan;
Mr. John Leslie Chamberlin, Marquardt Aircraft Company, Van Nuys, California;
Mr. Ward Cheney, University of Kansas;
Mr. Ralph Theodore Dames, Willow Run Research Center, Ypsilanti, Michigan;
Mr. William Frank Darsow, DePaul University;
Dr. William E. Davis, Hercules Powder Company, Wilmington, Delaware;
Professor Meyer Dwass, Northwestern University;
Mr. Ben Elwood Dyer, Ben Dyer, Consultants, New York, New York;
Mr. Daniel O'Connell Etter, Tulane University;
Mr. Arthur Goeffrey Eyles, Arthur G. McKee and Company, Cleveland, Ohio;
Mr. Royal Nathaniel Fitchett, Jr., Rome Air Development Center, Griffiss Air Force Base, Rome, New York;
Dr. Bernard Abraham Fleishman, Johns Hopkins University;
Mrs. Ruth M. Frisch, Syracuse University;
Mr. Arthur Oris Garder, United Gas Corporation, Shreveport, Louisiana;
Rev. Francis Joseph Ginivan, Saint Francis Xavier University, Antigonish, Nova Scotia;
Mr. Ben Theodor Goldbeck, Jr., University of Oklahoma;
Mr. Edward P. Graney, Willow Run Research Center, Ypsilanti, Michigan;
Mr. Glenn William Graves, Willow Run Research Center, Ypsilanti, Michigan;
Mr. Norman Greenspan, Polytechnic Institute of Brooklyn;
Mr. James Stonely Hall, McGill University;
Mr. Alan Howard Halpin, Willow Run Research Center, Ypsilanti, Michigan;
Dr. Charles Dewitt Harris, Carter Oil Company, Tulsa, Oklahoma;
Mr. John Henry Holland, International Business Machines Corporation, Poughkeepsie, New York;
Mr. Robert Edwin Kalaba, The Rand Corporation, Santa Monica, California;
Mr. Martin Krakowski, Carnegie Institute of Technology;
Mr. L. Clark Lay, John Muir College, Pasadena, California;
Mr. Alfred S. Lee, Magnolia Petroleum Company, Dallas, Texas;
Mr. Roger Joseph Lemelin, Willow Run Research Center, Ypsilanti, Michigan;
Mr. Harold George Loomis, Haller, Raymond and Brown, Inc., State College, Pennsylvania;

Dr. Theodore Arceola Love, Tennessee Agricultural and Industrial State College, Nashville, Tennessee;
Mr. Paul Joseph McCarthy, University of Notre Dame;
Mr. Matthew A. Medick, International Business Machines Corporation, New York, New York;
Mr. Tito Abella Mijares, University of Santo Tomas and University of the East, Manila, Philippine Islands;
Mr. Jack Culbertson Miller, Pomona College;
Mr. J. Sayer Minas, University of Illinois;
Mr. Norton Leonard Moise, University of California, Los Angeles, California;
Mr. Raphael Morena-Castro, Western Reserve University;
Mr. Robert Marvin Natkin, Chicago, Illinois;
Miss Donna May Neeb, University of Michigan;
Mr. Monroe Lawrence Norden, Johns Hopkins University;
Mr. Edward Joseph Pellicciaro, University of North Carolina;
Mr. John William Petro, University of Chicago;
Professor Edwin James George Pitman, University of Tasmania;
Mr. Ivan Paul Polonsky, University College, New York, New York;
Mr. Alfred Reichenthal, Arma Corporation, Brooklyn, New York;
Mr. Robert Edward Ross, Avco Manufacturing Company, Bethel, Connecticut;
Mr. Henry H. Ryffel, The Industrial Press, New York, New York;
Mr. Seymour Sherman, Bell Telephone Laboratories, Whippany, New Jersey;
Mr. Robert La Vern Slater, Jr., Chicago Midway Laboratories, Chicago, Illinois;
Mr. Victor Richard Staknis, Boston University;
Mr. Gene Thomas Thompson, Oregon State College;
Professor V. P. Venkatachari, Osmania University, Deccan, India;
Mr. Julius Widrewitz, Griffiss Air Force Base, Rome, New York;
Mr. Herbert Wilf, Columbia University.

It was reported that the following one hundred ninety-three persons had been elected to membership on nomination of institutional members as indicated:

University of Alabama: Miss Susie Lee Ward.

University of British Columbia: Mr. Wilfred Eaton Barnes, Mr. Charles Andrew Swanson, and Mr. Donald Alastair Trumpler.

Brown University: Mr. Harry Geoffrey Hopkins.

California Institute of Technology: Mr. William Daniel Dean, Mr. Wesley R. Guebert, Mr. Juris Hartmanis, Mr. John Beverley Johnston, and Mr. Rodrigo Alvaro Restrepo.

University of California, Berkeley: Mr. Chen-Chung Chang, Mr. Arthur Shapiro, and Mr. George Powell Steck.

University of California, Los Angeles: Mr. Ali Reza Amir-Moez, Miss Olive Jean Dunn, Mr. Harold Parkins Edmundson, Mr. Richard Carl Gilbert, Mr. Sheldon Green, Mr. Charles John August Halberg, Jr., Mr. Eugene Levin, Mr. Mervin F. Muller, and Mr. Chien Wenjen.

Carnegie Institute of Technology: Mr. Carlton Edward Lemke.

University of Chicago: Mr. Jacob Feldman, Mr. Malcolm Goldman, Mr. Samuel Shaheen Holland, Jr., Mr. Brindell Horelick, Mr. Royal Bruce Kellogg, Mr. Richard Leroy McKinney, Mr. Michael Darwin Morley, Mr. Edward Nelson, Mr. Robert Harvey Oehmke, Mr. Eugene Carroll Paige, Jr., Mr. Morris Schreiber, Mr. Jerome Spanier, and Mr. Elias M. Stein.

University of Cincinnati: Mr. James Massey Shaheen.

City College of New York: Mr. Harvey Philip Greenspan and Mr. Samson Mordecai Rosenzweig.

Columbia University: Mr. S. David Berkowitz, Mr. Douglas Grassel Dickson, Dr. Alexander Dinghas, Mr. Max Beerbohm Friedman, Mr. Donald Bashford MacMillan, Mr. Mark Robinson, and Miss Miriam Schapiro.

Cornell University: Mr. Raquel Rosa Heller, Mr. Martin Baer Herskovitz, Mr. David Hertzig, Mr. Leonard F. Kilian, Mr. Elliott Mendelson, Mr. Jerome Sacks, and Mr. David Edward Schroer.

Duke University: Mr. William Jeffrey Coles and Mr. William Rodger Smythe, Jr.

Haverford College: Mr. Robert Trull Ives.

University of Illinois: Mr. Samuel Eli Benesch.

Indiana University: Mr. Theodore Henry Miller Crampton, Mr. James A. Nickel, Mr. Joseph Francis Schell, Miss Jane Anna Uhrhan, and Mr. Robert Weiller.

Institute for Advanced Study: Dr. Shimshon Avraham Amitsur, Dr. Ram Prakash Bambah, Dr. Armand Borel, Dr. Morikuni Goto, Dr. Friedrich Ernst Peter Hirzebruch, Dr. Richard Lee Ingraham, Dr. Gopinath B. Kallianpur, Dr. Yukiyosi Kawada, Dr. Shigeo Sasaki, and Mr. Hidehiko Yamabe.

Iowa State College: Mr. Buchanan Cargal.

Johns Hopkins University: Mr. Robert W. Bass, Mr. Juri Vello Nou, Mr. Raymond Joseph Pipino, Mr. Mark E. Stern, Mr. Shlomo Zvi Sternberg, and Mr. August Martin Wildberger.

University of Kansas: Mr. Kenneth Robert Lucas, Mr. Isaac Namioka, and Dr. Shambhu Dayal Sinvhal.

Kenyon College: Mr. Robert George Busacker.

University of Maryland: Mr. Karl Heinz Dieter and Mr. Benjamin Yee-Chieh Koo.

Massachusetts Institute of Technology: Miss Evelyn Mary Bender, Mr. Harry Floyd Davis, II, Miss Phyllis Fox, Mr. William Brunner Kehl, Mr. Ernest Bronson Leach, and Mr. Jacob Joseph Levin.

Michigan State College: Mr. John Lucian Bagg, Mr. Alton Thomas Butson, Mr. William Gerald Franzen, Mr. Hugo Alexander Myers, Mr. Orrin Edison Taulbee, and Mrs. Patricia James Wells.

University of Michigan: Mr. William Price Brown, Mr. Walter Feit, Mr. Ronald Kay Getoor, Mr. James Raymond Munkres, and Mr. James Maxwell Osborn.

University of Minnesota: Mr. Glen Earl Baxter, Mr. Charles McMurray Braden, Mr. Leon Brown, Mr. Robert Ernest Fagen, Mr. Ernest Raymond Johnston, Mr. Jesse Marshall Shapiro, and Mr. David Zeitlin.

New York University: Mr. Halina Montvila, Mr. Paul David Ritger, and Mr. Peter Ungar.

Northwestern University: Mr. Robert Rolf Christensen.

Ohio State University: Mr. Albert George Fadell, Mr. Robert Morton Haber, Mr. Billy O. Hoyle, Miss Verna Frances Lair, Mr. Shen Lin, Mr. Christoph Johannes Neugebauer, Miss Phyllis Rubin, and Mr. Robert James Thompson.

Oklahoma Agricultural and Mechanical College: Professor Ladislaus J. Fila, Professor Franklin Arno Graybill, and Mr. Aboulghassem Zirakzadeh.

University of Oregon: Mr. Hubert Edwin Chrestenson.

University of Pennsylvania: Mr. Willard Ellis Baxter, Mr. Herbert Morton Gurk, Mr. Burrowes Hunt, Mr. Justin Jesse Price, Mr. Jerome Raymond Ravetz, and Mr. Richard Edmund Williamson.

Princeton University: Dr. Darrel Don Aufenkamp, Mr. Walter Lewis Baily, Mr. Albert Andrew Folop, Mr. John Blackman Fraleigh, Mr. Ralph Edward Gomory, Dr. Stefano Guazzone, Dr. Victor K. A. M. Gugenheim, Mr. Robert Clifford Gunning, Mr. Douglas Weir Hall, Dr. Harlan Duncan Mills, Mr. Sigurdur Helgason, Mr. Brian Hughs Murdoch, Dr. Albert Nijenhuis, Mr. Carl Reading Ohman, Mr. Charles David Parker, Mr. Franklin Paul Peterson, Dr. Clinton Myers Petty, Mr. Barth Pollak, Mr. Bruce Lloyd Reinhart, Mr. Kiron Chandra Seal, and Mr. Samuel James Taylor.

Purdue University: Mr. Robert Arnold Gambill.

Queens College: Mr. Maurice Leonard Richter and Miss Florence Spatz.

Rice Institute: Mr. Thomas Muir Gallie, Jr., and Mr. James Alexander Hummel.

Rutgers University: Mr. John Bender.

College of Saint Thomas: Mr. Paul John Nikolai.

Syracuse University: Mr. Sullivan Graham Campbell, Mr. Edwin Foote Gillette, Mr. Charles Stanley Ogilvy, and Miss Jacqueline Lax Zemel.

University of Tennessee: Miss Gertrude Ehrlich.

University of Texas: Mr. Clair Eugene Abraham, Mr. Steve Armentrout, Mr. James Warren Evans, Mr. John Theodore Mohat, Mr. Charles Stuart Stone, and Mr. James Newton Younglove.

University of Toronto: Mr. Irwin Guttman, Mr. Katsumi Okashimo, Mr. Seymour Schuster, Mr. David Arthur Sprott, Mr. Jakob Abraham Steketee, and Mr. Murray Scott Watkins.

University of Washington: Mr. James Harold McKay and Mr. Douglas Vern Newton.

Wellesley College: Miss Isabella Waldie.

Williams College: Mr. William Hollis Peirce.

University of Wisconsin: Mr. John West Addison, Jr., Mr. Edward H. Batho, Miss Anna Chandapillai, Mr. Nicholas D. Kazarinoff, Mr. Guydo Rene Lehner, Mr. Bruce Leon Lercher, Mr. Robert William McKelvey, Mr. Martin Herbert Pearl, Mr. Carl Joseph Vanderlin, and Mr. Roger Norman Van Norton.

Yale University: Mr. Philip Chadsey Curtis, Mr. Paul Ernest Klebe, Jr., and Mr. Thomas Allen Paley.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematikervereinigung: Dr. Arno Jaeger, University of Illinois, and Professor Wilhelm Süss, University of Freiburg; Société Mathématique de France: Dr. Louis Camille Maurice Nollet, Université de Liège, and Dr. Jamil Ahmad Siddiqi, Université de Paris; Svenska Matematikersamfundet: Mr. Göran Björck, University of Stockholm, Professor Lars Göran Borg, Royal Institute of Technology, Professor Otto Albin Frostman, University of Stockholm, and Mr. Arne Elis Pleijel, Kommunala Realskolan, Hagfors, Sweden.

The following appointments by the President were reported: as a committee to nominate officers and members of the Council for 1954: Professors Marston Morse (Chairman), J. C. Oxtoby, A. C. Schaeffer, A. E. Taylor, and H. S. Wall; as a member of the Committee on Applied Mathematics: Garrett Birkhoff (Committee now consists of M. H. Martin (Chairman), R. V. Churchill, F. J. Murray, Eric Reissner, Shizuo Kakutani, and Garrett Birkhoff); as a member of the committee to recommend to the Council the award of the Bôcher Memorial Prize: B. J. Pettis (Committee now consists of L. V. Ahlfors (Chairman), Philip Franklin, and B. J. Pettis); as a committee to select Gibbs lecturers for 1954 and 1955: A. A. Albert (Chairman), J. L. Doob, and C. B. Morrey; as a committee on corporate memberships: C. C. Hurd (Chairman), M. M. Flood, and B. P. Gill; as a committee on arrangements for the summer meeting of 1953: Norman Miller (Chairman), L. W. Cohen, H. A. Elliott, H. M. Gehman, Israel Halperin, R. L. Jeffery, Percy Lowe, and F. M. Wood; as a committee on arrangements for the meeting to be held at Wofford College on November 27–28, 1953: T. L. Jordan (Chairman), John Hill, G. May, E. H. Shuler, and W. M. Whyburn.

The following appointments to represent the Society were reported: Professor C. R. Wylie, Jr. at the inauguration of J. Richard Palmer as Seventh President of Westminster College on January 9, 1953; Professor Walter Strodt at the inauguration of Buell Gordon Gallagher as Seventh President of the City College of New York on February 19, 1953; Rev. J. J. Lynch at the Centenary Celebration of Manhattan College on April 25, 1953; and Professor R. L. Moore at the dedication of Benedict Hall on the campus of the University of Texas on April 13–14, 1953.

The following items were reported for the information of the Council: selection of J. L. Doob as Managing Editor of the Transactions and Memoirs Editorial Committee; Leo Zippin as Chairman of the Mathematical Surveys Editorial Committee; G. B. Price as Managing Editor of the Bulletin Editorial Committee; G. A. Hedlund as Chairman of the Proceedings Editorial Committee; William Feller as Chairman of the Mathematical Reviews Editorial Committee (R. P. Boas has been approved as a substitute for Professor Feller during the period February–August, 1953); A. A. Albert as Chairman of the Colloquium Editorial Committee; C. J. Rees as Chairman of the Committee on Printing and Publishing; J. W. Green to substitute for Professor Feller as Chairman of the Committee to Study Problems of Nominations by Petition; reciprocity agreements have been concluded with the Indian Mathematical Society, the Finnish Mathematical Society, and the Icelandic Mathematical Society; the Committee on Applied Mathematics has voted in favor of postponing the Seventh Symposium in Applied Mathematics to 1955; the McGraw-Hill Book Company has agreed to publish the Proceedings of the

Sixth Symposium in Applied Mathematics; Professor Wassily Leontief has accepted an invitation to deliver the Josiah Gibbs Lecture at the Annual Meeting in 1953; committees to select hour speakers have invited Professor Ernst Snapper to deliver an address at the Stanford, California meeting on May 2, 1953; Professor A. T. Lonseth, Missoula, Montana meeting on June 20, 1953; Professor Salomon Bochner at the Summer Meeting at Kingston, Ontario.

The following actions taken by mail vote of the Council were reported: election of Professors E. E. Moise and J. C. Oxtoby to serve as members of the Executive Committee of the Council for a period of two years beginning January 1, 1953; acceptance by the Council of an invitation from Wofford College, Spartanburg, South Carolina, to hold a meeting there on November 27–28, 1953; and acceptance by the Council of an invitation from the Johns Hopkins University to hold the Annual Meeting in 1953 in Baltimore.

The Council voted to approve the following dates of meetings of the Society: October 24, 1953 at Columbia University; February 27, 1954 in New York City; April 23–24, 1954 at Columbia University; and November 26–27, 1954 at the State University of Iowa.

The Executive Director reported that the National Science Foundation has made a grant to the Society of $6300 to defray expenses of an experimental project to explore the possibilities of publication and distribution of the results of research in the field of mathematics. The Council voted to authorize the President to appoint a committee to advise with the Executive Director concerning this project.

The Secretary reported that the Policy Committee for Mathematics would be asked to nominate a member of the committee to evaluate the present functions and operations of the Bureau of Standards in relation to the present national needs.

The Council voted to approve holding the Sixth Symposium in Applied Mathematics at the City College in Santa Monica, California.

The Council voted to co-sponsor with the Committee on Training and Research in Applied Mathematics of the National Research Council a Conference on Training in Applied Mathematics at the time of the October 1953 meeting in New York and a Conference on Topics in Applied Mathematics at the time of the November 1953 meeting in Evanston, Illinois.

The Council voted to cooperate with the National Science Foundation on a roster of mathematicians and voted to authorize the President to appoint a committee to make recommendations to the Council concerning the size and scope of this roster.

The Council voted to remove its restriction on the exhibition of books at Summer and Annual Meetings that these books be at the graduate level or higher.

The Symposium consisted of the following papers, invited by the Office of Ordnance Research:

(1) M. J. Lighthill: *Mathematical methods in compressible flow theory.*

(2) G. F. Carrier: *Boundary layer problems in applied mathematics.*

(3) Garrett Birkhoff: *Fourier synthesis of homogeneous turbulence.*

(4) G. S. S. Ludford and M. H. Martin: *Non-isentropic flows in a shock tube.* (Presented by Professor Ludford.)

(5) J. H. Giese: *Approximate methods for computing flow fields.*

(6) Lipman Bers: *Results and conjectures in the mathematical theory of subsonic and transonic flows.*

(7) Alexander Weinstein: *The singular solutions and the Cauchy problem in the hodograph method.*

(8) P. Germain: *Remarks on the theory of partial differential equations of mixed type and applications to the study of transonic flows.*

(9) R. v. Mises: *Discussion on transonic flow problems.*

(10) John von Neuman: *Role of computing machines.*

(11) Mark Lotkin: *Some problems on computing machines.*

(12) P. D. Lax: *Generalized solutions of non-linear equations and their numerical calculation.*

(13) L. H. Thomas: *Computation of one dimensional flows including shocks.*

(14) G. E. Hudson: *Deformation of a metallic shell of non-uniform thickness by a detonating wave.*

(15) Hubert Schardin: *Measurements of spherical shock waves.*

Presiding at the sessions of the Symposium were Dr. C. C. Bramble, Dr. C. W. Lampson, Mr. C. L. Poor, Dr. T. E. Stern.

Abstracts of the papers presented are listed below. Where a paper has more than one author, that author whose name is followed by "(p)" presented it. Those papers with "*t*" following their numbers were read by title. Professor Fraïssé was introduced by Professor Alfred Tarski, Dr. Fleishman by Professor F. A. Ficken, Mr. Berg by Professor Lipman Bers, and Mr. Siry by Professor D. W. Hall.

Algebra and Theory of Numbers

323t. I. T. A. C. Adamson: *Cohomology groups of coset spaces.*

Let G be a group, H a subgroup, not necessarily normal. The left cosets of G modulo H are denoted by $A_\nu=\bar{\sigma}_\nu H$. A "coset space complex" K is constructed, the q-chain group of which, $C_q(K)$, is freely generated by the $(q+1)$-tuples of cosets

$[A_0, \cdots, A_q]$; these groups are made into G-modules by setting $\sigma[A_0, \cdots, A_q] = [\sigma A_0, \cdots, \sigma A_q]$ and extending the action of G to $C_q(K)$ by linearity. The boundary homomorphisms are defined by setting $\partial[A_0, \cdots, A_q] = \sum_{\nu=0}^{q} (-1)^\nu [A_0, \cdots, A_{\nu-1}, A_{\nu+1}, \cdots, A_q]$, and extending by linearity. The complex is augmentable and acyclic, but not free. If A is a G-module, the q-cochains of K are the G-homomorphisms of $C_q(K)$ into A; cohomology groups are defined in the usual way. The following results are obtained: (1) If $0 \to A' \to A \to A'' \to 0$ is an exact sequence of G-modules such that $H^1(U, A') = 0$ for all subgroups U of H, then there is an exact sequence $\to H^r(K, A) \to H^r(K, A'') \to H^{r+1}(K, A') \to H^{r+1}(K, A) \to$ for all r; (2) If A is a G-module such that $H^r(U, A) = 0$ for $r = 1, 2, \cdots, n-1$, and all subgroups U of H, then the sequence $0 \to H^n(K, A) \to H^n(G, A) \to H^n(H, A)$ is exact, where the homomorphisms are those induced by injection, inflation, and restriction respectively. (Received March 9, 1953.)

324*t*. I. T. A. C. Adamson: *Cohomology groups for non-normal fields.*

Let k be a field for which local or global class field theory holds. Let E be a finite extension of k, not necessarily normal. Let $A(E)$ be a group invariantly attached to E—e.g. the multiplicative group of E or the group of idèles or idèle classes of E. Let F be a normal extension of k containing E, G its Galois group, H the subgroup of G which cuts out E, K the coset space complex of G modulo H. Then the cohomology groups $H^r(A(E))$ are defined to be $H^r(K, A(F))$; they do not depend upon the normal extension F; when E is normal with Galois group G', $H^r(A(E)) = H^r(G', A(E))$. Let $A(E)$ be the multiplicative group of E (local case) or the group of idèle classes of E (global case); then (1) $H^1(A(E)) = 0$; (2) $H^2(A(E))$ is cyclic of order equal to the degree of E over k, generated by c^m, where c is the canonical generator of $H^2(G, A(F))$ and m is the index of H in G; (3) If I is the ideal of the integral group ring of G consisting of elements with sum of coefficients zero, then for $r > 0$, $H^r(A(E))$ is isomorphic to $H^{r-1}(K, I)$. (Received March 9, 1953.)

325*t*. Leonard Carlitz: *q-Bernoulli and Eulerian numbers.*

Frobenius (Sitzungsberichte der Preussischer Akademie der Wissenschaften, 1910, pp. 809–847) showed that many properties of the Bernoulli and related numbers can be derived from the Eulerian function $H_m(x)$ defined by $(H+1)^m = xH^m$, $m \geqq 1$. In the present paper we show that much the same can be done for the q-Bernoulli numbers (Duke Math. J. vol. 15 (1948) pp. 987–1000) by means of a generalized function $H_m(x, q)$ defined by $(qH+1)^m = xH^m$, $m \geqq 1$. In particular if we place q equal to a rational number, then we obtained analogues of the Staudt-Clausen theorem and of Kummer's congruences. (Received February 25, 1953.)

326*t*. Leonard Carlitz: *Some congruences of Vandiver.*

Results of the following kind are proved. Let $\{a_{i,m}\}$, $i = 1, \cdots, k$, denote sequences of integers such that $a_i^m(a_i^{p-1} - 1)^r \equiv 0 \pmod{p^m, p^r}$, where after expansion a_i^m is replaced by $a_{i,m}$ and p is a prime; also let $\lambda_1 + \cdots + \lambda_k \equiv 0 \pmod{p}$. Then $a_1^{m_1} \cdots a_k^{m_k}(\lambda_1 a_1^{p-1} + \cdots + \lambda_k a_k^{p-1}) \equiv 0 \pmod{p^{m_1}, \cdots, p^{m_k}, p^r}$. These results generalized certain congruences proved by Vandiver (Bull. Amer. Math. Soc. vol. 43 (1937) pp. 418–423). (Received February 25, 1953.)

327*t*. Leonard Carlitz: *Some hypergeometric congruences.*

By specializing the parameters in known hypergeometric identities, a large number of congruences are obtained which may be thought of as congruential analogues of various formulas of Ramanujan. For example we cite $(1/p) \sum_0^m (8r+1)$

$\cdot(1\cdot 5 \cdots (4r-3)/4\cdot 8 \cdots 4r)^4 \equiv (1\cdot 3\cdot 5 \cdots (4m-1)/(m!)^2$ (mod $p=4m+1$), $\sum_0^m (-1)^r(4r+1)\ (1\cdot 3 \cdots (2r-1)/2\cdot 4 \cdots 2r)^3 \equiv -2^{-1}(1\cdot 3 \cdots (2m-1)/2\cdot 4 \cdots 2m)$ (mod $p=4m+3$), where the moduli are primes. (Received February 25, 1953.)

328*t*. Leonard Carlitz: *The class number of an imaginary quadratic field.*

Let $h(d)$ denote the class number of the imaginary quadratic field $R(d^{1/2})$ of discriminant d. Let p be an odd prime divisor of d and $n \geqq 1$. In this paper we find the residue of $h(d)$ (mod p^n) in terms of Bernoulli polynomials. (For $n=1$ see A. Hurwitz, *Mathematische Werke*, vol. 2, Basel, 1933, pp. 208–235). (Received February 25, 1953.)

329*t*. Leonard Carlitz: *The coefficients of singular elliptic functions.*

Let sn $x=$ sn (x, k^2) be an elliptic function that admits of complex multiplication and let the period quotient belong to the imaginary quadratic field of discriminant d. Put sn $x=\sum_0^\infty \alpha_{2m+1}x^{2m+1}/(2m+1)!$, $x/\text{sn } x=\sum_0^\infty \beta_{2m}x^{2m}/(2m)!$. Then if p is an odd prime such that $(d/p)=-1$, we show that $\alpha_m \equiv 0$ (mod p^r) for $m \geqq pr$; $\beta_m \equiv 0$ (mod p^{r+s}) for $p^s | m$, $m > pr$, $p^2-1 \nmid m$; $\beta_m \equiv 0$ (mod p^{r-s-1}) for $p^s | m$, $p^{s+1} \nmid m$, $m > pr$, $p^2-1 | m$. (Received February 25, 1953.)

330*t*. Leonard Carlitz: *The Schur derivative of a polynomial.*

Let $f(x)=f(x_1, \cdots, x_k)$ denote a polynomial in k indeterminates with integral coefficients. Define $\Delta f^{p^m}(x)=f'_m(x)=(f^{p^{m+1}}(x)-f^{p^m}(x^p))/p^{m+1}$, $\Delta^{r+1}f^{p^m}(x)=f_m^{r+1}(x)$ $=(f_{m+1}^{(r)}(x)f^{p^{m+1}}(x)-f_m^{(r)}(x)f^{p^{m+r}}(x^p))/p^{m+1}$. Generalizing the results of Schur it is shown that $\Delta^r f^{p^m}(x)$ has integral coefficients for $1 \leqq r \leqq p-1$. Moreover the residue of $\Delta^r f^{p^m}(x)$ (mod p^m) is determined. (Received February 25, 1953.)

331*t*. Leonard Carlitz: *Weighted quadratic partitions.*

Let p be a prime >2, and put $S=\sum \exp\{2\pi i(2\lambda_1 x_1+\cdots+2\lambda_t x_t)/p^r\}$ summed over all integers $x_1, \cdots, x_t$ (mod p^r) such that $a_1x_1^2+\cdots+a_tx_t^2 \equiv c$ (mod p^r). Assuming that the λ's are integers and that the a_i are prime to p it is shown that S can be expressed in terms of Gauss and Kloosterman sums. If $(\lambda_1, \cdots, \lambda_t, p)=1$, the results are particularly simple. (Received March 6, 1953.)

332*t*. K. T. Chen, R. H. Fox, and R. C. Lyndon: *On the quotient groups of the lower central series.*

A workable algorithm is developed for computing the lower central quotients $Q_n(G)=G_n/G_{n+1}$ of a group $G=G_1$. This rests upon finding, for a free group F, a basis for Hom $(Q_n(F), Z)$, where $Z=$ integers, consisting of certain higher differential operators in the free (Fox) calculus. Bases for Hom $(Q_n(F), Z)$ and, simultaneously, for $Q_n(F)$ are derived from combinatorial results on lexicographical order and the operation of "shuffling" two sequences. By-products: a direct proof of the Magnus-Witt theorem that F_n is isomorphic with the nth dimension group [W. Magnus, J. Reine Angew. Math. vol. 177 (1937) pp. 105–115, E. Witt, ibid. pp. 152–160]; determination of all finitary relations that hold identically among the coefficients in a Magnus power series of a group element. (Received March 3, 1953.)

333. Sarvadaman Chowla (p) and W. E. Mientka: *The rational points on a cubic curve.*

It is proved that the curve $y^2=x^3-Ax-B$ has no "exceptional" points if $A \equiv 0$ (2), $B\equiv 2$ (4). (Received March 9, 1953.)

334t. Harvey Cohn: *Density of Abelian cubics.*

The author shows that the number of *abelian* cubic fields of discriminant $\leqq x$ is $\sim$ const. $x^{1/2}$, so that, with repetitions counted in proper multiplicity, the discriminants are about as dense as perfect squares. This result is shown by the use of conventional "zeta-function methods," applied to $\prod(1-2p^{-s})=f(s)$ (over $p\equiv 1$ mod 3). It complements unpublished results kindly communicated by Dr. H. Davenport, to the effect that the number of *all* cubic fields of absolute discriminant $\leqq x$ is $O(x)$, and by Dr. H. Heilbronn to the effect that the number of *real* non-abelian cubics of discriminant $\leqq x$ is at least as large as const. x. (Research sponsored by the Army Office of Ordnance Research.) (Received March 9, 1953.)

335t. D. W. Dubois: *Primitivity, strict reality and the condition of Clifford in partly ordered fields.*

Let F be partly ordered with positive cone P. F is said to be (a) *primitive* if no proper subfield of F contains P; (b) *strictly real* if $x^2\geqq 0$ holds for all $x\in F$; (c) *Archimedean in the sense of Clifford* (p.o.a.c.) if $nx<b$, $n=1, 2, \cdots$, implies $x\leqq 0$. Among others, the following theorems are proved. (1) F is primitive if and only if it is the field of quotients of its ring B where B is the set of all $x\in F$ for which there are integers m, n with $m<x<n$; (2) F is strictly real if and only if its cone P is the intersection of a nonempty family $\mathfrak{P}$ of positive cones belonging to simple orderings of F; (3) F is primitive and p.o.a.c. if and only if F is strictly real with a family $\mathfrak{P}$ having the property that if x belongs to any $P'\in\mathfrak{P}$ then there is a positive rational $\gamma=\gamma(x)$ such that $x-\gamma$ belongs to some $P''\in\mathfrak{P}$. Corollaries to (2) are: (2a) Every formally real field G has a simple ordering; (2b) In a formally real field G, the totally positive elements are just those expressible as sums of squares. (Received February 23, 1953.)

336t. Trevor Evans: *An embedding theorem for semigroup with cancellation.*

This note is concerned with the following question. Is there a finite integer n such that any countable cancellation semigroup can be embedded in a cancellation semigroup generated by n elements? From previous theorems of this type one is led to expect that $n=2$ but this is not the case. An example is given of a countable cancellation semigroup which cannot be embedded in a two-generator cancellation semigroup. However, the following theorem can be proved. Any countable cancellation semigroup not possessing subsemigroups which are groups (apart from a possible unit element) can be embedded in a cancellation semigroup generated by two elements. A plausible conjecture for the case of countable cancellation semigroups containing subgroups is that such semigroups can be embedded in cancellation semigroups generated by six elements. The remainder of the paper is concerned with the embedding of countable rings in rings generated by two elements and some remarks on the manner in which loops can be embedded in monogenic loops. (Received March 9, 1953.)

337. Roland Fraïssé: *On certain relations which generalize ordering relations of type η.*

Let $A(x_1, \cdots, x_n)$ be an n-ary relation of base E (a function defined on E^n assuming two values). We say that A is *homogeneous* if, given any two restrictions A_1, A_2

of A, each with finite base, any isomorphism of A_1 onto A_2 can be extended to an automorphism of A. Write $A > B$ or $B < A$ if B is isomorphic to a restriction of A. Let Γ_A be the family of relations R with finite base such that $R < A$. All relations considered are assumed to have finite or denumerable bases. The following are proved: (1) There exists a homogeneous n-ary relation H with $H > R$ for every n-ary relation R; H is unique up to isomorphism. (2) If H is homogeneous and $\Gamma_A \subseteq \Gamma_H$, then $A < H$. (3) If H and K are homogeneous and $\Gamma_H = \Gamma_K$, then H and K are isomorphic. (4) A family Γ of relations with finite bases is a Γ_H (with H homogeneous) iff (i): if $A \in \Gamma$ and $B < A$, then $B \in \Gamma$, and (ii): if A, $B \in \Gamma$ and have the same restriction to the intersection I of their bases, then there exist $C \in \Gamma$ and isomorphisms ϕ and ϕ' of A and B resp. onto restrictions of C, such that $\phi(x) = \phi'(x)$ for $x \in I$. (Received March 11, 1953).

338. Emil Grosswald: *On diophantine approximations.*

Let α be an irrational number, $\alpha = [\theta_1, \theta_2, \cdots, \theta_k, \cdots]$ its expansion in a hermitian continued fraction, p_k/q_k the convergent preceding θ_k; let F_k be the set of fundamental regions R_{kj} of the modular group, of common cusp p_k/q_k. The line $L: x = \alpha$ meets infinitely many F_k and in each of them $n_k = \theta_k + 1$ fundamental regions R_{kj}. Let $V_{kj}\{z\} = (q_j' z - p_j')/(q_k z - p_k)$ be the modular transformation that maps R_{kj} onto $R_0(|z| \geqq 1, |\Re z| \leqq 1/2)$, L_{kj} the segment of L in R_{kj}, $z = \alpha + iy_{kj}$ a point of L_{kj}. Then $V_{kj}\{L_{kj}\} = \gamma_{kj}$, an arc of circle in R_0, and $V_{kj}\{z\} = x'_{kj} + iy'_{kj}$ with $y'_{kj} = y_{kj}/Q$, $Q = (q_k\alpha - p_k)^2 + q_k^2 y_{kj}^2 \geqq 2|y_{kj}q_k(q_k\alpha - p_k)|$ so that (1) $|\alpha - p_k/q_k| \leqq 1/2q_k^2 y'_{kj}$. Using properties of V_{kj} it follows that for proper choice of z, (2) $y'_{kj} \geqq 2^{-1}\{(\theta_k - 1)^2 + m^2\}^{1/2}$, where $m \geqq 3^{1/2}$ and, under certain conditions, $m \geqq 2$. From (1) and (2) follows: (3) $|\alpha - p_k/q_k| \leqq q_k^{-2}\{(\theta_k - 1)^2 + m^2\}^{-1/2}$. Observing that (a) $\theta_k \geqq 1$ always holds; (b) $\theta_k \geqq 2$ infinitely often; (c) $\theta_k \geqq 3$ infinitely often, unless $\alpha \sim (-1 + 5^{1/2})/2$, (3) with $m = 2$ yields (a) the characteristic property of hermitian convergents; (b) and (c) two theorems of Hurwitz (Math. Ann. vol. 39 (1891) pp. 279–284). In the general case, (3) is the analogue for hermitian continued fractions of a classical inequality for regular continued fractions. It is conjectured that $m = 2$ infinitely often for every fixed, infinitely repeated θ_k of an irrational α. (Received March 2, 1953.)

339t. D. G. Higman: *Indecomposable representations at characteristic p.* Preliminary report.

Let S be a subgroup of the finite group G, Ω a set. By M_S we denote the S-Ω-module induced by a given G-Ω-module M, and by M_S^G the G-Ω-module induced by M_S. The elements of M_S^G are the formal linear combinations $\sum_{x \in L} x \cdot u_x$, where L is a set of left representatives for G over S, and u_x is in M. The mapping α: $\sum x \cdot u_x \to \sum x u_x$ is a G-Ω-homomorphism of M_S^G onto M, whereas β: $u \to \sum x \cdot x^{-1}u$ for u in M is a G-Ω-isomorphism of M onto a G-Ω-submodule $\tilde{M}$ of M_S^G. Furthermore, $u\beta\alpha = (\sum x \cdot x^{-1}u) = \sum u = [G:S]u$ [cf. B. Eckmann, Bull. Amer. Math. Soc. Abstract 58-3-278]. If S contains a p-Sylow subgroup of G, Ω is a field of characteristic p, and M has finite dimension over Ω, then $\alpha(\beta\alpha)^{-1}\beta$ is a decomposition operator of M_S^G, giving $M_S^G \simeq M \oplus \{M_S^G - \tilde{M}\}$. Hence, by the Remak theorem, each indecomposable representation of G over Ω is a component of a representation induced by an indecomposable representation of a p-Sylow subgroup of G. Corollaries: If G has cyclic p-Sylow subgroups, then the number of inequivalent indecomposable representations of G over Ω is finite. If G has non-cyclic p-Sylow subgroups and Ω is infinite, then G has infinitely many inequivalent indecomposable representations. The degrees of the indecomposable representations of G are bounded if and only if the p-Sylow subgroups of G have this property. (Received March 12, 1953.)

340. L. Aileen Hostinsky: *Loewy chains and uniform splitting of lattices.*

An ascending Loewy η-chain is defined for an endomorphism η and a complete modular lattice L satisfying the additional property (*): $a \sum_\alpha p_\alpha = \sum_\alpha a p_\alpha$ for a and the ascending chain p_α elements of L. The following theorem is proved: The endomorphism η of $p/0$ is a uniformly splitting endomorphism if there exists an ascending Loewy η-chain the sum of which is p. In the method of proof much use is made of the modular law, property (*), and complete induction. (Received February 25, 1953.)

341. Bjarni Jónsson: *Modular lattices and normal subgroups.* Preliminary report.

The following proposition holds in every lattice of normal subgroups of a group and, more generally, in every lattice of commuting equivalence relations: If a_0, a_1, a_2, x are any elements, $y=(a_1+x)(a_2+x)$, $b_0=(a_0+y)(a_1+a_2)$ and cyclically, $c_0=(a_1+a_2)\cdot(b_1+b_2)$ and cyclically, then $c_0+c_1+c_2=c_1+c_2$. Applied to the lattice of a projective space this condition implies a special case of Desargues Theorem, and is known to fail in certain projective planes. This solves Problem 27 of G. Birkhoff's *Lattice theory* (rev. ed., 1948). It also shows that a free modular lattice with four generators is not isomorphic to a lattice of normal subgroups or of commuting equivalence relations. (Received March 10, 1953.)

342t. R. C. Lyndon: *On Burnside's problem.* Preliminary report.

A method of K. T. Chen, R. Fox, and the author is applied to the problem of Burnside. For $q \geqq 2$, p prime, let B be the group on q generators defined by the identical relation $w^p=1$, and let $p^{\beta(n)}$ be the order of the lower central quotient B_n/B_{n+1}. Results: (1) for $n<p$, $\beta(n)=\psi(n)$ where [E. Witt] $\psi(n)=n^{-1}\sum \mu(n/d)q^d$ for d/n; (2) $\beta(p)=\psi(p)-C_{p+q-1,q-1}+q$; (3) for $q=2$ henceforth, $\beta(p+1)=\psi(p+1)-p$; (4) for $p=5$, $\beta(7)=4$, and for $p\geqq 5$, $\beta(p+2)\geqq\psi(p+2)-7p+13$; (5) for $n\leqq 2p-5$, $\beta(n)>0$. [(2), (3), (4) are contained in an unpublished work of P. Hall, and (5) in a result of J. A. Green.] (Received March 11, 1953.)

343. G. de B. Robinson: *On the modular representations of the symmetric group.*

The modular representation theory of the symmetric group has been the subject of much study in recent years. J. H. Chung suggested a method for determining the D-matrix. The p-graph of a Young diagram, due to D. E. Littlewood and the consequent notion of r-inducing (Robinson, Proc. Nat. Acad. Sci. U.S.A. vol. 38 (1952) pp. 129–133, 424–426) has made it possible to limit and keep track of the processes involved. Chung's linkages can be interpreted as determining a partial order, so that an indecomposable representation of the regular representation now appears as a *lattice* of its ordinary irreducible representations. The 0, 1 elements of the lattice are easily recognized, and the indecomposable is uniquely determined when either one of these two representations is given. (Received March 26, 1953.)

344. A. L. Whiteman: *The 16th power residue character of 2.*

On the basis of extensive tables Cunningham [Proc. London Math. Soc. (1) vol. 27 (1895) pp. 85–122] conjectured the following criterion for the 16th power residue character of 2. Let $p=a^2+b^2=c^2+2d^2$, a and c odd, be a prime of the form $16n+1$. If $2^{(p-1)/8}\equiv 1 \pmod p$, then $2^{(p-1)/16}\equiv(-1)^{b/16+d/4} \pmod p$. The first proof of this result was given by Aigner [Deutsche Mathematik vol. 4 (1939) pp. 44–52]. Aigner's method employs class-field theory. In the present paper a second proof, based upon the theory of cyclotomy, is given. An alternative formulation of Cunningham's criterion is as follows. Let u and v denote odd numbers. The number 2 is a 16th power residue of a prime p of the form $16n+1$ if and only if p is simultaneously representable in the forms $p=a^2+1024b^2=c^2+128d^2$, or in the forms $p=a^2+256u^2=c^2+32v^2$. (Received February 2, 1953.)

345t. K. G. Wolfson: *The alegbra of bounded operators on Hilbert space.*

Let K be a B^*-algebra which contains an identity element and minimal right ideals. A left ideal of K is called a left annulet if it is the totality of left annihilators of a subset of K. Then, there exists a unique Hilbert space H, such that K is isomorphic (in a norm and * preserving manner) to the algebra $B(H)$ of *all* bounded operators on H, if and only if (1) K contains a smallest closed two-sided ideal (not the zero ideal). (2) If J_1 and J_2 are left annulets which satisfy $J_1J_2^*=0$, then J_1+J_2 is a left annulet. The proof depends on the existence of a lattice isomorphism between the totality of left annulets of $B(H)$ and the totality of (closed) subspaces of H. (Received March 9, 1953.)

Analysis

346. Shmuel Agmon, Louis Nirenberg (p), and M. H. Protter: *A maximum principle for a class of hyperbolic equations.*

A maximum principle is proved for solutions of certain linear hyperbolic equations in second order in two independent variables. It contains as a special case the maximum principle for Tricomi's equation due to Gernau and Rader. Applications to boundary value problems are made. The methods are elementary. (Received March 10, 1953.)

347t. Joseph Andrushkiw: *On the power series whose partial sums have zeros in a sector of the complex plane.*

Let $f_n(z)=1+\sum_{k=1}^{n} a_kz^k$ be the nth partial sum of the power series $f(z)=1+\sum_{k=1}^{\infty} a_kz^k$ (a_k complex numbers) and $z_j=\alpha_j+\beta_j$ ($j=1, 2, \cdots, n$) its zeros. If $\theta_1 \leqq \arctan \beta_j/\alpha_j \leqq \theta_2$, $\theta_2-\theta_1=(\pi/2)-2\eta$, $0<\eta\leqq\pi/4$, the inequality $|a_n|^{2/n}=(c/2)\cdot((A_1+\bar{A}_1)^2-2(A_2+\bar{A}_2+A_1\bar{A}_1))n^{-1}$ holds. There is $c=\csc 2\eta$, $A_1=a_1e^{i\vartheta}$, $A_2=a_2e^{2i\vartheta}$, $\bar{A}_1$ and $\bar{A}_2$ conjugate of A_1 and A_2 and $\vartheta=\theta_1+(\theta_2-\theta_1)/2$. It follows from the above inequality that $f(z)$ represents an integral function of the order not greater than 2. The example $g(z)=e^{z^2}(z^2-1)$ shows that the converse theorem is not true. (Received February 25, 1953.)

348t. H. A. Antosiewicz: *A boundedness theorem for a nonlinear differential equation.*

The differential equation $\ddot{x}+(f(x)+g(x)\dot{x})\dot{x}+h(x)=e(t)$, which can be written as the system $\dot{x}=(y-b(x))/a(x)$, $\dot{y}=-a(x)(h(x)-e(t))$ where $a(x)=\exp(\int_0^x g(u)du)$, $b(x)=\int_0^x a(u)f(u)du$, is considered under the assumptions that $f(x)$, $g(x)$, $h(x)$ are continuous and satisfy a Lipschitz condition for all x and $e(t)$ is continuous and bounded for $t\geqq 0$. It is shown that if (i) $f(x)\geqq 0$ for all x; $xh(x)>0$ for $x\neq 0$, $|h(x)|\to\infty$ and $\int_0^x a^2(u)h(u)du\to\infty$ with $|x|$; (ii) given any constant $M>0$, there exist constants $X(M)$

>0, $m>0$ such that $b^2(x)-M|\int_0^x a^2(u)du|\geqq m$ for $|x|>X(M)$; then every solution ultimately satisfies $|x(t)|<B_1$, $|\dot{x}(t)|<B_2$ where B_1, B_2 are positive constants independent of the particular solution considered. This result is not included in previous investigations of nonlinear differential equations of the second order. (Received March 4, 1953.)

349. Jerome Blackman: *The inversion of the generalized Fourier transform by Abelian summability.*

The space L_p^k, $1\leqq p<\infty$, consists of those functions for which the norm $(\int_{-1}^1|\phi(x)|^p dx+\int_{|x|>1}|\phi(x)/x^k|^p dx)^{1/p}=\|\phi\|_{k,p}<\infty$. For $\phi\in L_1^k$ or $\phi\in L_p^{k-1}$, $p>1$, the generalized Fourier transform of kth order, $E(k, t)$, has been defined by Bochner. Using the notation of Bochner let $\phi(x)=(1/(2\pi)^{1/2})\int_0^\infty e^{-ixt}e^{-\epsilon t}d^kE(k, t)+(1/(2\pi)^{1/2})\cdot\int_{-\infty}^0 e^{-ixt}e^{\epsilon t}d^kE(k, t)$. If $\phi\in L_1^k$, $\lim_{\epsilon\to 0}\|\phi_\epsilon(x)-\phi(x)\|_{k,p}=0$; $\lim_{\epsilon\to 0}\|\phi_\epsilon(x)-\phi(x)\|_{k-1,p}=0$ if $\phi\in L_p^{k-1}$. Similar results hold in the case corresponding to $p=\infty$. If $1<p\leqq 2$ and $\phi\in L_p^k$, the kth transform $E(k, t)$ may be defined by mean convergence and exists in a normed space with norm $\|E(k, t)\|^{k,p'}$ such that $\|E(k, t)\|^{k,p'}<C_p\|\phi\|_{k,p}$ where $1/p+1/p'=1$. (Received March 6, 1953.)

350*t*. E. K. Blum: *A uniqueness theorem for the Euler-Poisson-Darboux equation.*

We consider the equation $E(k)$: $u_{xx}-u_{tt}-(k/t)u_t=0$ for $k=-(2n+1)$, n a positive integer or zero. Let $f(x)$ be any function having a continuous derivative of order $n+3$ on an interval (C, B) of the x-axis. Let T be the characteristic triangle having (C, B) as base. A function $u^{[k]}(x, t)$ is said to be a solution of the Cauchy problem for $E(k)$ if (1) $u^{[k]}(x, t)$ is a regular solution of $E(k)$ in the interior of T, (2) $\lim_{t\to 0}u^{[k]}(x, t)=f(x)$, and (3) $\lim_{t\to 0}\partial u^{[k]}/\partial t=0$. We prove the following theorem: Let $\psi(x, t)\equiv\int_0^1 f[x+(1-2s)t]s^{-1/2}(1-s)^{-1/2}\log[ts(1-s)]ds$. Then every solution of the Cauchy problem for $k=-(2n+1)$ is of the form $u^{[k]}(x, t)=\sum_{r=1}^{n+1}b_{rn}t^r\partial^r\psi/\partial t^r+t^{2n+2}u^{[2n+3]}$ where the b_{rn} are constants determined by the recursion formula $b_{r,n+1}=b_{r-1,n}-b_{rn}(2n+2-r)$, $b_{0n}=0$, $b_{n+1,n}=1$ and $u^{[2n+3]}$ denotes a solution of $E(2n+3)$ which is regular in the interior of T and such that $\lim_{t\to 0}t^n u^{[2n+3]}=0$ and $\lim_{t\to 0}t^{n+1}\partial u^{[2n+3]}/\partial t=0$. (Sponsored by O.N.R.) (Received March 2, 1953.)

351. F. E. Browder: *The asymptotic distribution of the eigenvalues and eigenfunctions of the self-adjoint elliptic differential operator with variable coefficients.*

Let K be a self-adjoint linear elliptic differential operator of order $2m$ with suitably differentiable coefficients on the bounded domain D of Euclidean n-space. Suppose that $2m>n$. Let $\{\phi_i\}$ be a complete sequence of complex-valued eigenfunctions of $(-1)^mK$, orthonormalized in $L_2(D)$ and arranged in nondecreasing order of eigenvalues $\{\lambda_i\}$. If $a(x, \xi)$ is the characteristic form for K at x in D, let $\rho(x)=\int_{a(x,\xi)<1}d\xi$. Then as $t\to\infty$, the number of eigenvalues λ_i less than t is asymptotic to $(2\pi)^{-n}t^{n/2m}\int_D\rho(x)dx$; $\sum_{\lambda_j\leqq t}|\phi_j(x)|^2=(2\pi)^{-n}\rho(x)t^{n/2m}(1+o(1))$; $\sum_{\lambda_j\leqq t}\phi_j(x)\phi_j(y)=o(t^{n/2m})$ for $x, y\in D$. The proof uses the method of Carleman. The theorem is a generalization of results of Courant and Pleijel for the vibrating plate and Gårding for elliptic operators with constant coefficients. (Received March 11, 1953.)

352*t*. P. L. Butzer and Waclaw Kozakiewicz: *On the Riemann derivatives for integrable functions.*

The central difference of order s, $\nabla_{2h}^{s}f(x)$, corresponding to a number $h>0$, is defined inductively by the relations $\nabla_{2h}^{1}f(x)=f(x+h)-f(x-h)$, $\nabla_{2h}^{s+1}f(x)=\nabla_{2h}^{1}\nabla_{2h}^{s}f(x)$. It is established that if $f(x)\in L\{a, b\}$ [i.e. $f(x)$ is Lebesgue integrable in every closed subinterval of the open interval (a, b)] and if there exists a sequence of positive numbers $\{h_n\}$ converging to zero and a function $\sigma(x)\in L\{a, b\}$ such that $\lim_{n\to\infty} \sigma_n(x) \equiv\lim_{n\to\infty} [\nabla_{2h_n}^{l+1}f(x)]/(2h_n)^{l+1}=0$ almost everywhere in (a, b) with $\sup_n |\sigma_n(x)| \leqq\sigma(x)$ for $a<x-(l+1)h_n<x+(l+1)h_n<b$, then there exists a polynomial $P_l(x)$ of degree l in x such that $f(x)=P_l(x)$ for almost every x in (a, b). A corresponding result can be stated for the left and also the right differences [see also the authors' paper entitled *A theorem on the generalized derivatives*, Bull. Amer. Math. Soc. Abstract 59-3-284]. (Received March 11, 1953.]

353. Sarvadaman Chowla and Robert Osserman (p): *The e-rank of polynomials.*

Let $P(x)$ be a polynomial of degree n. Let $e^{P(x)}=\sum_{n=0}^{\infty} a_nx^n$. Consider the infinite matrix (a_{i-j}) where $a_n=0$ if $n<0$. By a theorem of Edrei this matrix must have negative minors of some order if $n\geqq 2$. Define the e-rank of $P(x)$ as the greatest integer r such that all minors of order less than or equal to r are non-negative. The question arises whether for all polynomials of given degree n, there exists a maximum value $r_0=r_0(n)$ of r. It is proved that $r_0(2)=2$. The conjecture $r_0(n)=n$ is unanswered ($n>2$). (Received March 9, 1953.)

354. M. H. Clarkson (p) and H. J. Ettlinger: *On obtaining solutions to the nonhomogeneous wave equation which satisfy boundary conditions of mixed type.*

Use is made of the Riesz Transform to find solutions of the nonhomogeneous wave equation in two and three dimensions which satisfy boundary conditions of mixed type. (Received March 11, 1953.)

355. Ruth M. Davis: *On the Cauchy problem for the Euler-Poisson-Darboux equation.*

Let $u(x, t)$ denote $u(x_1, x_2, \cdots, x_m, t)$. A solution to the equation (1) $\Delta u=u_{tt}+kt^{-1}u_t$ with given $u(x, 0)$ and $u_t(x, 0)=0$, where k is a real parameter, has been given by A. Weinstein in C. R. Acad. Sci. Paris vol. 234 (1952) pp. 2584–2585. For $k<m-1$, $k\neq-1,-3, -5, \cdots$, a new explicit form of the solution is obtained in the present paper. Let $\omega_s=2(\pi^{1/2})^s[\Gamma(s/2)]^{-1}$, let $m-k$ not be an odd positive integer, and let $L(x, t) =(\omega_{2s+2})(\omega_{k+2n+1})^{-1}\cdot\int\cdots\int_2 f(x_i+\alpha_i t)(1-\alpha_i^2)^s d\alpha_1\cdots d\alpha_m$, where $2s=(k+2n-1-m)$ and $\alpha_1^2+\cdots+\alpha_m^2\leqq 1$. Then the solution to (1) may be written as (2) $u(x, t)=[(k+1)(k+3)\cdots(k+2n-1)]^{-1} \sum_{j=0}^{n} B_{nj}t^{n-j}L_{(n-j)t}$ where $B_{nj} =(-1)^{n-j}2^n[(n-j)!]^{-1}\sum_{r=0}^{n-j} (-1)^rC_{n-j,r}\Gamma[(k+2n+1+r)/2]\{\Gamma[(k+1+r)/2]\}^{-1}$. If $m-k$ is an odd positive integer, $L(x, t)$ must be replaced by Poisson's mean value M. In this case (2) becomes the solution already obtained by J. B. Diaz and H. F. Weinberger, Bull. Amer. Math. Soc. Abstract 59-1-12. This paper was sponsored by the Office of Naval Research. (Received March 9, 1953.)

356. J. B. Diaz (p) and H. F. Weinberger: *The exceptional cases in the Euler-Poisson-Darboux equation.*

Let x denote $(x_1\cdots x_m)$. A solution of the singular Cauchy problem (*)$\Delta u-u_{tt}-Kt^{-1}u_t=0$, with $u(x, 0)=f(x)$, $u_t(x, 0)=0$, is found for the exceptional values

$K=-1, -3, \cdots$, excluded in a previous abstract (Bull. Amer. Math. Soc. Abstract 59-1-12). It is found that in the solution $u(x, t)$ previously obtained for all other values of K, the singularities occurring at these exceptional values can be removed by subtracting an appropriate function of the form $t^{1-K}V(x, t)$, where v satisfies (*) with K replaced by $2-K$. The initial value of v is a multiple of an iterated Laplacian of f, and so v can be found by the previous methods. Passage to the limit provides a solution of (*) for K an odd negative integer. The $(1-K)$th derivative of u is logarithmic in t at $t=0$ except when f is polyharmonic of order $(1-K)/2$. This work was sponsored by the Office of Naval Research. (Received February 13, 1953.)

357. Avron Douglis: *Hadamard's conjecture.*

A complete, relatively simple proof is given of a conjecture of Hadamard that the linear hyperbolic, partial differential equations of second order in an even number of independent variables for which Huygens' Principle is valid are just those which result from the wave equation after arbitrary transformations of three possible kinds: change of independent variables, multiplication of the dependent variable by a variable factor, and multiplication by another variable factor of each term of the equation. The proof is based on the author's method of approach to Cauchy's problem for hyperbolic equations of the second order which has been reported in Bull. Amer. Math. Soc. Abstract 58-4-354. (Received March 17, 1953.)

358. R. J. Duffin: *Continuation of biharmonic functions by reflection.*

Let R be a region contained in the right half-plane and bounded on one side by a segment of the y axis. Let the function $w(x, y)$ be biharmonic in R; that is, $\Delta\Delta w=0$. On the segment of the y axis let w satisfy the clamped plate boundary conditions; that is, $w=\partial w/\partial x=0$. Then it is shown that the formula $w(-x, y)=w-x^3\Delta(w/x)$ continues w so as to be biharmonic in the region obtained by reflecting R in the y axis. The same formula applies to the analogous three-dimensional problem. Other continuation formulae are found for circular boundaries and various other boundary conditions. (Received March 11, 1953.)

359. W. F. Eberlein: *A higher order connection formula.*

The methods of Langer applied to the equation $u''+k^2Pu=0$, where P has a zero of order n at $x=0$ and k is a large parameter, yield explicit approximate solutions w with errors roughly $O(k^{-1})$. When $n=1$, Langer [Trans. Amer. Math. Soc. vol. 67 (1949) pp. 461–490] has extended the procedure to obtain w's approximating the u's to higher powers of k^{-1}, but the new method breaks down in general if $n>1$. In this paper a new approximating equation is introduced in the case $n>1$ to obtain approximations v to the u's with errors essentially $O(k^{-2})$ in the complement of an arbitrary neighborhood of $x=0$. (Received March 11, 1953.)

360. Albert Edrei: *Meromorphic functions with three radially distributed values.*

Let $f(z)$ be a meromorphic function of the complex variable $z=r\exp(i\theta)$. Assume that all but a finite number of the roots of the three equations (1) $f(z)=0$; (2) $f(z)=\infty$; (3) $f^{(l)}(z)=1$ $[l\geqq 0, f^{(0)}\equiv f]$ are distributed on the q radii $r\exp(i\theta_1)$, $r\exp(i\theta_2), \cdots$, $r\exp(i\theta_q)$ $[r\geqq 0; 0\leqq\theta_1<\theta_2<\cdots<\theta_q<2\pi]$. Denote by $\delta^{(l)}(a)$ the defect of the a's of $f^{(l)}(z)$ and assume $\delta(0)+\delta^{(l)}(1)+\delta(\infty)>0$ $[\delta^{(0)}=\delta]$. Then, the order ρ, of (z), is necessarily finite and $\rho\leqq\sup\{\pi/(\theta_2-\theta_1), \pi/(\theta_3-\theta_2), \cdots, \pi/(\theta_{q+1}-\theta_q)\}$ where $\theta_{q+1}=2\pi+\theta_q$. This result lends itself to the following application. Consider the entire function $g(z)$

$=P(z) \exp(Q(z))$ where $Q(z)$ is any entire function and $P(z)$ an entire function of finite order ρ. Assume that all but a finite number of the roots of the three equations $g(z)=0$, $g'(z)=0$, $g''(z)=0$ are real. Then the order ρ_1, of $Q(z)$, is necessarily finite. Furthermore, either $\rho_1 \leqq \rho$ or else $\rho < \rho_1 \leqq 1$. This proposition easily yields extensions of known results of Pólya and Saxer. (Received March 11, 1953.)

361. H. J. Ettlinger and C. E. Abraham (p): *On the Holmgren-Riesz transform.*

In one dimension, Riemann defined the transform $F[\alpha; a, x|f(t)]$, where $f(t)$ has continuous derivatives and α is a real parameter [*Versuch einer allgemeinen Auffassung der Integration und Differentiation*, Collected Works, Leipzig, 1876, pp. 331–344]. M. Riesz generalized the transform to complex values of α and to higher dimensions [*L'integral de Riemann-Liouville et le problème de Cauchy*, Acta Math. vol. 81 (1949) pp. 1–223]. H. J. Holmgren independently developed in one dimension an equivalent transform by means of a different definition [*Om differentialkalkylen med indices of hvilken natur som helst*, Kongl. Svenska Vetenskaps-Akademies Hedlinger, vol. 5 (1865) pp. 1–83]. The properties of this transform are developed systematically under general conditions. (Received March 11, 1953.)

362t. Herbert Federer: *The fundamental inequality between the Lebesgue areas of a surface and its projections.*

Consider a finitely triangulable subset X of the plane and a continuous mapping f of X into Euclidean n-space E_n, with $n \geqq 2$. For any two integers i and j such that $1 \leqq i < j \leqq n$, let $P_{i,j}$ be the projection of E_n onto E_2 which maps any $x=(x_1, \cdots, x_n) \in E_n$ onto $(x_i, x_j) \in E_2$. It is proved that $L(f) \leqq \sum_{1 \leqq i < j \leqq n} L(P_{i,j}) \circ f)$, where L is the 2-dimensional Lebesgue area. For the special case in which $n \leqq 3$ and X is a 2-cell, this inequality was first established by L. Cesari [Annali della R. Scuola Normale Superiore di Pisa (2) vol. 10–11 (1941–42)]. The method of the present paper is new even when applied to the previously known special case. As a consequence of the general inequality now proved, the whole theory of 2-dimensional Lebesgue area becomes equally as extensive for surfaces in n-space as for surfaces in 3-space. (Received March 3, 1953.)

363t. Herbert Federer: *A new formula for Lebesgue area.*

Assuming that X is a compact, locally connected, finitely connected subset of the plane, let f be a mapping of X into E_n with monotone-light factorization $f=l \circ m$ and middle space M. Consider the set P of all $n-1$ dimensional planes of E_n (a factor space of the group of all rigid motions of E_n) and suppose μ is a Haar measure over P. For each $p \in P$ let $S(p)=M \cap \{\alpha | l(\alpha)=p\}$ and let $T(p)$ be the set of all those points of $S(p)$ at which $S(p)$ has positive topological dimension. For $p \in P$ and $y \in p$, let $N(p, y)$ be the number (possibly ∞) of points $\alpha \in T(p)$ such that $l(p)=y$. Furthermore suppose that $\mathcal{H}_n^1$ and $\mathcal{J}_n^1$ are the 1-dimensional Hausdorff and integralgeometric measures over E_n and that H_f^1 is the 1-dimensional Hausdorff measure over M (see *Measure and area*, Bull. Amer. Math. Soc. vol. 58 (1952) pp. 306–378). Then the three integrals $\int_P \int_p N(p, y) d\mathcal{H}_n^1 y d\mu p$, $\int_P \int_p N(p, y) d\mathcal{J}_n^1 y d\mu p$, $\int_P H_f^1[T(p)] d\mu p$ are equal to each other and to c times the 2-dimensional integralgeometric stable area of f (loc. cit.), where c depends only on n and μ. [The proofs given in the paper use a particular Haar measure μ and yield the corresponding value of c explicitly in terms of n.] Moreover if X is a 2-cell then the integralgeometric stable area of f is equal to the Lebesgue area of f; thus results a formula expressing the Lebesgue area of a surface in terms of the lengths of the positive-dimensional parts of its sections by $n-1$ planes. (Received March 3, 1953.)

364. F. A. Ficken: *Bounded linear adjoints for certain nonlinear transformations between Banach spaces.* Preliminary report.

Let X be a complex Banach space with elements x and norm $\|x\|$. Let Φ contain all real-valued continuous isotonic $\phi(r)$ $(r\geqq 0)$ with $\phi(0)>0$. With $\phi\in\Phi$, let X_ϕ^* be the Banach space of complex-valued functionals $x^*(x)$ that are continuous on X and have $\|x^*\|_\phi=\sup_x[|x^*(x)|/\phi(\|x\|)]<\infty$. Let Y be a complex Banach space and define Y_ψ^* similarly with $\psi\in\Phi$. Let $x\rightarrow y=Tx$ be a continuous closed mapping of X into Y for which there exist ϕ and ψ (henceforth fixed) such that $\beta=\sup_x[\psi(\|Tx\|)/\phi(\|x\|)]<\infty$, and say then that T is ϕ, ψ-bounded. For each $y^*\in Y_\psi^*$ and $x\in X$ define $x^*(x)=T^*y^*(x)=y^*(Tx)$. Then T^* maps Y_ψ^* linearly into X_ϕ^* and $\|T^*y^*\|_\phi\leqq\beta\|y^*\|_\psi$. Several theorems on the adjoint of a linear transformation (as in Banach's book) carry over to T^*. Example: If $x^*=T^*y^*$ has a solution $y^*\in Y_\psi^*$ for each $x^*\in X_\phi^*$, then T has an inverse T^{-1}; conversely, if T^{-1} exists and is ψ, ϕ-bounded, then T^* maps Y_ψ^* onto X_ϕ^*. If X and Y are complete metric spaces with distances $d(x, x')$ and $\delta(y, y')$, one can retain much of the argument by replacing $\phi(\|x\|)$ and $\psi(\|y\|)$ by $\phi(d(p, x))$ and $\phi(\delta(q, y))$ with $p\in X$ and $q\in Y$ fixed arbitrarily. (Received March 9, 1953.)

365. B. A. Fleishman: *Periodic solutions of a nonlinear wave equation.*

Consider the nonlinear wave equation of Duffing type $u_{xx}=u_{tt}+2\kappa u_t+\alpha u+\epsilon u^3+B(x, t)$ $(0<x<L, t>0)$ with positive damping $(\kappa>0)$ and p-periodic forcing term: $B(x, t+p)=B(x, t)$ and $B(0, t)=0=B(L, t)$. Conditions are obtained under which this equation has at least one solution $u(x, t)$ that meets the boundary conditions $u(0, t)=0=u(L, t)$ and is p-periodic in t. The author first finds, by solving an equivalent integral equation, the solution $u(x, t; f, g)$ of the above differential equation and boundary conditions with initial data $u(x, 0; f, g)=f(x)$, $u_t(x, 0; f, g)=g(x)$. Then, under suitable assumptions, the functional equations expressing the periodicity conditions $u(x, p; f, g)=f(x)$, $u_t(x, p; f, g)=g(x)$ are solved by an iterative method in a suitable Banach space, yielding unique (appropriately bounded) initial data $f(x)$, $g(x)$ such that $u(x, t; f, g)$ is p-periodic in t. The restrictions on the parameters are: $e^{-p\kappa}\max[Q_1(\kappa), Q_2(\kappa)]<1$, where $Q_1(\kappa)$ and $Q_2(\kappa)$ are quadratics with positive coefficients depending only on L; $\eta\equiv pL|\alpha-\kappa^2|e^{p\kappa}<1/2$; and, if $\beta=\|B(x, t)\|$ and R(fixed) $>3Lp\beta e^{p\kappa}/2(1-\eta)$, then $3\in R^2\leqq|\alpha-\kappa^2|$. (Received February 6, 1953.)

366t. M. P. Gaffney, Jr.: *A special Stokes's theorem for complete Riemannian manifolds.*

Let M be an orientable, complete, Riemannian manifold. The theorem stated in vector analysis terminology is: If v is a C^1 vector field on M such that both $|v|$ and div v are integrable functions, then $\int_M \operatorname{div} v\,dV=0$. The equivalent statement for differential forms is: Let α be an $n-1$ form of class C^1 such that both α and $d\alpha$ are in L_1. Then $\int_M d\alpha=0$. (β is in L_1 if $\int(*(\beta*\beta))^{1/2}*1$ is finite.) This generalizes Stokes's theorem for exact n forms on a compact manifold. Let d be restricted to C^1 forms α for which $\|\alpha\|$ and $\|d\alpha\|$ are finite; similarly δ ($\|\beta\|^2\equiv\int\beta*\beta_1$). The equations $0=\int_M d(\alpha*\beta)=(d\alpha, \beta)-(\alpha, \delta\beta)$ yield the corollary $(d\alpha, \beta)=(\alpha, \delta\beta)$. Thus M has negligible boundary. Let $\Delta=d\delta+\delta d$ be restricted to α for which the norms of α, $d\alpha$, $\delta\alpha$, $\delta d\alpha$, and $d\delta\alpha$ are all finite (the natural Hilbert space definition of Δ). Then Δ is symmetric, and hence of Weyl's limit point type on M. Thus the formalism of the compact case is valid on M. A previous result is that for negligible boundary the closure of Δ is self-adjoint ($=\Delta^*$). (Received April 20, 1953.)

367. F. W. Gehring: *A study of α-variation.*

A function $f(x)$ is said to have bounded α-variation over the interval $a \leqq x \leqq b$ and belong to W_α if the sums $\{\sum_{n=1}^{N} |f(x_n)-f(x_{n-1})|^{1/\alpha}\}^\alpha$, $0 \leqq \alpha \leqq 1$, are bounded for every subdivision $a=x_0 < \cdots < x_{10}=b$. Functions in W_1 have bounded variation. L. C. Young (Acta Math. vol. 67) has proved that a Stieltjes integral $\int_a^b f(x)dg(x)$ can be defined if $f(x)$ and $g(x)$ are in W_α and W_β with $\alpha+\beta>1$. A moment problem is discussed for the class W_α $(0<\alpha\leqq 1)$ and necessary and sufficient conditions are obtained for a sequence, $\{a_n\}$, to be represented $a_n=\int_0^1 x^n dg(x)$ where $g(x)$ is in W_α. The concept of α-variation can be applied to the study of infinite series. An absolutely convergent series is one whose partial sums have bounded variation in n. A scale for Cesaro and Abel summability is discussed along with consistency results and a Tauberian theorem for absolute Abel summability. (Received March 16, 1953.)

368. J. J. Gergen and F. G. Dressel (p): *Mapping for elliptic equations.*

The following extension of the Riemann mapping theorem is obtained. Let D and T be finite plane domains whose boundaries D^* and T^* are simple, closed Jordan curves. Let α, β, γ, δ be real, bounded functions of (x, y) of class C' on D such that $0<\alpha$, $0<m<\alpha\delta-(\beta+\gamma)^2/4$ on D, where m is a constant. Let $z^{(j)}$ $(j=1, 2, 3)$ be distinct points on D^*, and let $Z^{(j)}$ $(j=1, 2, 3)$ be distinct points on T^* in the same order on T^* as the points $z^{(j)}$ on D^*. Then there exists a 1:1 and continuous mapping $X=u(x, y)$, $Y=v(x, y)$ of $D+D^*$ into $T+T^*$ which carries D, D^*, $z^{(j)}$ into T, T^*, $Z^{(j)}$ and is such that u, v are of class C' and satisfy $\alpha u_x+\beta u_y=v_y$, $\gamma u_x+\delta u_y=-v_x$, $0<u_x v_y-u_y v_x$ on D. The analysis is based on the work of Douglas, Courant, Morrey, and earlier results of the authors. (Received January 9, 1953.)

369. Murray Gerstenhaber: *A characterization of the modular group and certain similar groups.*

Two realizations G' and G'' of an abstract group as a group of properly discontinuous transformations of the upper half-plane onto itself will be considered not essentially distinct, if G'' is conjugate to G' in the full group of conformal transformations of the upper half-plane onto itself. Let $Z_2 * Z_n$ (n finite) denote the free product of a cyclic group of order two and one of order n. Then for $n>2$, $Z_2 * Z_n$ can be realized in one and only one way as a properly discontinuous group of conformal transformations of the upper half-plane onto itself with a fundamental domain having finite hyperbolic area; $Z_2 * Z_2$ can not be realized in such a way at all. As the modular group is algebraically isomorphic to $Z_2 * Z_3$, this gives a characterization of the modular group. The theorem is proved by showing that the commutator subgroup of $Z_2 * Z_n$, acting as a group of conformal transformations of the upper half-plane, is the fundamental group of a Riemann surface possessing certain conformal transformations onto itself. These characterize the surface in question and it determines the realization of $Z_2 * Z_n$. (Received February 13, 1953.)

370. Samuel Goldberg: *Infinite series solutions of a singular diffusion equation.*

The partial differential equation $u_t=(au)_{xx}-(bu)_x$, $0<x<1$, $t>0$, for a probability density function $u=u(t, x)$ is studied, where $a=x(1-x)$, $b=\beta-(\alpha+\beta)x$, α and β are positive constants. One is led to an eigenvalue problem for a modified hypergeometric differential equation which, transformed to a corresponding integral equation, reduces the problem to that of finding the Green's functions for $L(X)=0$ where $L(X)=d[x^{2-\beta}(1-x)^{2-\alpha}X']/dx$. Boundary conditions with X and X' finite or infinite at $x=0$ and $x=1$ are considered. In the unbounded case, the boundary condition is $x^pX'/X\to k(0<p<1)$ as $x\to 0$. In each of these cases we have found the Green's functions and the eigenfunctions of the integral equation. These are combinations of hypergeometric functions. The solutions of the diffusion p.d.e. are obtained as infinite series expansions in terms of these functions. (Received March 11, 1953.)

371. Wilfred Kaplan: *Close-to-convex schlicht functions.*

It follows from known theorems that if $w=\phi(z)$ is a schlicht mapping of $|z|<R$ onto a convex domain and $w=f(z)$ is an analytic function for $|z|<R$ such that $f'(z)/\phi'(z)$ has positive real part, then $f(z)$ is schlicht. The schlicht functions $f(z)$ obtainable in this way are called close-to-convex. It is shown that $f(z)$ is close-to-convex if and only if $\arg f'(re^{i\alpha})-\arg f'(re^{i\beta})$ is less than $\pi+\beta-\alpha$ for $\alpha<\beta$; from this criterion a geometric description of $f(z)$ is obtained. It is shown that the class of close-to-convex functions includes the class of mappings of $|z|<R$ onto a star-shaped domain and includes those f for which, for some real constants α and β, $(z-Re^{i\alpha})(z-Re^{i\beta})f'(z)$ has positive real part. It is shown that a function $f(z)$ having a Poisson integral representation in terms of a function $h(\theta)$ which is monotone increasing for $0\leqq\theta\leqq\pi$ and monotone decreasing for $\pi\leqq\theta\leqq 2\pi$ is necessarily close-to-convex. (Received March 9, 1953.)

372*t*. Hyman Kaufman and R. L. Sternberg: *A two-point boundary problem for ordinary self-adjoint differential equations of even order.* I.

Consider the boundary problem: (1) $[p_0(x)u^{(n)}]^{(n)}-[p_1(x)u^{(n-1)}]^{(n-1)}+\cdots+(-1)^np_n(x)u=0$, (2) $u(x_r)=u'(x_r)=\cdots=u^{(n-1)}(x_r)=0$ $(r=1, 2)$, where $n\geqq 2$, $p_0(x)>0$, $p_i(x)\in C^{n-i}$ on an interval $[a, \infty)$ and all elements are real. Call $u=u(x)\in C^n$ a solution of (1)–(2) for $a\leqq x_1<x_2<\infty$ if $p_i(x)u^{(n-i)}\in C^{n-i}$ on $[a, \infty)$ and $u(x)$ satisfies (1)–(2) with (1) an identity on $[a, \infty)$. If $a>0$ and $p_0(x)\geqq k_0x^{2n-2}$ on $[a, \infty)$ for some $k_0>0$ and if the integrals $\int_a^\infty x^{2i-1}p_i(x)dx$ $(i=1, 2, \cdots, n)$ converge, then there exists an $a_0\geqq a$ such that for $a_0\leqq x_1<x_2<\infty$ the only solution $u=u(x)$ of (1)–(2) is the trivial solution $u(x)\equiv 0$ on $[a, \infty)$. The proof is obtained with the aid of an application of Theorem 5.3 of Sternberg (Duke Math. J. vol. 19 (1952) pp. 311–322). (Received March 11, 1953.)

373. I. I. Kolodner: *A method for solving the heat equation with moving boundary condition.*

Problem: find $\rho(t)$, $u(x, t)\in C^1$ $(D\colon t>0, x>\rho(t))$ such that (1) $u_{xx}=u_t$ in D, (2) $u(\rho, t)=f(t)$, $u_x(\rho, t)=g(t)$, $t>0$, (3) $u(x, 0)=0$, $x\geqq\rho(0)=0$, $u(\infty, t)=0$. Although a formal expression for u involving f, g, and ρ can be easily derived, it is in general of little use in proving the existence and uniqueness of solutions. Let $R(t)\in C^1$ $(t>0)$, $R(0)=0$, $\dot R(t)\neq 0$ $(\cdot=d/dt)$, and assume that $f\in C^1$, $g\in C$, and $|f|+|g|+|f\dot R|\leqq At^{-1+\epsilon}$, $\epsilon>0$. We show: i. There exists a unique square integrable solution $v^R(x, t)\in C^1$ $(D^R\colon t>0, r\neq R(t))$ of the problem (1) $v_{xx}=v_t$ in D^R, (2) $v(R+, t)-v(R-, t)=f(t)$, $v_x(R+, t)-v_x(R-, t)=g(t)$, $t>0$, (3) $v(x, 0)=0$, $|x|<\infty$, $v(\pm\infty, t)=0$. ii. If

$v^R(R-, t)=0$ has a (unique) solution R^* satisfying the conditions on R, then $v^{R^*}\equiv 0$ for $r<R^*$, and $\rho=R^*$, $u=v^{R^*}$, $x>R^*$ form the desired (unique) solution of the original problem. iii. If $v_x^R(R-, t)=0$ has a unique solution R^{**}, and $\dot{R}^{**}<0$, conclusions in ii follow. If R^{**} is unique, so is R^* and $R^*\equiv R^{**}$. Tools used were: Laplace transforms to derive a formal expression for v^R, and energy integral methods in proving uniqueness. An application arises in the determination of evaporation of a liquid sphere in presence of its vapor. In this case $f(t)=\rho(t)+1$, $g(t)=(\rho+1)(\alpha\rho+1)$, $\alpha>0$, and application of iii leads to an integro-differential equation for ρ which can be handled very efficiently. The method applies also to cases when the boundary condition (3) is more complicated. (Received March 11, 1953.)

374. C. N. Moore: *On convergence factors for double series summable (N) whose partial sums do not remain bounded.*

In the author's colloquium volume (Amer. Math. Soc. Colloquium Publications, vol. 22) necessary and sufficient conditions were derived for convergence factors in double series, whose partial sums remained bounded, and which are summable by Norlund means. It is not difficult to modify the treatment there given to cover the case where the partial sums become infinite in some definite manner. The conditions thus obtained enable one to infer summability (A) of the restricted type for the series in question. In several recent articles published in Doklady Akad. Nauk some of the Russian mathematicians have treated the case where the summability considered was confined to the case $(C, 1)$. Presumably these writers had some reason not to take account of the wide generality obtained in the modern treatment of convergence factor theorems. (Received April 25, 1953.)

375. Cathleen S. Morawetz: *A uniqueness theorem for an equation of mixed type.*

Consider the equation (*) $K(y)u_{xx}+u_{yy}=0$, with $K'(y)\geqq 0$, $K(0)=0$. Let D be a domain bounded by five curves C_1, γ_1, γ_2, C_2, and C_0. γ_1 and γ_2 are two characteristic arcs issuing from the origin. C_0 is a smooth arc in $y\geqq 0$ joining two points $(a_1, 0)$ and $(a_2, 0)$, $a_1\leqq 0\leqq a_2$. C_1 and C_2 are smooth arcs in $y\leqq 0$ issuing from $(a_1, 0)$ and $(a_2, 0)$ respectively and on which $dx^2+Kdy^2\geqq 0$. C_0 is assumed to be star-shaped with respect to the origin. Theorem: A solution of (*) vanishing on $C_0+C_1+C_2$ vanishes identically. The following limiting cases are included: $a_1=0$; C_1 is a characteristic; C_1 and C_2 are characteristics. (Received March 10, 1953.)

376t. Harry Pollard: *Solution of Bernstein's approximation problem.*

Let $K(u)$ be a function continuous on $(-\infty, \infty)$ and vanishing more rapidly than any power of u as $|u|\to\infty$. The Bernstein problem is to decide when $u^nK(u)$, $n=0, 1, \cdots$, is fundamental in the space C_0 of functions continuous on (∞, ∞) and vanishing at $\pm\infty$. A survey of the literature can be found in Carleson (Proc. Amer. Math. Soc. vol. 2 (1951) pp. 953–966). The present paper gives the first complete solution, as follows. It is necessary and sufficient that (i) $K(u)\neq 0$, (ii) $\int(1+u^2)^{-1}\log|K(u)|du=-\infty$, and (iii) there exists a set of polynomials $p_n(u)$ such that $p_n(u)K(u)\to 1$ and $|p_n(u)K(u)|\leqq C$. The proof depends on Loomis' work on Hilbert-Stieltjes transforms (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 1082–1086.) (Received February 4, 1953.)

377. M. H. Protter: *An existence theorem for the generalized Tricomi problem.*

A boundary value problem is treated for the equation (*) $K(y)u_{xx}+u_{yy}=0$ when $K(y)$ is a monotone increasing function of y possessing a continuous third derivative with $K(0)=0$, $K'(0)>0$. The domain considered is the following: let Γ be a simple rectifiable arc lying in the upper half-plane with end points on the x-axis at A and B $(A<B)$. Let Γ_1 and Γ_2 be real characteristics of equation (*) emanating from A and B respectively and which intersect. Suppose that γ is an arc lying in the triangle bounded by Γ_1, Γ_2 and the segment AB. It is further assumed that (i) Γ contains the "normal curve" in its interior and in some neighborhood of A and B coincides with this curve. (ii) γ passes through the point A, has a positive slope, intersects Γ_2, and in some (arbitrarily small) neighborhood of A coincides with Γ_1. Let D be the domain bounded by Γ, γ, and Γ_2. We consider the boundary value problem for (*) in which values are prescribed along Γ and γ. For this problem it is shown that the Fredholm alternative holds. This generalizes a recent result of Frankl. If γ coincides with Γ_1, then the problem reduces to the Tricomi problem and generalizes results of Tricomi and Gellerstedt. (Received March 11, 1953.)

378. Walter Rudin: *H_p classes in general domains.*

In the unit circle, the class H_p $(p>0)$ of analytic functions f for which $\int_0^{2\pi}|f(re^{i\theta})|^p d\theta$ is bounded as $r\to 1$ has been extensively studied. The author defines a conformally invariant extension for arbitrary plane domains D: $f\in H_p$ in D if f is analytic and single-valued in D, and if $|f|^p$ has a harmonic majorant in D. The following results are obtained. (1) If Z is a closed set of capacity zero, and if $f\in H_p$ in $D-Z$, then f can be defined on Z so that $f\in H_p$ in D. (2) Fixing a point $t\in D$, let $\|f\|_p=\{u(t)\}^{1/p}$, where u is the least harmonic majorant of $|f|^p$. If $p\geqq 1$, this makes H_p into a Banach space (H_2 is a Hilbert space). Although the norm depends on the choice of t, the induced topology does not. (3) If T is a linear functional of norm ρ on H_p and $p>1$, there is a unique $F\in H_p$ such that $\|F\|_p=1$ and $TF=\rho$. (4) If the boundary C of D is analytic, the boundary behavior of the functions under consideration is as in the unit circle. In particular, H_p can then be regarded as a subspace of L_p, the space of complex functions f on C, normed by $\|f\|_p=\{(1/2\pi)\int_C|f(z)|^p\partial G/\partial n ds\}^{1/p}$, where G is the Green's function of D with pole at t; if $p\geqq 1$, the Cauchy formula holds with C as path of integration. (Received March 17, 1953.)

379*t*. Walter Rudin: *Schwarz's lemma in H_1.*

Let D be a domain whose boundary C consists of n simple closed nonintersecting analytic curves. Fixing a point $t\in D$, let H^0 be the set of functions $f\in H_1$ for which $f(t)=0$ and $\|f\|_1\leqq 1$, where the norm is defined as in the preceding abstract. Let $\rho=\sup|f'(t)|$ for $f\in H^0$, and let $\alpha=\sup|f'(t)|$ under the stronger restriction $|f(z)|<1$ in D. There always exists at least one function $F\in H^0$ for which $F'(t)=\rho$, and a unique function g of the form $g(z)=(z-t)^{-1}+\sum c_k(z-t_k)^{-1}+w(z)$ (where $t_1,\cdots t_{n-1}$ are the critical points of the Green's function of D with pole at t, and $w(z)$ is analytic and single-valued on $D\cup C$), such that $|g(z)|\equiv\rho$ on C; moreover, $F(z)g(z)\geqq 0$ on C. Two cases are to be distinguished. (1) If the Szegö kernel function $K(z,t)$ has its zeros at $z=t_1,\cdots,t_{n-1}$, then g has n poles and no zeros in D, F has n zeros in D, and $\rho=\alpha$. (2) Otherwise, g has at least one zero in D, F has fewer than n zeros in D, and $\rho>\alpha$. If D is the ring $0<r<|z|<1$, then (1) occurs if $|t|=r^{1/2}$; in this case there are infinitely many extremal functions F; the ratio of any two of them is an elliptic function of $\log z$. If $|t| \neq r^{1/2}$, then (2) occurs, and the extremal function F is unique. (Received March 17, 1953.)

380. Jacob Samoloff: *Mixed means, extensions, and multiple iterative sequences.*

An investigation is presented of convergence properties in the case of multiple iteration processes of the form $x_{k,n+1} = f_k(x_{1,n}, \cdots, x_{p,n})$, $k = 1, 2, \cdots, p$; $n = 0, 1, \cdots$, where the f_k are means, i.e., they satisfy the inequalities $\max(x_i) \geqq f(x_1, \cdots, x_p) \geqq \min(x_i)$, for $a \geqq$ all $x_i \geqq b$. Sufficient conditions for uniform convergence are given, independently of continuity or differentiability of the functions. Elementary methods are used. In all cases under discussion the sequences $\{x_{k,n}\}$, $k = 1, 2, \cdots, p$, converge to a common limit $F(x_1, \cdots, x_p)$, where $x_k = x_{k,0}$, and F is found to be also a mean, which is uniquely defined by the characteristic property $F(x_1, \cdots, x_p) = F[f_1(x_1, \cdots, x_p), \cdots, f_p(x_1, \cdots, x_p)]$. Applications are to mixed means (e.g. arithmetic-geometric mean of Gauss) and to a method of defining a mean of $p+1$ variables corresponding to a given mean of p variables (called extension). Explicit expressions are found for F in special cases. (Received March 9, 1953.)

381. Samuel Schecter: *On a class of harmonic mappings.*

A harmonic mapping of a domain D in the z-plane is defined by a pair of real-valued functions $u(z)$, $v(z)$ harmonic in D. Let a one-to-one mapping of an annulus $0 < q < |z| < 1$ onto a convex domain with the origin deleted be given by $w = \int ce^{-h(z)}dz - \overline{\int ce^{h(z)}z^{-2}dz}$ where $h(z)$ is analytic in the annulus. Estimates are obtained for $\arg w'(\theta) = \phi(\theta)$ on $|z| = 1$ when suitable conditions are prescribed for $\operatorname{Re}\{h(e^{i\theta})\} = A(\theta)$. It is shown for instance that if $A \in L_p(0, 2\pi)$, $p > 1$, then $\phi \in L_p(0, 2\pi)$ and inequalities relating their p-norms are obtained. If $A \in \operatorname{Lip} \alpha$, $0 < \alpha < 1$, then $\phi \in \operatorname{Lip} \alpha$. The method of proof depends on an extension of the theorems of M. Riesz, Privaloff, and Warschawski on conjugate functions to harmonic functions defined in an annulus. Results in this direction have been obtained by C. Saltzer (Bull. Amer. Math. Soc. Abstract 58-2-195). The following has been shown for harmonic mappings of the unit disk onto itself: Let $\phi(\theta)$ define a homeomorphism of $z = e^{i\theta}$ onto itself and let $\phi(\theta) - \theta = 0$ for $0 \leqq \alpha \leqq \theta \leqq \beta < 2\pi$, $\phi(\theta) - \theta \neq 0$ for $\beta < \theta < 2\pi$, $\phi(2\pi) - \phi(0) = 2\pi$. Then the one-to-one harmonic mapping extension of this homeomorphism into $|z| < 1$ cannot leave the origin fixed. (Received March 11, 1953.)

382. Albert Schild: *On a class of functions, schlicht in the unit circle.*

Let S_p be the class of functions having $|z| = 1$ as radius of schlichtness, and let $f_p(z) = z - \sum_{n=2}^{N} a_n z^n$, having all a_n real and non-negative for $n = 2, 3, \cdots, N$, $N \geqq 2$. It is shown that a necessary and sufficient condition for $f_p(z) \in S_p$ is that $1 - \sum_{n=2}^{N} n a_n = 0$. Certain other properties of this class of functions are studied. Also new estimates of the bound of convexity, of the shortest distance of the boundary of the map of $|z| = 1$ by $f_p(z)$ from origin, and of other quantities connected with the conformal mapping of schlicht functions are obtained. The methods used are of an elementary nature. (Received March 5, 1953.)

383t. H. M. Schwartz: *A class of integrals arising in some problems of quantum mechanics.*

Special cases of the integrals $\int_0^\infty e^{-kx} x^p T_m^n(ax) T_\mu^\nu(bx)dx$ $[T_m^n(x) = (-1)^{n+m}(1/(n+m)!^2)$

$L^m_{n+m}(x)$, see H. Bateman, *Partial differential equations*, p. 450] have been discussed in the literature [for example, H. Bateman, loc. cit., E. Schrödinger, *Collected papers on wave mechanics*, p. 100, W. Gordon, Ann. d. Phys. vol. 2 (1929) p. 1031]. The following general result has been obtained by two alternative very simple methods: These integrals, with obvious restrictions on the parameters, equal $a^n b^\nu/(n+m)!(\nu+\mu)!k^{n+\nu+p+1} \cdot \sum_{i=0}^{n}\sum_{j=0}^{\nu}(-1)^{i+j}(n+\nu+p-i-j)!(C_{n+m,i})(C_{\nu+\mu,j})\cdot k^{i+j}/(n-i)!(\nu-j)!\ a^i b^j$. More compact expressions are also obtained. It is expected to submit this note for publication in the American Mathematical Monthly. This work was supported in part by the United States Atomic Energy Commission. (Received March 10, 1953.)

384*t*. H. M. Schwartz: *On the construction of potential functions for given asymptotic phase in a Schrödinger wave equation.*

It is known that $u''(x)+(k^2-U(x))u(x)=0$, $u(0)=0$ imply the existence of the finite limit of $u(x)/\sin(kx-d(k))$ as $x\to\infty$ for sufficiently rapid decrease of $U(x)$ for $x\to\infty$. Prompted by some interest in connection with quantum mechanical interpretation of the differential equation and of the function $d(k)$ in scattering theory, the question of the determination of the function $U(x)$ over $(0,\infty)$ when $d(k)$ is assigned for $0\leqq k\leqq\infty$ has been subject to some recent discussion [N. Levinson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. vol. 25, no. 9 (1949), C. E. Fröberg, Arkiv Mat. Astron. Fys. vol. 34A, no. 28 (1948); vol. 36A, no. 11 (1949); Arkiv Fys. vol. 3, no. 1 (1951), R. Jost and W. Kohn, Phys. Rev. vol. 87 (1952) p. 977] and preliminary results by the author were presented to the Arkansas Academy of Science [Proc. Ark. Acad. of Sci. (1951)]. In the present paper the construction of $U(x)$ is carried out by two methods both involving an iteration process in which occur nonlinear integrals of the first kind over the interval $(0,\infty)$. One of these methods is based on an explicit expression for $d(k)$ [see Courant and Hilbert, *Methoden der mathematischen Physik*, vol. I, p. 287]. The convergence of the process and uniqueness of solution is discussed under a number of restrictive conditions. It is expected to submit this paper for publication in the Proceedings of the American Mathematical Society. This work was supported in part by the U. S. Atomic Energy Commission. (Received March 10, 1953.)

385*t*. Helen M. Sternberg and R. L. Sternberg: *A two-point boundary problem for ordinary self-adjoint differential equations of even order*. II. Preliminary report.

For the preliminary case $n=2$: $(1_2)\,[p_0(x)u'']''-[p_1(x)u']'+p_2(x)u=0$, $(2_2)\,u(x_r)=u'(x_r)=0$ $(r=1,2)$ of the boundary problem (1)–(2) considered in (I) (Bull. Amer. Math. Soc. Abstract 59-4-372) the following partial converse is obtained. If $p_0(x)\leqq k_0$, $p_i(x)\leqq 0$ $(i=1,2)$ on $[a,\infty)$ and if there exists an $a_0\geqq a$ such that for $a_0\leqq x_1<x_2<\infty$ the only solution $u=u(x)$ of (1_2)–(2_2) is the trivial solution $u(x)\equiv 0$ on $[a,\infty)$, then the integrals $\int_a^\infty x^{2i-2}p_i(x)dx$ converge and $\lim\sup_{x\to\infty}[x\int_x^\infty t^{2i-2}|p_i(t)|dt]\leqq 2^{2i-1}k_0$ $(i=1,2)$. The proof is similar but not equivalent to that of Theorem 5.2 of the paper of Sternberg referred to in (I). (Received March 11, 1953.)

386*t*. Walter Strodt: *A lemma on homogeneous and isobaric differential polynomials with constant coefficients.*

Let $P(y(x), y'(x),\cdots, y^{(n)}(x))$ be a homogeneous isobaric differential polynomial, of degree D and positive weight W. Let $m=W/D$. Then the differential polynomial $Q(v(u), v'(u),\cdots, v^{(n)}(u))$ which is obtained from P by the substitution

$y(x)=v(u)e^{mu}$, $x=e^u$ effectively involves terms of weight less than W. This lemma is essential in showing the existence of logarithmic monomials which are approximate solutions of differential equations. Its proof is obtained by noting that the denial of the conclusion implies that $P(t_0, t_1, \cdots, t_n)\equiv Q(t_0, t_1, \cdots, t_n)$, so that whenever $f(x)$ is a solution of $P=0$ so is $x^m f(\log x)$. But the constancy of the coefficients implies that if $f(x)$ is a solution so is $f(x+a)$. From these two remarks it follows easily that for every positive integer k and every set of complex numbers $a_0, a_1, \cdots, a_k$ the function $[H_0(x)H_1(x)\cdots H_k(x)]^m$ is a solution of $P=0$, where $H_0(x)\equiv a_0+x$, and $H_{j+1}(x)\equiv a_{j+1}+\log H_j(x)$. Thus the manifold of $P=0$ involves $k+1$ arbitrary constants which are easily seen to be essential, and this is absurd since k is arbitrary. The result remains valid, and the method of proof changes only slightly, if m is any nonzero complex number. (Received January 22, 1953.)

387t. Walter Strodt: *Logarithmic monomials which are approximate solutions of algebraic differential equations with constant coefficients.*

Let $P_0(y(x), y'(x), \cdots, y^{(n)}(x))$ be a differential polynomial with constant coefficients, effectively of positive degree, and such that $P_0(0, 0, \cdots, 0)\neq 0$. Then there exists a logarithmic monomial $M(x)=Cx^{r_0}(\log x)^{r_1}(\log\log x)^{r_2}\cdots(\log_s x)^{r_s}$, where C is complex and the r_i are real, such that $P_0(M(x), M'(x), \cdots, M^{(n)}(x))\to 0$ as $x\to\infty$ (in any sector of the complex plane in which M is analytic). The method of proof provides an alogorithm for finding M. For each term effectively present in P_0 the ratio of weight to degree is calculated and r_0 is the minimum of these ratios. The substitution $y=ve^{r_0 u}$, $x=e^u$ transforms P_0 into $P_1(v(u), v'(u), \cdots, v^{(n)}(u))+Q_1(u, v(u), v'(u), \cdots, v^{(n)}(u))$ where every coefficient in Q_1 tends to zero as $u\to\infty$. P_1 is treated the way P_0 was treated, and thereby r_1 is determined. Repetition of this procedure leads to a finite sequence of differential polynomials $P_0, P_1, \cdots, P_s$, the terminal polynomial P_s being characterized by the property that it effectively involves terms of weight zero. The coefficient C in M is then any one of the obviously existing constants which satisfy the equation $P_s=0$. Assurance that the process terminates in finitely many steps, and a bound for the number of steps required, is given by the author's lemma on homogeneous isobaric differential polynomials, stated in the foregoing abstract. (Received January 22, 1953.)

388t. Walter Strodt: *Principal logarithmic monomials for algebraic differential equations.*

Let a system of simply-connected neighborhoods of ∞ in the complex domain be given. The notation $f(x)<1$ is to mean that for any non-negative integers j and k, and any positive ϵ, there is a neighborhood of ∞, $W(j, k, \epsilon)$, such that $|d^j f(e_k(x))/dx^j|<\epsilon$ in $W(j, k, \epsilon)$, where $e_0(x)\equiv x$ and $e_{j+1}(x)\equiv\exp(e_j(x))$. As usual $f\prec g$ means $f/g\prec 1$ and $f\sim g$ means $f-g\prec g$. A function f is called trivial if $f\prec x^{-N}$ for every positive N. A function f is called logarithmic of rank s if f is trivial or if $f\sim$ some logarithmic monomial $Cx^{r_0}\cdots(\log_s x)^{r_s}$. A set E of functions is called a logarithmic domain of rank s if every sum $M_1f_1+M_2f_2+\cdots+M_kf_k$, in which the M_j are logarithmic monomials of rank s and the f_j belong to E, is logarithmoid of rank s. If $P(x, y, y', \cdots, y^{(n)})$ is a differential polynomial with coefficients in a logarithmic domain, and if $P(x, 0, \cdots, 0)$ and at least one other coefficient are nontrivial, then there is a logarithmic monomial M, obtainable by an algorithm like the one in the author's foregoing abstract, and called a principal (logarithmic) monomial for P, such that $P(x, M, M', \cdots, M^{(n)})\prec P(x, 0, \cdots, 0)$. Equations with rational coefficients are included as a very special case. (Received January 22, 1953.)

389*t*. Walter Strodt: *Principal solutions of algebraic differential equations of first order.*

A solution of $P(x, y, y', \cdots, y^{(n)})=0$ is called principal if it is asymptotically equivalent to a principal monomial of P, as defined in the author's foregoing abstract. Let M, L, and K be principal monomials for $P(x, y, y')$, $\partial P(x, y, y')/\partial y'$, and $P(x, y, y') - y\partial P(x, 0, 0)/\partial y - y'\partial P(x, 0, 0)/\partial y'$, respectively. If $M \prec L$ and $M \prec K$ then P is called quasi-linear, and $M[\partial P(x, 0, 0)/\partial y']/P(x, 0, 0) \sim$ some logarithmic monomial $Tx^{t_0}(\log x)^{t_1} \cdots (\log_q x)^{t_q}$. Let the coefficients of P belong to a logarithmic domain where the neighborhoods of ∞ fit a sector $\mathcal{S}$ of the complex x-plane. $\mathcal{S}$ is divided into sectors $\mathcal{U}_1, \cdots, \mathcal{U}_k$ of uniqueness, and sectors $\mathcal{N}_1, \cdots, \mathcal{N}_h$ of non-uniqueness, according to the negativity and positivity of $\mathcal{R}[T \exp (it_0\theta - i\theta)]$ ($\theta = \arg x$) (unless $t_0=1$ and $\mathcal{R}(T)=0$). For each $\mathcal{U}_j$ there exists a unique solution $y(x, \mathcal{U}_j)$ of $P=0$ which is principal if neighborhoods of ∞ are redefined to fit $\mathcal{U}_j$. If $j\neq k$, $y(x, \mathcal{U}_j)$ is sometimes, but not always, different from $y(x, \mathcal{U}_k)$. For each $\mathcal{N}_j$ there exists a one-parameter family of principal solutions in $\mathcal{N}_j$. The proof is by successive approximations, after detailed preliminary reductions utilizing the theory of principal monomials. (Received January 22, 1953.)

390*t*. C. T. Taam: *On the solutions of second order self-adjoint linear differential equations.*

Let $P(x)$ and $Q(x)$ be complex-valued, $r(x)(>0)$ and $f(x)$ real-valued functions defined for $x \geqq 0$, the functions $1/P$, Q, $1/r$, and f belonging to $L(0, R)$ for every positive R. The differential equation (A): $(PW')'+QW=0$ is called disconjugate on an interval I if no solution of (A) other than the trivial solution ($\equiv 0$) possesses more than one zero on I. Let (B) denote what results in (A) if P and Q are replaced by r and f respectively. Write $P=p_1+ip_2$, $Q=q_1+iq_2$, where p_1, p_2, q_1, and q_2 are real. In this note a general criterion for (A) disconjugate on I is obtained and from which the following comparison theorem is proved: Suppose that (B) is disconjugate on a closed or open interval I_0. If $jp_1+kp_2 \geqq r$, $jq_1+kq_2 \leqq f$ almost everywhere on I_0 for some real constants j and k, then (A) is disconjugate on I_0. These results generalize those obtained previously by the author for the case $P=1$. (Received March 9, 1953.)

391. E. W. Titt: *On a theory of the linear second order partial differential equation—integration of potentials for hyperbolic exterior with n independent variables.*

In a previous paper [Bull. Amer. Math. Soc. Abstract 58-6-663] integrating factors for hyperbolic interior were developed from a weighted non-Euclidean area of a portion of a fixed noncharacteristic splitting plane, the weight function being a new type of potential called retarded. This multiple integral, when reduced to a single integral, takes on the nature of a transform of the original potential and the study of the kernel involved in the transform has proved to be an important key in the theory. The integrating factor having a jump discontinuity is obtained by applying a certain number of differential operations to the transform and it turns out that in even dimensions the same integrating factor can be used in both interior and exterior, proper attention being paid to the difference in geometrical meaning of the quantities involved. An instance of our integrating factor was used by d'Adhémar in $n=4$ exterior. In odd dimensions an alteration in the kernel, suggested by previous work, proves necessary to obtain the exterior theory. After the Green's theorem stage further differential operations would yield a formula based on logarithmetic quantities which extend quantities appearing in the work of Volterra and Tedone. (Received March 12, 1953.)

392t. J. L. Walsh: *An interpolation problem for harmonic functions.*

Complex-variable methods of orthogonal functions developed by Kakeya, Takenaka, Malmquist, Walsh, and Lokki yield a solution of the interpolation problem for functions $u(z)$ of class H, namely harmonic in B: $|z|<1$ with $\int[u(re^{i\theta})]^2 d\theta$ bounded in B and norm defined by $\|u\|^2=\int[u(e^{i\theta})]^2 d\theta$. The function $u_\alpha(z)$ of H of minimum norm with $u_\alpha(\alpha)=1$, $|\alpha|<1$, is (*) $[(1-|\alpha|^2)/(1+|\alpha|^2)]$ Re $[(1+\bar{\alpha}z)/(1-\bar{\alpha}z)]$. Given distinct points $\alpha_0, \alpha_1, \cdots$, in B, a suitable linear combination $\psi_k(z)$ of the functions (*) corresponding to $\alpha_0, \alpha_1, \cdots, \alpha_k$ has the property $\psi_k(\alpha_0)=\cdots=\psi_k(\alpha_{k-1})=0$, $\psi_k(\alpha_k)\neq 0$; the $\psi_k(z)$ are mutually orthogonal on C: $|z|=1$; every $u(z)$ of H possesses a unique formal expansion $\sum a_k\psi_k(z)$ found by interpolation to $u(z)$ in the α_k or alternately by the usual orthogonal-function method on C. This expansion represents in B the function of H of least norm equal to $u(\alpha_k)$ in the points α_k. For real α_k with $\alpha_0=0$, the $\psi_k(z)$ are the real parts of the rational functions $\phi_k(z)$ orthogonal on C defining an interpolation series $\sum C_k\phi_k(z)$ formally expanding an arbitrary function analytic in B by interpolation in the points α_k. (Received April 9, 1953.)

393t. J. L. Walsh and Lawrence Rosenfeld: *On the boundary behavior of a conformal map.*

A function $\phi(u)$ defined for $u_1\leqq u<+\infty$ is said to possess *property* B at $u=+\infty$ if, uniformly in every interval $|U|\leqq U_0$, we have $\lim_{u\to+\infty}\phi[U\phi(u)+u]/\phi(u)=1$. Here $\phi(u)$ need not be single-valued but we require this relation for any choice of values of $\phi(u)$. Let S be the simply-connected infinite strip in the plane $w=u+iv$ defined by $\phi_-(u)<v<\phi_+(u)$, $u_1<u<\infty$, where $\phi_+(u)$ has property B and where some segment $u_2<u<\infty$, $v=0$ lies in S. Let $w=f(z)$ with $z=x+iy$ map the infinite strip Σ: $|y|<\pi/2$ onto S so that $u(z)\to+\infty$ when z is in Σ and $x\to+\infty$. If x_n is any sequence of real points such that $x_n\to+\infty$ and if $\lim_{u\to+\infty}\phi_-(u)/\phi_+(u)=\lambda$, then $\lim_{n\to\infty}[f(z+x_n)-f(x_n)]/\theta(u(x_n))=z/\pi$ for z in Σ, uniformly on any closed bounded subset of Σ, where $\theta(u)=\phi_+(u)-\phi_-(u)$. The same conclusion is valid if $\phi_-(u)$ has property B and the condition $\lim\phi_-(u)/\phi_+(u)=\lambda$ is replaced by $0<m\leqq|\phi_-(u)/\phi_+(u)|\leqq M<\infty$. (Received April 25, 1953.)

394. C. Y. Wang: *Polynomial approximations to solutions of variational problems in the complex domain.*

Let C be a closed analytic Jordan curve which bounds a region D. Let $\mathcal{J}$ be the class of analytic functions $f(z)$ in D such that $f(z_0)=0$, $f'(z_0)=1$, $z_0\in D$, and $f'(z)\in H(2, C)$. It is well known that there exists $\phi(z)\in\mathcal{J}$ which minimizes $\{(1/L)\cdot\int_C|\phi'(z)|^2|dz|\}^{1/2}=\|\phi'\|$, where L is the arc length of C. In this paper we approximate $\phi(z)$ by the polynomial $Q_n(z)\in\mathcal{J}$ of degree $\leqq n$ which minimizes $\|Q_n'\|$ and get an estimate for this approximation. We make use of the fact that Hilbert space is a uniformly convex Banach space, and a rate of convergence is obtained. By means of this rate we use Szegö polynomials and Féjer-Riesz theorem to obtain the estimate $|\phi(z)-Q_n(z)|\leqq M\rho^n$ where $0<\rho<1$, ρ depends on the analyticity of the curve C, and M is some constant. Furthermore the following problem is solved. Let $L_j(f)$, $j=1, 2, \cdots, m$, be the given linear functionals in $H(2, C)$, let $f\in\overline{E}\subset H(2, C)$ such that $L_j(f)=0$ for all j, and let $f_0\in H(2, C)-\overline{E}$ such that not all $L_j(f_0)$ vanish. We find $g\in\overline{E}$ such that $\|f_0-g\|=$ minimum. Then let P_n be the polynomial of degree $\leqq n$ in

E such that $L_j(P_n)=0$ for all j and $\|f_0-P_n\|=$minimum. We show that $P_n\to g$ and obtain an estimate of the rate of convergence. (Received February 20, 1953.)

395t. Alexander Weinstein: *On an elementary property of the Darboux operators and on a theorem of Friedrichs.*

Let $u^\alpha(x_1, \cdots, x_m, t)$, $-\infty<\alpha<\infty$, denote a solution of the equation (*) $L_\alpha w^\alpha \equiv w^\alpha_{tt}+\alpha t^{-1}w^\alpha_t-\Delta w^\alpha=0$. L_α is called a Darboux operator. According to Darboux (*Théorie des surfaces*, vol. 2, p. 63) $t^{-1}w^\alpha_t=w^{\alpha+2}$. Since $L_\alpha w^\beta=L_\beta w^\beta+(\alpha-\beta)t^{-1}w^\beta_t$, it follows immediately that $L_\alpha w^\beta=w^{\beta+2}$. Therefore, (**) $L_{\beta+2n}L_{\alpha_n}\cdots L_{\alpha_2}L_{\alpha_1}w^\beta=0$. The special case in which $m=1$, $\beta=0$, and $\alpha_1=\cdots=\alpha_n=2n$ yields $L^{n+1}_{2n}w^0=0$ and was found by Friedrichs by an elaborate computation (see Courant-Hilbert, vol. 2, p. 416 ff.). This work was sponsored by the Office of Naval Research. (Received March 9, 1953.)

396. František Wolf: *Spectral decomposition of a class of operators in Banach space.*

The generalized spectral decomposition $f(A)=\int_{\sigma(A)}f(\lambda)d^nE(\lambda)$, reported in Bull Amer. Math. Soc. Abstract 58-4-369, has as its essential features (i) it exhibits that the "support" (the smallest closed set such that $f(\lambda)=0$ on it implies $f(A)=0$) of the mapping $f\to f(A)$ is $\sigma(A)$, (ii) we can speak about the spectrum of an element of the space $\sigma_A(x)\subseteq\sigma(A)$, which is the support of the mapping $f\to f(A)x$, (iii) to an arbitrary closed set S we can construct an f, zero exactly on S and $f(A)x=0$ defines a subspace invariant under A such that $\sigma_A(x)\subseteq S$, (iv) if $S\cap\sigma(A)\neq S'\cap\sigma(A)$, then the corresponding subspaces are different. Under the conditions given in the report $f(A)$ is defined for $f\in C^n$. If A is more general, $f(A)$ will be defined for a more restricted class of f's. If A is too general, the class of f's becomes quasianalytic, and the concept of the support of the mapping evidently collapses. But as long as the class is not quasianalytic, there exist "localizing" functions, and all the results still hold. A condition for this is the existence of $p(t)$ such that $p(-t)=p(t)$, $p(t+\tau)\leqq p(t)+p(\tau)$, $p'(t)\geqq 0$, $\lim_{t\to\infty} tp'(t)=\infty$, $\int_1^\infty(p(t)/t^2)dt<\infty$, $\|T^n\|\leqq l^{p(n)-\log(1+n^2)}$ (cf. Mandelbrojt, *Classes quasianalytiques*, Paris, 1935, p. 78 ff.). With similar conditions J. Wermer shows the existence of proper invariant subspaces (Duke Math. J. vol. 19 (1952) pp. 615–622). (Received March 12, 1953.)

397t. K. G. Wolfson: *On the separation of spectra.*

It is assumed that the differential equation $(px')'+(q+\lambda)x=0$ is of "limit point" type for both $t=-\infty$ and $t=\infty$. A separation theorem is proved relating the spectrum of this boundary value problem with the spectra of the boundary value problems defined on $[0, -\infty)$ and $[0, \infty)$ by the boundary condition $x(0)=0$. The method is based on the spectral theorem for self-adjoint operators on a Hilbert space. (Received March 9, 1953.)

398. D. M. Young and M. L. Juncosa (p): *On the degree of convergence of solutions of difference equations to solutions of the heat equation.* Preliminary report.

Let $u(x, t)=\sum_{n=1}^\infty a_n \sin n\pi x \exp(-n^2\pi^2 t)$ be a Fourier series solution of the heat equation $u_{xx}=u_t$ in R: $0<x<1$, $t>0$, with boundary conditions $u(+0, t)=u(1-0, t)=0$, $t>0$, and initial condition $u(x, +0)=f(x)$, $0<x<1$, where $f(x)$ is continuous except for a finite number of finite jumps and has a convergent Fourier series. For each integer M, let $u_M(x, t)$ satisfy the difference equation $u_M(x, t+p)-u_M(x, t) = r\{u_M(x+h, t)+u_M(x-h, t)-2u_M(x, t)\}$ $(h=M^{-1};\ p=rh^2)$, for all points (x, t) of R such that Mx and M^2tr^{-2} are integers. Let $\bar{u}_M(x, t)$ be obtained from $u_M(x, t)$ by bilinear interpolation. If $0<r\leqq 1/2$, then in any closed interior region of R, the convergence of $\bar{u}_M(x, t)$ to $u(x, t)$ is of the respective orders M^{-2}, M^{-1}, $M^{-\alpha}$ according as $f(x)$ has a derivative of bounded variation, or is itself of bounded variation, or satisfies a Lipschitz condition of order α, $0<\alpha\leqq 1$. If $f(x)$ is a step-function, the convergence is at least of the order M^{-1}; if M tends to infinity through a sequence of values $\{M_k\}$ such that, for each k, M_{k+1} is an integral multiple of M_k, and if the abscissa of each point of discontinuity is an integral multiple of M_k^{-1} for some k then the convergence is of the order M^{-2}. (Received March 12, 1953.)

Applied Mathematics

399t. Milton Abramowitz: *On regular and irregular Coulomb wave functions in terms of Bessel-Clifford functions.*

Expansions for the regular and irregular solutions of the differential equations $y''+(1-2\eta/\rho-((L+1)/\rho^2)y=0$ are obtained in terms of the functions $t^{n/2}In(2t^{1/2})$ and $(-1)^n t^{n/2}Kn(2t^{1/2})$ where $t=2\eta\rho$. The expansions have the important property that the coefficients are the same in both. The expression for the regular solution is shown to be uniformly convergent for all finite values of t. However, the expansion for the irregular solution is shown to be asymptotic in the sense of Poincaré. (Received February 19, 1953.)

400t. Milton Abramowitz: *On the differential equation in problem of heat convection of laminar flow through a tube.*

The solutions of the differential equation $y''+x^{-1}y+\beta^2(1-x^2)y=0$ subject to the boundary condition $y(0)=1$, $y(1)=0$ is solved in terms of the Bessel functions $z^nJ_n(z)$. The first five characteristic values β_s are determined with the aid of this expansion employing numerical methods. (Received February 19, 1953.)

401. P. W. Berg: *The existence of Helmholtz flows of compressible fluids.*

Plane steady barotropic, subsonic potential flows of a gas about a prescribed open obstacle are considered. The question concerns the existence of flows which are uniform at infinity and such that the streamlines leaving the obstacle are free streamlines extending downstream to infinity. It is shown that if the inclination of the obstacle (with respect to the flow direction at infinity) is a Hölder-continuous function of arc length which remains between the bounds 0 and π, then such a flow exists for each prescribed subsonic value of the maximum speed. The problem is reduced successively to a mapping problem and to an integral equation. The existence of a solution of the integral equation is demonstrated using the Leray-Schauder theorem. (Received March 10, 1953.)

402. Lipman Bers: *Existence and uniqueness of a subsonic gas flow past a given profile.*

Let P be a simple closed curve possessing a Hölde-continuously turning tangent except at one point z_T which is a protruding corner or cusp. Theorem: There exists a number $M^*\leqq 1$ such that for every M_∞, $0<M_\infty<M^*$, there is one and only one po-

tential gas flow around P with stream Mach number M_∞ and horizontal velocity vector at infinity. The flow depends continuously on M_∞ and the maximum local Mach number $M_{\max}$ takes on all values between 0 and 1 as M_∞ goes from 0 to M^*. If P is symmetric about the x-axis, $M_{\max}$ is a monotone function of M_∞. The proof uses pseudo-analytic functions, quasi-conformal transformations, and the Schauder-Leray degree. (The case of P smooth with the circulation prescribed was treated by Schiffman, Journal of Rational Mechanics and Analysis, Vol. 1 (1952), by entirely different methods.) (Received March 10, 1953.)

403. Peter Chiarulli: *Diffraction of a sound pulse by a moving obstacle.*

The problem considered is that of a finite flat plate moving parallel to itself at subsonic speeds and encountering at some angle a plane acoustic pulse. G. F. Carrier (unpublished) by means of a Weiner-Hopf approach has found in closed form the resulting diffraction pattern from the leading edge and from the trailing edge. It is of interest to determine the diffraction pattern for those times for which there is an interaction of leading and trailing edge effects and in particular to determine that time for which the effect of the pulse on the obstacle is no longer important; but for these time values the boundary conditions become too complicated to be amenable to the Weiner-Hopf technique. Moving coordinates are introduced and the resulting problem is equivalent to finding the linearized steady supersonic flow over a straight-edged flat wing with one supersonic edge. Standard techniques are available for solving this problem and in particular the Evvard procedure is used to determine the diffraction pattern for those time values which are of interest. (Received March 11, 1953.)

404*t*. R. C. Di Prima and G. H. Handelman: *Vibrations of rotating beams.*

This paper is concerned with certain aspects of the problem of determining the lowest natural frequency of vibration of a twisted, non-uniform rotating beam. Through an extension of the methods of W. Prager [*Theory of structures*, Brown University lecture notes] for the static case, the equations of a vibrating twisted, nonrotating beam in intrinsic coordinates has been developed. Corresponding minimum and orthogonality principles have been developed. Approximate frequencies have been found by a Rayleigh-Ritz, Galerkin procedure for a uniform beam with constant angle of twist both for the case in which the vibration is restricted to one principle direction and in general. A study of the dependence of frequency upon end restraint and rotation speed has been carried out for the straight, rotating beam. The general vector differential equations and corresponding minimum principle have been established for the non-uniform, twisted, rotating beam with various forms of end restraint. (Received March 11, 1953.)

405. David Gale: *A theory of n-person games with perfect information.*

Let Γ be an n-person game, Σ_i the set of pure strategies σ_i for player i, and P_i the ith player's pay-off function. Then σ_i is called *equivalent* to σ_i' $(\sigma_i \sim \sigma_i')$ if $P_i(\sigma_1, \cdots, \sigma_i, \cdots, \sigma_n) = P_i(\sigma_1, \cdots, \sigma_i', \cdots, \sigma_n)$ for all $(\sigma_1, \cdots, \sigma_{i-1}, \sigma_{i+1}, \cdots, \sigma_n)$, and σ_i *dominates* σ_i' $(\sigma_i \succ \sigma_i')$ if $P_i(\sigma_1, \cdots, \sigma_i, \cdots, \sigma_n) \geqq P_i(\sigma_1, \cdots, \sigma_i', \cdots, \sigma_n)$ for all $(\sigma_1, \cdots, \sigma_{i-1}, \sigma_{i+1}, \cdots, \sigma_n)$ and σ_i not $\sim \sigma_i'$. Let $A(\Gamma)$ be the game obtained from Γ by replacing every maximal set of equivalent pure strategies by a uniform mixed strategy, and let $D(\Gamma)$ be the game obtained from Γ by deleting all dominated strategies. If $\Gamma=\Gamma_0$ we define Γ_i inductively by $\Gamma_i=D(A(\Gamma_{i-1}))$. Assuming Γ_0 finite we must eventually have $\Gamma_i=\Gamma_{i+1}=\Gamma_\infty$ (definition). Theorem: If Γ_0 is the normal form of a game with perfect information, then Γ_∞ contains exactly one strategy for each player. These unique strategies may be considered as mixed strategies for Γ_0 and are then defined to be the optimal strategies for Γ_0. Examples are given to justify the definition. (Received March 4, 1953.)

406. M. M. Gutterman and C. M. Price (p): *On the approximation of product of functions.*

For $i=1, \cdots, n$, let α_i and β_i be real-valued functions defined on some set W_i such that β_i is non-negative and $\lambda_i=\beta_i\circ\alpha_i^{-1}$ is well defined on $\alpha_i(W_i)$. Suppose that each λ_i is concave (convex) on its domain $\alpha_i(W_i)$; that T_i is a subset of W_i and T_i contains two elements y_i and z_i such that $\alpha_i(y_i)\leqq\alpha_i(x_i)\leqq\alpha_i(z_i)$ for all x_i in T_i; and that a is a real number and b a non-negative real number. Then SUP $(INF)_{x_i\in T_i}[a\prod\alpha_i(x_i)-b\prod\beta_i(x_i)]$ $=\text{MAX}\ (\text{MIN})_{u_i=y_i \text{ or } z_i}[a\prod\alpha_i(u_i)-b\prod\beta_i(u_i)]$. Another such result is obtained. Formulas are developed which permit rapid calculation of the SUP and INF in the case, of some interest in practical probability work, when each $\alpha_i(x)=e^{-x}$ and each $\beta_i(x)=1-x$, $0\leqq x<1$. (Received March 9, 1953.)

407. W. S. Jardetzky: *On general solutions of partial differential equations with constant coefficients.*

To write the general solution of (1) $\sum A(\partial^n\phi/\partial x_1^a\partial x_2^b\cdots\partial x_m^g)=0$ $(a+b+\cdots+g=n,\ A=\text{const.})$ use can be made of the expression (2) $\chi=\sum_1^m v_i(x_i-\hat{x}_i)$, where v_i and $\hat{x}_i$ are parameters. An arbitrary function $f(\chi)$ satisfies (1) and the equation of characteristics provided (3) $\sum_{a+\cdots+g=n}Av_1^av_2^b\cdots v_m^g=0$. With respect to at least one parameter, say v_m, (3) can be written in the form (4) $\sum_{i=0}^n B_iv_m^i=0$. Thus, there are n expressions $\chi^{(k)}$ and n arbitrary functions $f^{(k)}(x^{(k)})$. Therefore, the general solution of (1) takes the form (5) $\phi=\sum_{(k)}\int(2m-1)\int\int f^{(k)}(\chi^{(k)};\ v_1,\cdots,v_{m-1},\ \hat{x}_1\cdots\hat{x}_m)dv_1\cdots dv_{m-1}d\hat{x}_1\cdots d\hat{x}_m$. The limits of integration have to be adjusted to conditions of a problem. The well known Whittaker's form of solutions of Laplace- and wave equation are particular cases of (5). A generalization of the Bateman's form is also possible. (Received February 26, 1953.)

408. G. S. S. Ludford: *Extensions in the applicability of Riemann's formula.*

The object of this paper is to investigate the Cauchy initial values problem for the linear hyperbolic differential equation $L(u)\equiv u_{xy}+au_x+bu_y+cu=f$, when the initial data has a finite number of discontinuities (even infinite) and when some characteristics meet the initial curve in more than one point, this latter not necessarily implying that there is a tangent characteristic. With the usual continuity conditions on the coefficients in $L(u)$, and a slight generalization of the concept of regularity, it can be shown that there is a unique regular solution of the former problem provided the initial data is absolute integrable, and that there is a unique regular solution of the latter on a certain sheeted surface associated with the original coordinate plane and initial curve. The method used in both cases is a modification of the classical method of Riemann. It shows that if a regular solution exists it is given by Riemann's formula, when that formula is suitably interpreted. Direct verification shows that the formula gives a solution of the desired kind. Problems of this kind occur both in one-dimensional gas dynamics and plasticity. (Received March 5, 1953.)

409. G. W. Morgan (p) and J. L. Fox: *On the stability of some flows of an ideal fluid with free surfaces.*

This work is an extension of an investigation begun by W. D. Hayes and C. M. Ablow (O.N.R. Contract N70nr-35807, NR-062 090; Technical Report No. 1) of small perturbations of flows with free surfaces, the primary object being the determination of the stability of the basic steady state flow. The assumption of plane irrotational motion permits the use of complex functions in the analysis. All relations are linearized with respect to the perturbation variables. The perturbation f of the complex potential is assumed to have the form $f=G_1(w_0)\exp(\lambda t)+G_2(w_0)\exp(\bar{\lambda}t)$ where w_0 is the complex velocity of the basic flow. Considerations of analyticity and the application of the boundary conditions lead to complicated eigenvalue problems for the functions G_1, G_2 and the eigenvalues λ. The sign of the real part of all admissible λ determines the stability. The problems treated are the hollow vortex bounded by a circular wall, a class of orifice flows, the flow due to the impinging of two equal and opposite jets, the flow due to the impinging of a jet on a finite plate perpendicular to the jet, and the Helmholtz plate flow. The results indicate that the impinging of opposite jets may represent an unstable flow; all other flows (with one special exception) are neutrally stable or stable. (Received March 11, 1953.)

410. L. E. Payne and H. F. Weinberger (p): *Upper and lower bounds for virtual mass.*

Let $\phi_i(x_1, x_2, x_3)$ be the velocity potential of an infinite incompressible fluid disturbed by the motion of a body with unit velocity in the x_i direction. The virtual mass W_i of the body is defined as the Dirichlet integral of ϕ_i, and is proportional to the kinetic energy of the fluid. Several upper and lower bounds for W_i are obtained. Some of the lower bounds depend on the inequality $W_iP_i \geqq V^2$ given by Schiffer and Szegö (Trans. Amer. Math. Soc. vol. 67 (1949) pp. 130–205). Here, V is the volume of the body and P_i its polarization in the x_i direction. Upper bounds for P_i were previously obtained by the authors (Bull. Amer. Math. Soc. Abstract 59-3-310). Upper bounds for W_i depend on the following principle: Let a body A be cut in two by a plane parallel to the x_i-axis. Let B and C be the two symmetric bodies obtained by reflecting the two parts of A in the cutting plane. Then $2W_i(A) \leqq W_i(B)+W_i(C)$. This upper bound can, by repeated applications, be reduced to the sum of virtual masses of axially symmetric bodies. These virtual masses are related to polarizations (L. E. Payne, Bull. Amer. Math. Soc. Abstract 59-2-225). This work was sponsored by the Office of Naval Research. (Received March 9, 1953.)

411. F. Virginia Rohde: *Large deflections of a simply supported beam.* Preliminary report.

The large deflections of a simply supported beam with concentrated load off center are considered. This case differs from that in which the load is placed in the middle (Conway, *The large deflection of simply supported beam*, Philosophical Magazine vol. 38 (1947) pp. 905–911) in two respects: the position of maximum deflection is not under the load, and the reactions are not both perpendicular to the elastic curve of the beam. Since the results are in terms of elliptic functions involving the angles at the ends of the beam and under the load, the relations between these angles cannot be found in explicit form. Therefore values of the angles must be assumed, from which the position of the load and ratios of the horizontal projection of the elastic curve and of the maximum deflection to the total length of the beam are found. These ratios, together with that of the maximum deflection to maximum deflection obtained by elementary theory, are plotted against the dimensionless ratio PL^2/EI. The results obtained by small and large deflection theory diverge considerably over most of the range of values considered, elementary theory giving the larger deflections. (Received March 4, 1953.)

412. L. V. Robinson: *Some comparisons between solutions of ordinary and partial differential equations.* Preliminary report.

Assuming the p_k to be differentiable functions of the x_k ($k=1, 2, \cdots, n$), the inverse of an operator, $\lambda \equiv p_1\ \partial/\partial x_1 + p_2\ \partial/\partial x_2 + \cdots + p_n\ \partial/\partial x_n$, is found in terms of transforms of the general type, exp (gD_k), where the g are also functions of the x_k and $D_k = \partial/\partial x_k$. It is then shown that λ and λ^{-1} in a generalized calculus bear essentially a one-to-one correspondence to $D=d/dx$ and D^{-1} in ordinary calculus. Similarly, every ordinary differential equation has its counterparts in an infinity of partials, permitting the latter to be solved in much the same manner as the former and leading to a very close parallelism between properties of their solutions. (Received March 9, 1953.)

413. K. M. Siegel and F. B. Sleator (p): *Bessel functions of nearly equal argument and order.*

An upper bound for the function $J_\nu(\nu x)$ has been given by Watson which holds for $0<x\leqq 1$ but becomes infinite as $x\rightarrow 1$. A better bound on the function over a certain range is obtained here which approaches the value 1 as $x\rightarrow 1$. This is an improvement on the previous bound for the range $[1-(2\pi\nu)^{-2}]^{1/2}<x\leqq 1$. Examples are given and corresponding expressions developed for a range of $x \geqq 1$ and for certain modified Bessel functions. (Received March 13, 1953.)

414*t*. R. L. Sternberg: *A general solution of the two-frequency modulation product problem.* II. Preliminary report.

By replacing the region R: $\cos u + k \cos v \geqq h$ of the square $0 \leqq u, v \leqq \pi$ by a suitable polygonal region, formulas are obtained suitable for evaluating to six decimals the integrals $A_{mn}(h, k) = (2/\pi^2)\int\int_R(\cos u + k \cos v - h) \cos mudu \cos nvdv$ ($m, n=0, 1$), $|h| \leqq 2$, $0<k\leqq 1$, which arise in the problem referred to in the title (see Bull. Amer. Math. Soc. Abstract 58-4-374 and W. R. Bennett, *The biased ideal rectifier*, Bell System Technical Journal vol. 26 (1947) pp. 139–169). Tables of these functions accurate to one unit in the sixth place for $h=-2(0.2)2$ and $k=0.001$, 0.01, 0.1 and 1.0 are near completion. (Received March 3, 1953.)

415*t*. John Todd: *The condition of the finite segments of the Hilbert matrix.*

The finite segments H_n of the Hilbert matrix $H=((i+j-1)^{-1}))$ enjoy a reputation of ill condition. Various measures of the condition of these matrices are estimated and the results fully confirm the bad reputation. These estimates are obtained by using the well known explicit inverse of H_n. The results have been used in the practical evaluation of methods for inversion of matrices on high speed automatic digital computing machines. (Received March 11, 1953.)

Geometry

416. H. S. M. Coxeter: *A simple way to enumerate the five parallelohedra.*

A parallelohedron is a convex polyhedron, in real affine space, which can be repeated by translation to fill the whole space without interstices. It is centrally symmetrical and has centrally symmetrical faces: say f pairs of opposite faces, v pairs of opposite vertices, and n sets of parallel edges. An argument suggested by Voronoi [J. Reine Angew. Math. vol. 133 (1907) p. 107; vol. 134 (1908) p. 278] shows that $f \leqq 7$ and that the faces can be only parallelograms or symmetrical hexagons. Such a polyhedron is represented in the real projective plane by a configuration of n lines intersecting by twos or threes in f points to form v regions [Coxeter, *Regular polytopes*, 1948, pp. 28, 30]. The most complicated case (with $f \leqq 7$) is the complete quadrangle, and every other case can be derived from this by omitting one or more of the six lines. The possible values of v are 4, 6, 7, 9, 12, corresponding to the parallelepiped, hexagonal prism, rhombic dodecahedron, elongated dodecahedron, and truncated octahedron [cf. Fedorov, Mineralogicheskoe Obshchestvo, Leningrad vol. 21.4 (1885)]. (Received March 10, 1953.)

Logic and Foundations

417. J. R. Shoenfield: *Enumerations of recursive functions.*

A recursive function enumeration (rfe) is a partial recursive $F(e, x)$ such that for every recursive function $f(x)$, there is an e, called the F-number of f, such that $f(x) = F(e, x)$ for all x. An rfe is effective if for every recursive $G(x, y)$, the F-number of $G(f(x), y)$, as a function of x, is a recursive function of the F-number of $f(x)$ and y. Effective rfe's are known to exist. If F is an rfe, the set of F-numbers of recursive functions is not recursively enumerable; in particular, F cannot be defined for all arguments. If F is an effective rfe, then the class of F-numbers of any recursive function is not recursive. It follows that there is no recursive method of deciding when two F-numbers are F-numbers of the same function, or when they are F-numbers of functions enumerating the same set. The set of F-numbers of functions which enumerate recursive (or finite) sets is not recursive. The set of F-numbers of functions which vanish infinitely often is not recursive. Several other theorems of a similar nature hold. (Received March 11, 1953.)

418t. Alfred Tarski: *Two general theorems on undefinability and undecidability.*

This paper contains a generalization of ideas known from works of Gödel and other authors. See specifically Mostowzki, *Sentences undeciable* · · · , Amsterdam, 1952; R. M. Robinson, Proceedings of the International Congress of Mathematicians, 1950, vol. 1; Tarski, Studia Philosophica vol. 1. Assumptions: $\mathfrak{T}$ is any formalized theory; S is a set of $\mathfrak{T}$-formulas including all axioms of predicate calculus with identity and closed under rules of inference; $\Delta_0, \Delta_1, \cdots, \Delta_n, \cdots$ are $\mathfrak{T}$-terms containing no variables; $\sim(\Delta_0 = \Delta_1)$ is in S; x, y are fixed $\mathfrak{T}$-variables. A function F on and to the integers is called S-definable (relative to Δ_n) if, for some formula Φ and every integer n, $(x = \Delta_n) \rightarrow [(y = \Delta_{F(n)} \leftrightarrow \Phi]$ is in S. Consider a quite arbitrary one-one correlation between $\mathfrak{T}$-expressions Ψ and integers n; $Nr\ (\Psi)$ is the integer correlated with Ψ, Ω_n is the expression correlated with n. Let $D(n) \equiv Nr[(x = \Delta_n) \rightarrow \Omega_n]$; let $P(n)$ be 0 if Ω_n is in S, and 1 otherwise. Theorem I. *If S is consistent, then functions D and P*

are not both S-definable. New assumptions: $G(n) \equiv Nr(\Delta_n)$ and $H(n, p) \equiv Nr(\Omega_n^\frown \Omega_p)$ (where $^\frown$ is the concatenation symbol) are general recursive functions. Then Theorem I implies: Theorem II. *If all general recursive functions are S-definable, then S is inconsistent or essentially undecidable.* (Received January 16, 1953.)

Statistics and Probability

419t. G. B. Kallianpur and Herbert Robbins: *Ergodic property of Brownian motion process.*

Let $X(t)$ denote the linear Brownian motion (Wiener) process, let $f(x)$, $g(x)$ be bounded and summable for $-\infty < x < \infty$, and set $\bar{f} = \int f(x)dx$, $\bar{g} = \int g(x)dx$. Theorem: if $\bar{f} \neq 0$ then, for every u, $\lim_{T\to\infty} Pr[(1/\bar{f}T^{1/2}) \int_0^T f(X(t))dt \leqq u] = G(u)$, where $G(u) = (2/\pi)^{1/2} \int_0^u e^{-y^2/2} dy$ for $u \geqq 0$, $=0$ for $u < 0$, if $\bar{g} \neq 0$, $\lim_{T\to\infty} \int_0^T f(X(t))dt / \int_0^T g(X(t))dt = \bar{f}/\bar{g}$ in probability. The same theorem holds for two-dimensional Brownian motion $X(t) = (X_1(t), X_2(t))$ with independent components, with the factor $1/\bar{f}T^{1/2}$ replaced by $2\pi/\bar{f} \log T$ and with $G(u) = 1 - e^{-u}$ for $u \geqq 0$, $=0$ for $u < 0$. (Received April 15, 1953.)

420. Eugene Lukacs: *On strongly continuous stochastic processes.*

A condition is derived which assures that the increments of a stochastic process are normally distributed. It is assumed that the increments over non-overlapping time intervals are completely independent and it is shown that the normality of the increments follows from a certain continuity property. A characterization of the Wiener process in terms of this continuity property is given. (Received March 9, 1953.)

Topology

421. R. D. Anderson: *Continuous collections of continuous curves.*

By a modification of the argument used by the author in his paper *Monotone interior dimension-raising mappings* (Duke Math. J. vol. 19 (1952) pp. 359–366) it can be shown that there exists a continuous collection G of mutually exclusive compact nondegenerate continuous curves filling up a one-dimensional compact continuum such that G with respect to its elements as points is homeomorphic to the compact Hilbert cube H. As any compact metric closed point set can be imbedded in H, it follows immediately that for any compact metric closed point set K there exist a one-dimensional compact closed point set M_K and a monotone interior mapping f of M_K onto K such that for each point k of K, $f^{-1}(k)$ is locally connected. (Received March 23, 1953.)

422. J. D. Baum: *An equicontinuity condition for transformation groups.*

The following necessary and sufficient condition for equicontinuity in transformation groups (dynamical systems) is proved: Let (X, T) be a transformation group, X compact and T_2 and minimal under T, and T abelian. Then T is equicontinuous if and only if there exists $x \in X$ such that for every continuous mapping f of X into the reals, the function $f_x(t) \equiv f(xt)(t \in T)$ is almost periodic in the left uniform functional transformation group over T to R, the reals. This result is a generalization of an unpublished theorem of Kakutani's and is proved using methods due to Gottschalk and Hedlund. Particular use is made of the equivalence of almost periodicity and equicontinuity proved by Gottschalk (Bull. Amer. Math. Soc. vol. 52 (1946) p. 633). (Received February 5, 1953.)

423. Morton Brown: *A countable connected Hausdorff space.*

The points are the positive integers. Neighborhoods are sets of integers $\{a+bx\}$, where a and b are relatively prime to each other ($x=1, 2, 3, \cdots$). Let $\{a+bx\}$ and $\{c+dx\}$ be two neighborhoods. It is shown that bd is a limit point of both neighborhoods. Thus, the closures of any two neighborhoods have a nonvoid intersection. This is a sufficient condition that a space be connected. (Received March 12, 1953.)

424. Haskell Cohen: *Cocantorian manifolds.* Preliminary report.

A locally compact Hausdorff space is an n-cocantorian manifold if its codimension (Bull. Amer. Math. Soc. Abstract 58-1-70) is n and it cannot be disconnected by a closed subset of codimension less than $n-1$. After several preliminary lemmas it is shown that every locally compact Hausdorff space of codimension n has a compact n-cocantorian manifold. (Received March 11, 1953.)

425*t*. M. L. Curtis: *Monotone deformation-free mappings.*

If B is an open subset of the n-sphere S^n and B is uniformly locally connected in terms of Čech cycles with integer coefficients and also in terms of Čech cocyles with reals (mod 1) as coefficients in dimensions $0, \cdots, n$, then $A=S^n-B$ is uniformly locally connected in terms of Čech cycles with integer coefficients in the same dimensions. If M is a continuum separating S^n, and M is deformable into a domain A of S^n-M with a deformation $h:M\times I\rightarrow\bar{A}$ such that $h(M\times t)\cap M=0$ for $t>0$, then $B=S^n-A$ satisfies the hypothesis of the above result so that A is uniformly locally connected in terms of integral Čech cycles. If X and Y are connected and locally connected by arcs, Y is simply connected and $f:X\rightarrow Y$ is monotone and onto, then the induced map $f_*:\pi_1(X)\rightarrow\pi_1(Y)$ is onto. These three lemmas are combined to show that if the deformation of M is monotone and M is locally simply connected, then A is ULC^n (=uniformly locally connected in the homotopy sense in dimensions $0, \cdots, n$). (Received March 11, 1953.)

426. M. L. Curtis: *Universal spaces for fibre spaces.* Preliminary report.

A fibre space consists of spaces E, B and a map p of E onto B such that the covering homotopy theorem is satisfied for arbitrary topological spaces. Following a definition given by R. Thom, one says that two fibre spaces (E_1, B_1, p_1) and (E_2, B_2, p_2) are fibre homotopy equivalent if $B_1=B_2$ and there exist fibre mappings $\alpha:E_1\rightarrow E_2$ and $\beta:E_2\rightarrow E_1$ such that $\alpha\beta\simeq 1$ and $\beta\alpha\simeq 1$, all maps and homotopies inducing the identity on the base. Given a topological space B and a point x in B, a class $C(B, x)$ of fibre spaces is defined. (E, K, p) belongs to $C(B, x)$ if: (1) K is covered by a collection $\{V_\nu\}$ of open sets such that for each V_ν a definite fibre homotopy equivalence $\phi_\nu: p^{-1}(V_\nu)\rightarrow V_\nu\times\Omega$ is given, where Ω is the space of loops at x in B. (2) on $V_\nu\cap V_\mu$, ϕ_ν and ϕ_μ are fibre homotopic. If B is locally contractible and E_x is the space of paths from x in B, then (E_x, B, p) is a universal space for fibre spaces in $C(B, x)$ which have a finite complex as base space. The classification theorem also holds for this class of spaces. Principal fibre bundles with compact connected Lie group G as fibre belong to $C(B_G, x)$ where B_G is the base of the contractible universal bundle for G. This leads to the result that such principal bundles are equivalent if and only if they are fibre homotopy equivalent. (Received March 11, 1953.)

427. Robert Ellis: *Continuity and homeomorphism groups.*

Let X be a T_2-space, let Φ be a group of homeomorphisms of X onto X, let $\pi: X\times\Phi\to X$ be the map such that $(x, \phi)\pi = x\phi$ ($x\in X$, $\phi\in\Phi$), and for each $x\in X$ let $\pi_x:\Phi\to X$ be the map such that $\phi\pi_x = (x, \phi)\pi = x\phi$. The group Φ is *rigid* provided that if $\mathcal{J}$ is a filter base on Φ and x, y, $z\in X$ are such that $x\mathcal{J}\to z$ and $y\mathcal{J}\to z$, then $x=y$. The following two theorems and related results are proved. (1) Let X be locally compact, let Φ be abelian and rigid, let $\mathcal{T}$ be a first countable topology on Φ which makes each $\pi_x(x\in X)$ continuous, and let $x_0\in X$ be such that the closure of $x_0\Phi$ is X. Then π is continuous at (x_0, ϕ) for all $\phi\in\Phi$. (2) Let X be a minimal orbit closure under Φ, let X be first countable compact, let Φ be abelian, let each $\pi_x(x\in X)$ be one-to-one, and let each $\pi_x^{-1}\pi_y$ (x, $y\in X$) be homeomorphic. Then Φ is equicontinuous. (Received March 11, 1953.)

428. Mary E. Estill: *A biconnected set having no widely connected subset.*

In footnote number 8 of *The closure of types of connected sets* which appears in the Proceedings of the American Mathematical Society vol. 2 (1951) pp. 178–185, P. M. Swingle raises a question raised with him by E. W. Miller. The question he intended to raise was that of the existence of a biconnected set in the plane without a dispersion point such that no subset of it is widely connected. There is such a set and an example is given. (Received March 12, 1953.)

429*t*. F. A. Ficken: *Closed linear spaces of continuous functions orthogonal to closed subsets of a completely regular space.* Preliminary report.

Let X be a completely regular topology space. Let Y be a complex (or real) linear space of complex- (or real-) valued functions $y(x)$ continuous on X, let Y contain all bounded continuous $y(x)$, and let Y be so topologized that the operations of addition and multiplication by scalars are continuous. For each closed $F\subset X$ (or closed linear $L\subset Y$) define $F^{\perp}$ (or $L^{\perp}$) to consist of those $y\in Y$ (or $x\in X$) such that $y(x)=0$ for all $x\in F$ (or $y(x)=0$ for all $y\in L$). Let $\mathcal{J}$ (or $\mathcal{L}$) denote the complete lattice of closed subsets F of X (or closed linear subspaces L of Y). The correspondence $F\to F^{\perp}$ maps $\mathcal{J}$ into $\mathcal{L}$, and $F^{\perp\perp}=F$. The correspondence $L\to L^{\perp}$ maps $\mathcal{L}$ into $\mathcal{J}$, but L may be properly contained in $L^{\perp\perp}$. The correspondence $L\to L^{\perp\perp}$ is a closure operation on $\mathcal{L}$. Let $\mathcal{P}$ denote the complete lattice consisting of those $L\in\mathcal{L}$ for which $L^{\perp\perp}=L$. Then $F\to F^{\perp}$ maps $\mathcal{J}$ into $\mathcal{P}$. The principal result of this note is that $F\leftrightarrow F^{\perp}$ is a lattice anti-isomorphism between $\mathcal{J}$ and $\mathcal{P}$. (Received March 9, 1953.)

430*t*. E. E. Floyd: *A theorem on maps from an n-sphere to n-space.*

Let f be a continuous map from an n-sphere S to n-space E. If x and y are points of S, it is proved that there exists a rotation r of S with $f(rx)=f(ry)$. This result is an extension of a well known theorem of Borsuk concerning antipodal points. (Received March 12, 1953.)

431. E. E. Floyd: *Orbit spaces of finite transformation groups.* I.

Let X be a finite simplicial complex, and let G be a finite transformation group operating simplicially on X. Let Y denote the space of orbits of X. It is proved that if X is either homologically trivial over the integers or an absolute retract, then so also is Y. (Received March 12, 1953.)

432. S. I. Goldberg: *Representation theory of Lie algebras.* Preliminary report.

In the paper (E) *Extensions of Lie algebras and the third cohomology group* (to appear in the Canadian Journal of Mathematics) the author developed certain aspects of representation theory and showed that the extension problem under consideration was susceptible to treatment by the cohomology theory of Lie algebras. Certain other notions in (E) are rephrased in the language of cohomology thereby further indicating the close contact between cohomology and representation theory. The following result is proved: "Let L be a Lie algebra over an arbitrary field. An extension $U = (L, V, W; \beta)$ of the L-module W by the L-module V determines a unique 1-cohomology class of L in the L-module Hom (V, W). Conversely, every 1-cohomology class of L in Hom (V, W) determines an extension U up to L-isomorphism leaving W elementwise fixed." The factor set β is actually an element of Z^1 $(L, \text{Hom } (V, W))$. A cup product is defined and it is then conjectured that H^q $(L, \text{Hom } (V, W)) \cong H^{q+2}(L, W)$. (Received March 9, 1953.)

433*t*. J. S. Griffin, Jr.: *Fibre spaces with totally disconnected fibres.*

Let $\{X, B, \pi\}$ be a fibre space in the sense of Hu (Proc. Amer. Math. Soc. vol. 1 (1950) p. 756) such that if $b \in B$ then $\pi^{-1}(b)$ is totally disconnected. Then if Y and A are topological spaces and p: $Y \rightarrow A$ is continuous onto, for any continuous functions f: $Y \rightarrow X$ and g: $A \rightarrow B$ such that $\pi f = gp$, each homotopy N of g may be uniquely covered by a homotopy K of f, i.e., there is exactly one homotopy K of f such that $\pi K = Np$. Immediate consequences of this lemma are first and second covering homotopy theorems for $\{X, B, \pi\}$, with no hypotheses on the domains of the functions being deformed, together with the uniqueness of these homotopies. If C is a topological space which is simply connected and locally arcwise connected and f: $C \rightarrow B$ is continuous, then for each $c \in C$ and $x \in \pi^{-1}f(c)$ there is a continuous function g: $C \rightarrow X$ such that $g(c) = x$ and $\pi g = f$; if C is connected this function is unique. Thus if B is simply connected and locally arcwise connected, $\{X, B, \pi\}$ has a cross section. (Received February 10, 1953.)

434. O. G. Harrold, Jr.: *The enclosing of simple arcs by simple closed surfaces in three-space.*

The set K in 3-space is called locally polyhedral at p if some neighborhood of p meets K in the null set or a finite polyhedron. The arc J is said to have property $\mathfrak{P}$ provided that to each point x of J and positive ϵ there is a set K in 3-space such that (i) K is a topological 2-sphere, (ii) x lies in the interior of K, (iii) diameter of K is less than ϵ, (iv) cardinal $K \cap J =$ order (Menger-Urysohn) of x in J, and (v) K is locally polyhedral at each point of the complement of J. Theorem I. If the arc J has Property $\mathfrak{P}$ and ϵ is positive, there is a polyhedral 2-sphere K whose interior contains J and which lies in the ϵ-neighborhood of J. Theorem II. The complement of J is an open 3-cell. (Received March 10, 1953.)

435*t*. V. L. Klee, Jr.: *A remark on compact convex sets.*

By using a theorem of Keller [Math. Ann. vol. 105 (1931) pp. 748–758], the following result is established. Theorem: Suppose K is an infinite-dimensional compact convex subset of a linear topological space E. Then each of the following implies that K is homeomorphic with the Hilbert parallelotope: (a) there is a countable set F of continuous linear functionals on E such that for any two distinct points x and y

of K there is an $f\in F$ for which $f(x)\neq f(y)$; (b) E is a normed linear space; (c) K is separable and E is a normed linear space in its weak topology. The result (b) solves the problem proposed in the author's *Convex bodies and periodic homeomorphisms in Hilbert space* [Trans. Amer. Math. Soc. vol. 74 (1953) pp. 10–43]. (Received February 23, 1953.)

436. E. E. Moise: *Affine structures in 3-manifolds.* VIII. *Invariance of the classical knot-types.*

Let P and P' be polygons in 3-space, and let f be a homeomorphism of 3-space onto itself, preserving orientation, and throwing P onto P'. It is shown that the knot-types of P and P' must then be the same. The basic definitions of knot-theory can therefore be generalized as follows: A knot is a tamely imbedded simple closed curve J. The knot-type of such a J is the set of all curves J' for which there is an orientation-preserving homeomorphism throwing 3-space onto itself and throwing J onto J'. The above theorem shows that the classical knot-types are subsets of the classical knot-types. The methods of proof are extensions of those used in *Affine structures in 3-manifolds.* V. *The triangulation theorem and Hauptvermutung* (Ann. of Math. vol. 56 (1952) pp. 96–114). (Received March 30, 1953.)

437. J. H. Roberts: *A topological mapping of the rational points of Hilbert space into the plane.*

An affirmative answer is given to the following question, raised by P. Erdös: Is the set of rational points of Hilbert space (a 1-dimensional set) topologically equivalent to a subset of the Euclidean plane? The proof is dependent upon a particular homeomorphism of Hilbert space into a subset of the Hilbert cube, under which, for any point, the number of irrational coordinates is fixed or else increases by one. The following more general theorem is proved: If M_k is the set of points of Hilbert space each of which has at most k irrational coordinates (M_k is $(k+1)$-dimensional), then M_k is topologically equivalent to a subset of E_{2k+2}. (Received April 6, 1953.)

438. J. W. Siry: *Chromatic polynomials of large maps.*

Birkhoff and Lewis (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 355–451) stated a strong form of the four-color conjecture in terms of conditions on chromatic polynomials and established, by determining the polynomials involved, that these conditions are satisfied for certain proper maps which are regular in the sense of Birkhoff (Amer. J. Math. vol. 35 (1912) pp. 115–128). In a program to extend this study to include the truncated icosahedron, Rudebock (Thesis, Maryland, 1948, and further unpublished results) and Hall (results as yet unpublished) obtained polynomials for numerous maps having 25 or fewer regions. Further progress has been greatly facilitated by the method of the present paper for determining chromatic polynomials of large maps. The method is based upon an analysis of the structure of sets of maps by means of a certain class of reflexive, symmetric binary relations and a related class of equivalence relations. When applied to a class, M, of maps each having a certain proper 9-ring and a homeomorph invariant under a transformation group isomorphic with the symmetric group of degree 3, the method yields an expression for each $P_n(\lambda)\in M$ in terms of a set of thirty-six $P_{n-k}(\lambda)$, where $k\geqq 6$. This brings within range the determination of the chromatic polynomial of the truncated icosahedron itself. (Received March 14, 1953.)

439. R. L. Wilder and J. P. Roth (p): *On certain inequalities relating the Betti numbers of a manifold and its subsets.*

Let M be a closed subset of a compact space S. Then, with a field as coefficient group, the homology group $H^r(M)$ is isomorphic to the direct sum of groups $H_0^r(S)$ and $H_1^{r+1}(S, M)$, subgroups of $H^r(S)$ and $H^{r+1}(S, M)$, respectively. Thus $p^r(M) \leqq p^r(S) + p^{r+1}(S, M)$. In particular, if S is an orientable n-gcm, then $p^r(M) \leqq p^r(S) + p^{n-r-1}(S-M)$, a relation which for $r=n-1$ was recently obtained by P. A. White [Proc. Amer. Math. Soc. vol. 3 (1952) pp. 488–498, Theorem 4]. Or, if U is an open subset, with boundary M, of an orientable n-gcm, then $p^r(M) \leqq p^r(\overline{U}) + p^{n-r-1}(U)$; so that if U is ulc^{n-r-1}, then $p^r(M) \leqq p^r(\overline{U}) + p^{n-r-1}(\overline{U})$, which is a generalization of a relation due to M. Morse [Proc. Nat. Acad. Sci. U.S.A. vol. 38 (1952) pp. 247–258]. Another inequality of Morse (loc. cit.) generalizes to the form $p^{n-r-1}(S-M) \leqq p^{n-r-1}(S) + p^r(M)$, where again S is an orientable n-gcm, M a closed subset of S. A third relation (Morse, loc. cit.) will be treated in a subsequent paper. (Received March 10, 1953.)

440. Hidehiko Yamabe: *On the conjecture of Iwasawa and Gleason.*

Let G be a connected locally compact group, and U be a compact neighborhood of the identity element. Suppose that a sequence of compact sets $\{D_n\}$ satisfies the following properties: (i) $\{D_n\}$ converges to a compact subgroup contained in U. (ii) $D_n^n \subset U$, $D_n^{n+1} \not\subset U$. (iii) For all rational numbers between 0 and 1, $\lim D_n^{[sn]} = D(s)$ does exist. Moreover $D(1) - \bigcup_{\mu<1} D(\mu) \neq \emptyset$. Then we can find a sequence $\{\theta_n\}$ of functions in $L_2(G)$, and a sequence $\{x_n\}$ of elements with $x_n \in D_n$, such that $\{n(x_n\theta_n - \theta_n)\}$ converges weakly to a function $\tau \neq 0$ in $L_2(G)$. Here n has only to range over a suitable subsequence of natural numbers. By making full use of above statement we can prove the following theorem: a locally compact connected group with no small normal subgroup is a Lie group. This gives an affirmative answer to the conjecture of Iwasawa and Gleason. The above theorem is known to be equivalent to the theorem: every locally compact group is a generalized Lie group. (Received March 26, 1953.)

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