freedom from bias which was previously commented on. In either case there is no "recurrent carelessness in maintaining a strict distinction" between use and mention, as charged in the editor's preface.) The English word "proposition" has the same connotation. Moreover, it is not necessary, in using it, to settle the question, mentioned in the editor's Note 1, as to what propositions are. But the word "sentence," particularly when accompanied by such a preface, suggests an insistence on the linguistic viewpoint. This imposes on the work a bias which was not there originally; and it interferes somewhat with the intuitiveness of the approach also. If, as the editor himself comments in his Note 1, it is not necessary to decide the nature of the objects of logical study, why insist on a terminology which commits one to a particular view of it? It is a pity that what is otherwise an excellent translation should be marred by such pedantry.

To sum up, the book of Hilbert and Ackermann is one of the classics of the logical literature. In spite of the fact that it reflects the state of the science twenty-five years ago, with some changes of detail but no thorough-going revision, it is still the best textbook in a Western European language for a student wishing a fairly thorough treatment. The English translation is well done. The reviewer regrets certain features of it, and in particular regrets that it was published without consultation of the surviving author; but that does not alter the fact that it is a real service to English speaking students of the subject. Its content does not differ from that of the latest German edition in any important way.

HASKELL B. CURRY

Introduction to the theory of games. By J. C. C. McKinsey. New York, McGraw-Hill, 1952. 10+371 pp. $6.50.

This book is intended as a textbook for advanced undergraduate and graduate students. It fills, perhaps uniquely at present, a wide existing need which the now classical book of von Neumann and Morgenstern cannot satisfy on this level. In addition to the normal interest of mathematicians in the theory of games there is also the great interest of economists and many applied mathematicians in the theory; much of what is now called operations analysis, military and otherwise, makes copious use of this theory. This textbook, therefore, which presupposes essentially a knowledge of advanced calculus, will be useful to students and workers in several fields. In the scope of about 360 pages it discusses the principal topics which, by general agreement, should fit into an introductory text. Most of the book is devoted to zero-sum two-person games, but there are several chapters on n-person games and non-zero-sum games. Separate chapters are devoted to games in extensive form, continuous games, separable games, and games with convex payoff functions. By way of applications there are chapters on linear programming and statistical inference. The final chapter is devoted to unsolved problems.

The author's graceful style is an attractive feature of the book. He is professor of a subject (philosophy) where style is sometimes more important than content, and he writes on a subject (mathematics) where content is always all-important. The combination of attractive features of both subjects is a very felicitous one. In the process of formulation of new theory the author passes back and forth in a non-confusing way from a discussion of practical problems to the construction of the mathematical models, to the greater clarification of the former and the richer motivation of the latter. In the development of the mathematical theory the need for the introduction of new definitions is justified by reference to the development of the theory, and the choice of a particular definition over several possible alternative definitions is justified by a discussion of their consequences. Even books on abstract mathematical disciplines could emulate this procedure with profit; the result is so much more readable than an unbroken succession of definitions and theorems.

The reviewer's chief criticism is that some of the author's proofs are too formalistically algebraic. The result is to make these proofs unnecessarily long and to conceal from the student the basic facts which underlie the proofs. (This is in such contrast to the author's general procedure.) We shall cite a few examples of this. (1) The proof of the fundamental theorem of game theory (page 34). Actually the discussion of a graphical method of solution on p. 52 et seq. can, with minor additions, be made to include a rigorous and perspicuous proof. Another possibility is the very elegant and short proof of Karlin (*Contributions to the theory of games*, ed. by Kuhn and Tucker, Annals of Mathematics Studies, no. 24, Princeton, N. J., p. 135). (2) The proof of the fundamental theorem for continuous games (p. 186). The proof due to Wald (e.g., *Statistical decision functions*, New York, Wiley 1950, p. 38, Theorem 2.2) is elementary and proves much more in a much smaller compass. (3) The proof that the characteristic function is superadditive (p. 307, Theorem 15.1, (III)). The characteristic function of a set of players is the total amount these players can expect to obtain if they form a coalition, even if their strategy is known to their opponents. Obviously, therefore, the characteristic function of a coalition of two sets of players cannot be less than the sum of the characteristic functions.

It is to be regretted that the chapter on statistical inference, which could have been one of the most interesting in the book, is not on the same high level as the rest of the book. It would have been easy to use simple, interesting, and meaningful examples; many such are available in the literature for the cases where minimax solutions are simple to give. Indeed, the proof itself that a given decision function for a meaningful example is minimax could be very instructive. The author correctly makes the point that one cannot regard Nature as a malevolent opponent of the statistician. The statistician who adopts a minimax strategy is simply following a conservative policy. Yet the student may be forgiven if he is puzzled by such expressions as (1) " . . . the possible mixed strategies which nature (sic) may use," (2) "*S* (the statistician) does not know exactly what mixed strategy nature uses," or (3) "the probability with which nature played her various strategies." Certainly many leading statisticians would deny that in many realistic problems the notion of an a priori distribution is tenable. The author must be well acquainted with the epistemological problem involved. As a matter of fact, modern statistics began with an emphatic rejection of the notion that statistical inference was possible only by use of a priori distributions. (This denial alone is, of course, not a sufficient condition for progress, and some argue that it is not necessary or even desirable.) On p. 283 the author makes the point that the statistician is not especially interested in whether the two-person game between Nature and the statistician is determined or not. At this point the perplexed reader may rightly ask what the connection is between game theory and statistical inference. The answer (unfortunately not given by the author) is that it is often of great technical help to the statistician to know that the "game" is determined. In many problems one finds a minimax decision function by first obtaining a least favorable a priori distribution (optimal strategy for Nature viewed as an opponent of the statistician). The determinateness of the game is also a powerful tool for proving complete class theorems (e.g., Wald, loc. cit., Section 3.6).

Incidentally, the notion of a "best" strategy (p. 84), as compared with merely "optimal" strategies, has long been known in statistics where Wald introduced the notion of "admissibility." A discussion of this idea and the notion of a complete class (due to Lehmann) would have enriched the chapter on statistical inference. These ideas form an interesting bond between the two theories.

It would also have been interesting and profitable to describe some of the contributions to the theory of games made by statisticians in the course of their researches in statistics. For example, elsewhere the author cites as a noteworthy phenomenon an example of a continuous game where every pure strategy is employed in the optimum strategy. Such examples are commonplace in the statistical literature; several interesting ones (designed for another purpose) are to be found in Ann. Math. Statist. (1950) p. 190. Other citable results are those on the equivalence of behavior strategies and mixed strategies under general conditions (Ann. of Math. (1951) p. 581), and nontrivial results on the elimination of randomization (Ann. Math. Statist. (1951) p. 1 and p. 112).

The above criticisms should be regarded as directed at minor blemishes of a highly meritorious piece of work. The mathematical public is indebted to the author for an excellent and highly readable book, which this reviewer read with pleasure.

J. WOLFOWITZ

Leçons d'analyse functionelle. By F. Riesz and B. Sz.-Nagy. Budapest, Akadémiai Kiadó. 8+448 pp. About $7.50.

This work is superb.

For the field which it covers, it cannot be approached now nor will be soon by other books. It is not presented as a treatise for specialists, the essential purpose of which is to report advanced and complex results. Nor is it written as a textbook for the young student. Its aims are much higher and much more elegant. And in accomplishing these aims its authors have put together a magnificent advanced treatise and a most excellent though not elementary text. The purpose of the work is to set down, within the spirit and context of the undertaking, a certain coherent and central portion of mathematics in final and definite form. And within the spirit of the undertaking, this version is final and correct. Whether it is the only possible such version is another question, the answer to which is not important at this point. The hallmark of the work is its balance and good taste: in the choice of subjects, in the extent and detail in which they are developed, in the methods used to present them, and in the critical question of style and exposition.

The subjects treated are the modern theory of integration and differentiation, and the theory of linear operators which is based upon these concepts. Thus we find discussion of the space L^2 of square integrable functions, of abstract Hilbert space, of the space C of continuous functions. The latter is connected to integration theory by the fundamental correspondence between linear functionals and measures. This leads to a brief treatment of the spaces L^p, $p \geqq 1$, of reflexive spaces, and finally, of Banach spaces. For these various