The Lebesgue integral. By J. C. Burkill. Cambridge University Press, 1951. 8+87 pp. $2.50.

This tract is an exposition of classical Lebesgue theory on the real line, written for "those who have no wish to plumb the depths of the theory of real functions." The treatment throughout is fairly standard. The chapters are as follows: I. Sets of points; II. Measure (after de la Vallée Poussin); III. The Lebesgue integral (using ordinate sets); IV. Differentiation and integration (based on the Vitali covering theorem); V. Further properties of the integral (change of variable, the Fubini theorem, and the L^p-spaces); VI. The Lesbesgue-Stieltjes integral.

The limitations which the author imposes on the scope of his treatment are severe. For example, general measure theory is not even mentioned, which automatically excludes such timely topics as the Radon-Nikodým theorem and the theory of Cartesian product measures. Nor does the concept of set function occur, even for the real line; monotone functions, functions of bounded variation, and absolutely continuous functions are treated as point functions only. It follows that the book will be of little direct use to students interested in such fields of modern analysis as the general theory of Hilbert space, integration on locally compact groups and its consequences, or the fundamentals of probability theory, to name a few.

Of course, it can always be argued that the study of general measure and integration theory can with benefit be preceded by the study of a short treatment of the classical case, such as is presented here. And the present tract is a very clear and concise exposition of those topics with which it is concerned.

L. H. Loomis

Measure theory. By P. R. Halmos. New York, Van Nostrand, 1950. 11+304 pp. $5.90.

In this book Professor Halmos presents an account of the modern theory of measure and integration in the generality required for the study of measure in groups. Thus finiteness conditions are imposed only where necessary, and algebraic and topological aspects are appropriately stressed. Although written primarily for the student, the many novel ideas in the book and its store of interesting examples and counter examples have already made it an indispensable reference for the specialist. The clarity of expression and the sprightly style which are characteristic of the author make the exposition a pleasure to follow.

In the later chapters a knowledge of general topology and topological groups is presupposed, but the first part is intended to assume only undergraduate algebra and analysis. However, some previous acquaintance with the Lebesgue integral in the real domain is almost indispensable for an appreciation of the more abstract theory set forth here. The book does not aim to be exhaustive, but many alternative developments and points of contact with other mathematical theories are indicated especially in the exercises at the end of each section, which constitute an extensive and important part of the book. Some of these are examples, corollaries, and variant formulations intended to sharpen the student's understanding, while others are really supplementary sections of the text, in condensed form. Many are interrelated, for instance those concerned with measure in semirings. With the aid of brief but well chosen hints a large amount of additional material is thus brought within the scope of the book. An adequately prepared reader who takes the trouble to work through many of these exercises can hardly fail to acquire a feeling for the subject and find it exciting.

The first half of the book contains the standard theory, but in a somewhat more general formulation than is usual. After discussing extension theorems and various types of convergence the integral is defined by a completion process, starting with simple functions. The integral of a general function f is defined as the limit of the integrals of a mean fundamental sequence of simple functions that converges in measure to f. This definition is technically convenient, and has the advantage of achieving full generality in one step, without the need for successive extension to unbounded functions, domains of infinite measure, or functions not of constant sign. A chapter on additive set functions, their decompositions and Radon-Nikodým derivatives, and one on product measure, giving a very simple treatment of Fubini's theorem, complete this part of the book. A chapter is then devoted to an introductory treatment of measurable transformations, function spaces, and isomorphisms of measure algebras, but there is no attempt to develop ergodic theory proper. A chapter on probability sets forth the basic theorems and definitions in measure-theoretic terminology, and includes an heuristic introduction along the lines of the author's well-known expository article on the foundations of probability [Amer. Math. Monthly vol. 51 (1944) pp. 493–510].

The last three chapters, about a quarter of the book, contain an elegant account of the theory of measure in locally compact Hausdorff spaces, Haar measure in groups, and Weil's duality theorem. For the most part the development here follows Kodaira and Kakutani and culminates in a proof of their theorem on the completion regularity of Haar measure. Much of this material appears here for the first time in book form. These chapters constitute the focal point of the whole development, a fact which explains to some extent the arrangement and choice of material in the earlier chapters. For instance, properties of Carathéodory outer measure, and most results pertaining specifically to measure in metric spaces, appear only among the exercises. For the same reason, many important results that belong properly to the theory of real functions are omitted altogether. There is no discussion of integration as inverse to differentiation, and the density theorem appears only in a generalized form applicable to Haar measure. The surprising thing, however, is that so much material is included without undue condensation. It seems likely that this book will come to be recognized as one of the few really good text books at its level. It can hardly fail to exert a stimulating influence on the development of measure theory.

J. C. OXTOBY

A decision method for elementary algebra and geometry. By Alfred Tarski. Prepared for publication with the assistance of J. C. C. McKinsey. Berkeley and Los Angeles, University of California Press, 1951. 3+63 pp. $2.75.

The results of this monograph were obtained by the author in 1930. The material in its full development was published privately (and hence not reviewed in the Bulletin) by the Rand Corporation in 1948. The present edition is simply a reprint of that edition with corrections and some supplementary notes.

By elementary algebra Tarski means that part of the theory of real numbers which can be expressed in the formal language described as follows: (1) Variables of this language stand for real numbers. (2) There are three constants, "0", "1", and "-1". (3) A term is defined (recursively) as a constant or variable, or else of the form $\alpha+\beta$ or $\alpha\cdot\beta$ where α and β are any terms. (4) An atomic formula is either of the form $\alpha=\beta$ or of the form $\alpha>\beta$ where α and β are terms. (5) A formula is either an atomic formula or made up from atomic formulas by means of truth functions and quantifiers in the usual manner of the first order function calculus. In this language we can discuss any integer and any polynomial we choose; but, although we can talk of all real numbers having a certain property, we cannot talk of *all* integers having a property or of *all* polynomials having a