Methods of algebraic geometry. Vol. 2. By W. V. D. Hodge and D. Pedoe. Cambridge University Press, 1952. 9+394 pp. $7.50.

Having carefully laid the algebraic foundations in volume I (see Bull. Amer. Math. Soc. vol. 55 (1949) pp. 315–316), the authors first define an algebraic *variety* to be the aggregate of points determined by the vanishing of a set of homogeneous polynomials over a given field K. The ground field K is quite general except that it is not allowed to have a finite characteristic. In elementary treatments of real analytic geometry, one often finds it convenient to make temporary use of complex coordinates. Here, similarly, algebraic extensions of K are often introduced for special purposes. Great care is taken in defining a *generic* point (van der Waerden's "allgemeine Nullpunkt") of an algebraic variety, and a generic member of a system of k-spaces S_k. It is proved that a generic S_{n-d} meets an irreducible d-dimensional variety V_d in a finite number of points, each of which is a generic point of V_d. The number of points is the *order* of V_d. Much use is made of the so-called *Cayley form*, which is the "zugeordnete Form" of van der Waerden and Chow (Math. Ann. vol. 113 (1937) pp. 692–704).

Chapter XI is a thorough treatment of algebraic correspondences, following van der Waerden and Weil. The general principles are illustrated by application to two classical problems: finding the transversals of four skew lines in S_3, and reducing the general ternary cubic form to $X_0^3+X_1^3+X_2^3+6\lambda X_0X_1X_2$.

In Chapter XII, the theory of intersection leads naturally to the theory of equivalence, which is defined as follows. Two varieties V_a and V_a' on a V_n (in projective space of more than n dimensions) are said to be "equivalent in the narrow sense" if they belong to the same continuous system. Two such varieties are said to be "equivalent in the wide sense" ($V_a \equiv V_a'$) if there exists another variety V_a'' such that V_a+V_a'' and $V_a'+V_a''$ are equivalent in the narrow sense. Equivalences $U_a \equiv U_a'$ and $V_a \equiv V_a'$ imply $U_a+V_a \equiv U_a'+V_a'$, but there does not necessarily exist a variety X_a such that $U_a+X_a \equiv V_a$. The desirable group property is achieved by inventing a *virtual* variety V_a-U_a and writing

$$V_a - U_a \sim V_a' - U_a'$$

when $V_a+U_a' \equiv U_a+V_a'$. The authors prove that, when the fundamental variety V_n is a flat n-space, any variety V_a of order g is equivalent to gS_a, where S_a is a flat a-space. They then mention the generalization "which has exercised the minds of many geometers:"

For any absolutely irreducible variety V_n, and for each value of a $(0 \leqq a \leqq n)$, there exists a *base* $V_a^{(1)}, \cdots, V_a^{(k)}$ such that any V_a on V_n satisfies an equivalence

$$\rho V_a \sim \rho_1 V_a^{(1)} + \cdots + \rho_k V_a^{(k)},$$

where $\rho, \rho_1, \cdots, \rho_k$ are integers $(\rho \neq 0)$. No attempt is made to prove this "theorem of the base" in its full generality, but various methods are suggested while proving it for quadrics (Chapter XIII) and for Grassmann varieties (Chapter XIV). These methods apply to varieties which admit transitive groups of automorphisms.

The chapter on quadrics is particularly welcome for its clarity and completeness. Here it is shown, for instance, that the S_d's on a non-singular quadric Q_{n-1} form a single irreducible system of dimension $(d+1)(2n-2-3d)/2$. The classical theory of "eight associated points" (the complete intersection of three ordinary quadrics Q_2) is generalized as follows: If $P_1=0, \cdots, P_r=0$ are the envelope equations of r points $(r \geqq 2n+2)$ such that any quadric Q_{n-1} through all but one of the points passes through the remaining one, then there exists an identity of the form

$$\lambda_1 P_1^2 + \cdots + \lambda_r P_r^2 = 0.$$

The chapter on Grassmann varieties includes applications to enumerative geometry. For instance, the theory immediately yields the interesting result (p. 366) that two quadrics Q_3 in S_4 have, in general, sixteen common lines.

The book closes with a few pages of Bibliographical Notes, giving due credit to Cayley, Castelnuovo, Enriques, Severi, Macaulay, Lefschetz, van der Waerden, Chevalley, Zariski, Weil and others. The authors have skilfully blended the work of these illustrious men with many original ideas of their own. Their generality of outlook inevitably makes the book somewhat difficult to read, and there are no diagrams. However, an excellent index enables the reader to find much of interest without going through all the details. The Cambridge University Press must again be congratulated on a superb job of printing.

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Brief Mention

Introduction to the theory of distribution. By Israel Halperin. Based on Lectures by Laurent Schwartz. (Canadian Mathematical Congress, Lecture Series, No. 1.) University of Toronto Press, 1952. 33 pp.