Finite deformation of an elastic solid. By F. D. Murnaghan. New York, Wiley, 1951. 6+140 pp. $4.00.

Apparently intended as a text, this book follows the growing custom of beginning with an introductory chapter containing pure mathematics neither necessary nor sufficient for the applications which follow. The student is deterred from accepting the results without question by frequent interjections of "(why?)," "(prove this)," and other commands, and the pedagogical usefulness of the works is attested by the numerous simple exercises. The pages, whose crowding with symbols and parenthetical expressions in the text suggest that the publisher confused the manuscript with some work on topology or algebra, are rather frightening to look at.

The author maintains that the theory of finite deformations is most easily presented and understood by the use of matrices. "Do not fall into the error of regarding them as a complicated device invented by mathematicians to make the theory of elasticity harder than it actually is," he warns. However, the book did not furnish recreational reading to the reviewer, who finds the author's former tensorial treatment in the American Journal of Mathematics, 1937, not only more complete but also by virtue of its compact explicitness easier to grasp. At one point the author reverts to tensors, although taking care to advise readers to skip this passage, and later in the solution of special problems in curvilinear coordinates he employs without derivation equations which would develop naturally if an invariant formulation had been given to start with.

About forty pages are required to derive the classical equations of finite elastic strain. Next the author develops what he calls "the integrated linear theory" of hydrostatic pressure. This theory is obtained by taking the expression for change of volume in the linearized theory, then supposing the Lamé constants are linear functions of pressure, then integrating the result. He shows that by suitable choice of the constants occurring it is possible to get very good agreement with some of Bridgman's experiments. The numerical details are presented in full.

To consider the author's theory, let us repeat part of his presentation of the classical proof of the existence of stress-strain relations (pp. 55–56). Writing T for the stress matrix, η for the strain matrix, ρ_a and ρ_x for the density before and after deformation, J for the matrix of gradients of deformed with respect to undeformed positions, ψ for the mass density of the energy of deformation, and using a star to denote transposition, he says: "Since [the principle of conservation of energy] must hold for an arbitrary volume V_x of our deformable medium, we have (why?)

$$\operatorname{Tr}\,(J^{-1}T(J^*)^{-1}\delta\eta) = \rho_x\,\delta\psi.$$

Since ψ is (by hypothesis) a function (written symmetrically) of η we have (by the very definition of a differential) $\delta\psi = \operatorname{Tr}\,((\partial\psi/\partial\eta)\delta\eta)$ (show this), and so

$$\operatorname{Tr}\,(J^{-1}T(J^*)^{-1}\delta\eta) = \rho_x \operatorname{Tr}\left(\frac{\partial\psi}{\partial\eta}\,\delta\eta\right).$$

Since this relation must hold for an arbitrary (symmetric) matrix $\delta\eta$, we have (why?) . . .

$$T = \rho_x J \frac{\partial\psi}{\partial\eta} J^*."$$

The reader notes that if ψ were allowed to depend on T here, the author's formula for $\delta\psi$ would become $\delta\psi = \operatorname{Tr}\,((\partial\psi/\partial\eta)\delta\eta + (\partial\psi/\partial T)\delta T)$, and the proof would fail. That is, the classical finite strain theory requires that for a given point in the medium at given temperature or entropy, ψ be determined by the strain η *alone*, independently of the existing stress. Thus the author's assumption that the Lamé constants are functions of pressure would appear inconsistent with the theory of finite strain, and some readers may prefer to regard his "integrated linear theory" as an isolated semi-empirical result. The foregoing remarks are not to be confused with the well known results, discussed by the author on p. 65, that if an isotropic elastic body is really subject to initial hydrostatic pressure p_0, but we choose to neglect that fact and treat it as if it were unstressed, we can get correct results for small strains if we simply replace λ by $\lambda + p_0$, μ by $\mu - p_0$.

The remainder of the book is devoted to the case when the strain energy is approximated as a cubic function of the strain components. From the general considerations of Reiner [Amer. J. Math. vol. 70 (1948) pp. 433–446], which are not discussed by the author, it can be shown that to this degree of approximation the classical formulae relating shear stress to angle of shear, twisting couple to angle of twist, etc., will not be altered, although the characteristic phenomena of nonlinear elasticity will appear in their simplest form as new stresses not present at all in the linear theory. The author works out the form of the cubic terms for the various types of crystals. In the following discussion of simple tension, simple shear, compression of a circular cylindrical tube and of a spherical shell, and torsion of a circular cylinder, no mention is made of the general solutions valid for arbitrary strain energy which have recently appeared in the literature [R. S. Rivlin, Philos. Trans. Roy. Soc. London. ser. A vol. 241 (1948) pp. 379–397, and other papers]. New, however, is the calculation of the second order change of dimensions in a state of simple shearing stress, as distinct from a simple shear displacement, and of the similar change of dimensions of a circular cylinder in torsion.

The only historical references in the book tell us that Jacobians are named after Jacobi, the Lamé constants after Lamé, besides giving the dates and nationalities of these two persons. Apart from a single reference to some experimental data, the only literature citations are to the author's other texts. While this practice has become the rule in volumes intended for the pedagogical and undergraduate market, its extension to serious works does not seem altogether commendable to this reviewer. The publishers present this book as an "authoritative exposition." Inclusion of the recent results in finite strain theory obtained by Signorini, Reiner, Rivlin, and Green and Shield, which seem deep and significant to the reviewer, would not have been unwelcome.

In the preface the author states: "If the mathematical treatment given here serves to stimulate the procurement of experimental knowledge of these phenomena we shall have attained our aim." Abundant and detailed experiments on the very large strain of rubber have been reported by Rivlin from 1947 onwards. In the reviewer's opinion, the results of these experiments fully confirm the predictions of the general theory of elasticity, while showing that the second order approximation employed by the author is insufficient.

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Lectures in abstract algebra. Vol. I. *Basic concepts*. By N. Jacobson. New York, Van Nostrand, 1951. 12+217 pp. $5.00.

This is the first volume of a projected three volume work designed to give a general treatment of abstract algebra. This volume gives a comprehensive introduction to abstract algebra and its basic concepts. The next two volumes will be more specialized in nature. The second one will deal with the theory of vector spaces and the final volume with field theory and Galois theory.

The present volume is well organized and excellently written. A considerable number of exercises are given that vary greatly in difficulty.