The last section provides a table of useful transforms of the various types discussed.

Some of the recent work involving the use of the integral equation of the Wiener-Hopf type is conspicuous by its absence. It is in these topics that Fourier methods come to the forefront because, for the most part, there are no other methods available. Such a discussion would also serve to accentuate the importance of the role of function-theoretic methods in the integral transforms discussed in this text.

ALBERT E. HEINS

Vorlesungen über Differential- und Integralrechnung. Vol. II. *Differentialrechnung auf dem Gebiete mehrerer Variablen.* By A. Ostrowski. Basel, Birkhäuser, 1951. 482 pp. 67 Swiss fr.

Volume I of this work appeared in 1945, and was reviewed by the present writer (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 798–799). The first volume was devoted to the structure of differential and integral calculus for functions of a single variable, and to the development of the rules of calculus as they apply to the standard elementary functions. This second volume carries the study of limits and continuity further than was done in the first volume, and deals with a variety of additional topics. There are eight chapters. A third volume is planned to complete the work. It will deal with integral calculus in relation to functions of several variables.

Chapter I is entitled *Infinite sets*. After a discussion of denumerable and nondenumerable sets, the chapter is mainly taken up with the concepts of point set topology for Euclidean space. Chapter II treats the theory of limits and continuity for real functions defined on sets in Euclidean space. Chapter III deals with infinite sequences and series, beginning with the concepts of limits inferior and superior for sequences. A prominent place is given to a useful but apparently little known theorem of Cauchy, which reads as follows: Suppose $0<A_1<A_2< \cdots$, $A_n\rightarrow\infty$ as $n\rightarrow\infty$, and let $\{a_n\}$ be any sequence. Then

$$\frac{a_{n+1}-a_n}{A_{n+1}-A_n}\rightarrow d \quad \text{implies} \quad \frac{a_n}{A_n}\rightarrow d.$$

This holds as well if $d=\pm\infty$. There are numerous applications of this theorem, among which is a proof of one of the forms of l'Hospital's rule (in Chapter IV). The treatment of series of constant terms follows standard lines. There is a generalized form of Raabe's test, but the very useful test of Gauss is omitted. The discussion of uniform convergence includes Dini's theorem and a general theorem on the exchange of order in repeated limits (essentially the theorem associated with Moore and Osgood). The chapter concludes with a proof of the Weierstrass approximation theorem by Lebesgue's method. This is the longest chapter in the book. There is a wealth of material in the numerous exercises.

In Chapter IV, entitled *Completion of differential calculus*, the majority of the space is devoted to partial differentiation. The author calls a function $z=f(x, y)$ "differenzierbar" if it has first partial derivatives, and "differentiable" if it has a total differential dz in the sense of an approximation to Δz, linear in Δx and Δy. A notion of being uniformly "differenzierbar" is introduced, and in terms of this notion a theorem on necessary and sufficient conditions for differentiability is stated. This notion is also used to give a sufficient condition that $f_{xy}=f_{yx}$. Chapter V contains a thorough treatment of implicit function problems for one or more equations, the attendant discussion of Jacobians, and some consideration of the theory of extrema. Sufficient conditions in the latter theory are discussed in the two variable case only. The chapter ends with a proof of the Hadamard determinant inequality, using the Lagrange multiplier rule. Chapter VI, on methods of numerical approximation, includes topics in interpolation, numerical differentiation and integration, and iterative methods of solving equations. Formulas for error terms are discussed in all cases, and convergence theorems for Newton's method are given. Chapter VII deals with arc length and, for plane curves, with curvature and related topics. Rectifiability is discussed in terms of continuous functions of bounded variation, but most of the chapter is on a more elementary level. The topics in Chapter VIII are: vectors, the Frenet formulas, envelopes of families of surfaces, and the first fundamental quadratic form for a surface.

The text should prove very valuable as a permanent reference for students taught from it. It could also be used to good advantage as a reference by teachers of advanced calculus in this country, particularly on account of the extensive collection of exercises, which occupy nearly 30% of the page space of the book. A large proportion of the exercises are new in the textbook literature. They contain much information of interest. In contrast to the situation in volume I, the diagrams are of a uniformly high standard of execution.

The only serious adverse criticism which the reviewer has to present deals with the first two chapters. The treatment of point set topology is unsatisfactory in several respects. The author beclouds the concept of a point of accumulation by introducing the notion of a point set whose elements need not all be distinct points and talking about points of accumulation of "sets" of this kind (he regards a sequence $\{P_n\}$ as such a set). Now, a "point set" in this latter sense is in reality a function (a sequence being a function defined on the positive integers). There is no need to introduce the concept "point of accumulation" except in the sense that is customary in topology. Strict adherence to this point of view makes for clarity and for less difficulty on the part of students. All that the author wishes to accomplish can be done by suitable discussion of points of accumulation of a set and their relation to convergent sequences chosen from the set. A second criticism relates to the author's definition of a closed set as one which contains all its points of accumulation *and is bounded.* This departure from the usual definition spoils the duality between open and closed sets, complicates the statements of many theorems, and has no advantages apparent to the reviewer. The topological definition of connectedness is not given; the definition which is given on p. 39 (connectedness by polygonal paths) is unsatisfactory, except for open sets, and the remarks about connected regions on p. 42 puzzled the reviewer. The intermediate value theorem for continuous functions is not presented as a connectedness theorem.

ANGUS E. TAYLOR

Conformal mapping. By Z. Nehari. New York, McGraw-Hill, 1952. 8+396 pp. $7.50.

This is a textbook that will fill two needs. The author has designed the first four chapters to serve as the basis for a one term introductory course in complex variables, while the remainder of the book can be used in a graduate course in conformal mapping. It is claimed in the preface that only a knowledge of advanced calculus is necessary to read this book. (Perhaps a slightly better knowledge of the properties of real numbers is actually assumed than is given in most courses in advanced calculus.)

In Chapter I, the properties of harmonic functions in the plane are developed. The author discusses the solutions of the boundary value problems of the first and second kinds, introducing the Green's and Neumann's functions, and the harmonic measure. The first chapter closes with a derivation of the Hadamard variation formula giving the dependence of the Green's function on the domain.

The complex number system is explained in the first part of Chapter II, culminating in a discussion of sequences and series of complex numbers. After an analytic function is defined to be one which has a derivative, the connection between analyticity, the