sional wave equation Kirchhoff's formula, Huygens' principle, retarded potentials, and similar matters are mentioned. The chapter concludes with a section on the partial differential equation $\Delta\Delta u = 0$ and its applications.

Integral equations are the subject of Chapter IV. After a brief introduction, Fredholm-type integral equations with degenerate ("polynomial") kernels are discussed. Neumann's expansion (in the general case), Fredholm's theory, and the theory of symmetric kernels, with the classical theorems, follow. The sections on estimates, approximations, and numerical methods will be particularly useful. The two final sections establish connections with boundary value problems and show a number of applications. Reading this chapter one wonders if some of its sections are not too theoretical and if it is justifiable to devote, in a book of this nature, to integral equations about twice as much space as to functions of a complex variable.

Chapter V is on the calculus of variations, and in spite of its brevity is amply illustrated by examples. Euler's differential equation and Legendre's condition are derived, and so is Jacobi's condition. The section on Ritz's method and applications is especially valuable for the readers for whom the book is designed.

Chapter VI is a brief chapter on linear difference equations with constant coefficients and systems of such equations. The section on applications includes an example to show how difference equations can be used for the approximate solutions of differential equations.

The Appendix is a brief summary of some topics, mostly belonging to advanced calculus. It is designed principally to refresh one's memory, although some of its parts could be used to fill in gaps in the mathematical education of the reader.

A. ERDÉLYI

The preparation of programs for an electronic digital computer. With special reference to the EDSAC and the use of a library of subroutines. By M. V. Wilkes, D. J. Wheeler, and S. Gill. Cambridge, Massachusetts, Addison-Wesley, 1951. 10+170 pp. $5.00.

The EDSAC, designed and constructed at the Cambridge University Mathematical Laboratory, was one of the earliest high speed automatic digital computing machines in operation. It has a mercury delay line storage for 1024 words of 17 binary digits with photo-electrically read teletype tape input and teleprinter output. It is a one-

address machine with a multiplication time of about 7 milliseconds. It has been used as a true general purpose machine, and the problems handled seem to have been largely at the choice of the Laboratory. Accordingly, it has been possible to set up, without external interference, an organization for problem handling which has been tried out successfully on problems making progressively more demands on the machine and the numerical analysts.

One principle of the organization is the use of a thoroughly checked library of sub-routines. For instance the evaluation of an integral of the form

$$S(x, y) = \int_0^x e^{-y \sec \theta} d\theta$$

could be carried out by a program consisting mainly of the sub-routines for quadrature, for the exponential function, and for the secant. The actual assembly of these sub-routines into the main program can itself be effected by a special sub-routine.

A catalogue of (some of) the sub-routines used is given, together with detailed coding for some of these.

Another principle which is emphasized is the use of checking sub-routines. One of these indicates the progress of a computation by printing the operation symbols for each instruction as it is carried out, going to a new line at a branch order. This gives a compact representation which can help to indicate at what stage trouble develops.

This book will be of considerable value to those concerned with high-speed automatic digital computing machines. Unfortunately many ideas of general interest are only accessible after the assimilation of the EDSAC code. A use of flow diagrams and a sub-division of the book into three parts: one for the beginner, one for the expert, and one a hand-book for EDSAC, might have been helpful.

It is, for instance, worth pointing out that when carrying out a recursive process by means of a variable order, the end of the process can be determined by that order, instead of setting up a separate tally. Again the discussion of methods for the incorporation of "closed" sub-routines into a program, so that the control can pass from the main program, through them, and back to the main program whenever necessary, is valuable. These are matters of importance to all machine users.

It is worth observing that the objections raised to the use of the Gaussian approximate quadrature (namely, the trouble (on desk machines) of calculating the integrand at awkward arguments) no

longer apply in the present circumstances. It is also worth recalling that there is a recurrence relation for the evaluation of a polynomial. But the detailed coding of such problems is of little interest save to the veriest beginner.

JOHN TODD

BRIEF MENTION

Programmgesteuerte digitale Rechengeräte (elektronische Rechenmaschinen). By H. Ruthishauser, A. Speiser, and E. Stiefel. (Mitteilungen aus dem Institut für angewandte Mathematik, no. 2.) Basel, Birkhäuser, 1951. 102 pp. 8.50 Swiss fr.

This booklet has been written after visits by the authors to various centers of high speed automatic digital computing in 1948–9. It is the first at all comprehensive report on its subject in the German language, and constitutes a very readable introduction. In addition to sections on history and bibliography there are careful discussions of the organization of machines, the representation of numbers, the handling of the elementary operations, possible address-systems, flow-diagrams, multiple-precision operations, checks. The concluding section discusses the physical realization of various components.

JOHN TODD

Colloque de topologie (Espaces fibrés). Centre Belge de Recherches Mathématiques. Liège, Georges Thone, and Paris, Masson, 1951. 129 pp. 1225 fr.

This booklet is a report of a colloquium on fibre spaces and fibre bundles held in Brussels in June, 1950. It contains the following papers: *Introduction à la théorie des espaces fibrés*, by H. Hopf; *Notions d'algèbre différentielle; applications aux groupes de Lie et aux variétés où opère un groupe de Lie*, by H. Cartan; *Les connexions infinitesimales dans un espace fibré différentiable*, by C. Ehresmann; *La transgression dans un groupe de Lie et dans un espace fibré principal*, by H. Cartan; *Sur un type d'algèbres différentielles en rapport avec la transgression*, by J. L. Koszul; *Espaces fibrés et homotopie*, by B. Eckmann; *Sur l'homologie des groupes de Lie, des espaces homogènes, et des espaces fibrés principaux*, by J. Leray; *Sur un formule de la théorie des espaces fibrés*, by H. Hopf; *Quelques relations entre l'homologie dans les espaces fibrés et les classes caractéristiques relative à un groupe de structure*, by G. Hirsch.

The first paper by Hopf is of an expository nature; it furnishes an