that it is false that p." This is not a good interpretation, for "$\sim p$" is a sentence of the object language, while "it is false that p" is a sentence of the syntax language. (Compare for instance Quine, *Mathematical logic*, p. 27.) In particular this use causes confusion when iterated and coupled with the interpretation of the word "true" in this book, for then "$\sim(\sim p)$" must be read: "it is not a theorem that it is not a theorem that p."

The second disagreement concerns the use of the word "categorical." This is defined on p. 44: " . . . In many deductive theories we wish the axioms to be categorical, that is, that the system should be adequate to decide the truth or falsehood of any proposition which can be formulated in the system. In the frame of A 1″–A 7″ (a formulation of the propositional calculus-rev.) we can give this demand the strong form that for every $p\in C$ (C is the class of wff's-rev.) either $\vdash p$ or $\vdash\sim p$" as this is contrary to the general usage of this word (cf. Fraenkel, *Einleitung i.d. Mengenlehre*, 3d ed., p. 349), the reviewer would in this case suggest the use of the word "complete." A similar objection applies to the use of the word "true." On page 94 we find the definition "A sentence q is said to be true if there is a proof of q." The reviewer would prefer the word "provable" in this connection. If "true" is used with Rosenbloom's meaning, every undecidable sentence is false. This contradicts the following statement on p. 179: "Thus any canonical language which is consistent and adequate for arithmetic will contain undecidable sentences expressing elementary arithmetical propositions. There will even be such sentences which we can prove to be true by an argument in the syntax language." The reviewer noted a few misprints, also a few misreferences (e.g. T9″ referred to on p. 44 could not be found, however T13 (p. 35) could be used here, also there is no Lemma 6 (cf. pp. 44–45), only Theorem 6 (on p. 22), also on p. 54, the ref. to A5″ seems incorrect).

I. L. NOVAK

Espaços vetoriais topologicos 1. By L. Nachbin. (Notas de Matematica, no. 4.) Rio de Janeiro, Boffoni, 1948. 2+100 pp. 70 Cruzeiros.

This is intended as the first volume of a self-contained treatise on topological vector spaces. Of the 9 chapters it contains, chapters 1 to 4 are devoted to algebraic and topological preliminaries (topological spaces, fields, topological fields, vector spaces); in addition chapter 7 discusses mainly absolute values on fields and their generalizations, so that only 4 chapters remain for topological vector spaces proper.

The author presents the theory from a very general point of view, making as few assumptions as possible on the field of scalars, which for most results is allowed to be an arbitrary topological field (very little labour would have been needed to extend almost all definitions and theorems to the case in which the scalars are in a noncommutative topological sfield). A topological vector space is defined in chapter 5 as a vector space E over a topological field K, with a topology on E such that $x+y$ and λx are continuous functions of (x, y) and (λ, x) respectively; such a topology is characterized by properties of a system of neighborhoods of 0, and the notions of continuous linear transformation and of quotient topological vector space are carefully studied. Chapter 6 is devoted to the definition and elementary properties of bounded sets in a topological vector space, such a set B being characterized by the fact that λB "tends to 0" with λ in an obvious sense.

Chapter 7 is the longest and most elaborate of the book. Together with the classical notion of absolute value, the author introduces quasi-absolute values $v(x)$, which satisfy the usual axioms with the exception of the triangle inequality, which is weakened to $v(x+y) \leqq m(v(x)+v(y))$ for a fixed $m>1$. After a discussion of archimedean and non-archimedean absolute values, he proves that for every quasi-absolute value v, v^h is an absolute value for sufficiently small $h>0$ (theorem of Artin). Next he introduces the topology defined on a field by an absolute value, and proves Shafarevich's criterion characterizing such topologies. Finally he introduces a new notion, that of *strictly minimal* topological field: this is a field K with a Hausdorff topology $\mathfrak{T}$ such that no Hausdorff topology strictly coarser than $\mathfrak{T}$ can make K into a topological vector space (the scalars retaining their original topology $\mathfrak{T}$). It turns out that that condition (which is satisfied by any field with an absolute value) is necessary and sufficient for the validity of the following theorem: in order that a hyperplane defined by an equation $f(x)=0$ in a topological vector space E over K be closed, it is necessary and sufficient that f be a continuous linear form.

Chapter 8 treats the *strong* topologies defined by Mackey: a topology on a vector space E is strong if there is no strictly finer topology giving the same bounded sets. This is equivalent to saying that any linear mapping sending bounded sets of E into bounded sets of a topological vector space F is always continuous. Metrizable vector spaces over metrizable fields are always strong.

Finally the last chapter is devoted to *weak* topologies, but practically does not go beyond their definition; in particular, the theory of weak duality, which alone can give a meaning to the notion of weak topology, is not touched at all. There are other topics which one misses in chapters 5 and 6, where they would have been in their proper setting, such as the discussion of finite-dimensional topological vector spaces, or of locally compact vector spaces. On the whole, in the reviewer's opinion, the book suffers from a lack of balance, due to the overemphasis laid on chapter 7, at the expense of more relevant matters. However, the author has done a very valuable service to mathematicians in bringing together in book form a large number of results which up to now were scattered in periodicals, and not always very explicitly. His style moreover deserves high praise for its remarkable clarity and thoroughness, so that the book genuinely vindicates its claim of being self-contained, although of course the motivation for the whole theory can only be understood with a considerable background of functional analysis.

J. DIEUDONNÉ

Rekursive Funktionen. By R. Péter. Budapest, Akademischer Verlag, 1951. 206 pp.

Although recursions have been used since Archimedes, and have played a part in foundational investigations by Dedekind (1888), Peano (1891), and Skolem (1923), the theory of recursive functions consists largely of two recent developments, which we call here the "special theory" and the "general theory."

The stimulus to the special theory came from Hilbert's lecture *Über das Unendliche* (published 1926) in which he proposed to attack the continuum problem of set theory by showing that there is no inconsistency in supposing that the number-theoretic functions are all definable by use of forms of recursion associated with the transfinite ordinals of Cantor's second number class. (This program has not yet been carried out, though Gödel in 1938 used an analogous idea to show the consistency of the continuum hypothesis within axiomatic set theory.) For Hilbert's proposal it was necessary to show that higher forms of recursion do give new functions; and the first demonstration of the existence of a function definable by a double recursion but not by use only of simple or "primitive" recursion was given by Ackermann in 1928 in a paper entitled *Zum Hilbertschen Aufbau der reellen Zahlen.* Beginning in 1932, Rósza Péter has published a series of papers, examining the relationship of various special forms of recursion, and showing the definability of new functions by successively higher types of recursion, which establish her as the