at the time of the outbreak of the war in 1939, developments made during the last decade are covered to only a limited extent.

The reader is referred to a review of the original edition of volume I by the present reviewer (Bull. Amer. Math. Soc. vol. 40 (1934) p. 787) for comments still largely applicable to the new edition, to a review of the new edition of volume I by J. H. Roberts (Mathematical Reviews vol. 10 (1949) p. 389) for a detailed comparison of the old and new editions and to a review of volume II by E. G. Begle (Mathematical Reviews vol. 12 (1951) p. 517) for detailed indication of the content of volume II. The two volumes comprise a historic contribution to mathematical and topological literature and will need to be a part of the library of every individual interested in or making use of the results of topology.

G. T. WHYBURN

The elements of mathematical logic. By P. C. Rosenbloom. New York, Dover, 1950. 4+214 pp. $2.95.

This book is intended for mature mathematicians with no previous knowledge of mathematical logic. Chapter I deals with Boolean algebras and includes the Stone representation theorem. Chapter II is entitled *The logic of propositions.* Truth tables are explained and there are three alternative formulations of the propositional calculus and a number of tautologies are proved. There is also a finitary formulation which incorporates part of the syntax in the object language which is therefore unusually rich. Next the relation between Boolean algebras and propositional calculus is explained and the final section of this chapter contains a very interesting discussion of many-valued and modal logics and of intuitionism. It includes some material on Post algebras and formulations of intuitionistic propositional calculus and Lewis' basic logic. Chapter III, on *The logic of propositional functions*, begins with an informal discussion of intuitive class theory and the Russell paradox. It shows that some restriction on the method of class formation is necessary and different methods of doing this are briefly mentioned. After this there is in §2 a formulation of the monadic first order functional calculus and an extension to polyadic functional calculus. The Peano axioms are then adjoined to get a system adequate for arithmetic. §3 begins with an exposition of the pure first order functional calculus, followed by a brief account of the theory of types, the system of Quine's new foundations and finally of Zermelo's system. Bernays' system is also mentioned, but von Neumann's only in the bibliographical notes (p. 202): "Other formulations of Zermelo's system have been given by von Neumann and Ackermann." It is debatable whether it is fair to consider the system of von Neumann merely a reformulation of Zermelo's. §4 contains an account of the work of Curry on the analysis of the notions of substitution and variable and the development of combinatory logics. It is proved that the logic presented is functionally complete. Church's λ-conversion is also introduced. §5 sketches a development of mathematics in the system of Quine's new foundations. This is followed in §6 by a very brief discussion of the Epimenides and Richard paradoxes. The 7th and last section of this chapter deals with the axiom of choice and gives several equivalent formulations. Chapter IV is called *The general syntax of language.* It depends largely on the work of Post. The first section discusses what is meant by a language and a simple language. §2 introduces the concepts of productions, axioms, theorems, canonical languages, extensions of languages, canonical classes of statements, and also contains an examination of the character of definitions. In §3 we find a definition of normal languages which are especially simple in that they have only one axiom and the productions are of a very symmetric form. A theorem of Post states (p. 170): "Every canonical language has a conservative normal extension." As almost all known systems of mathematical logic (those with no rules of inference with an infinite number of premises) can be formulated in terms of canonical languages, this theorem says that all problems of each of these systems can be reduced to problems of a language with a very simple structure. In order to conserve space, the proof of this theorem is however omitted since it is somewhat lengthy. The chapter concludes with a proof of Gödel's incompleteness theorem. The appendix contains canonical formulations of pure first order functional calculus, of Quine's system, and of Zermelo's system, and also a proof of Church's theorem. There are exercises throughout the book following each section and also a very illuminating set of bibliographical notes.

Professor Rosenbloom's book is a very interesting addition to the texts of logic now available, particularly since it contains material not found in others, for instance discussions of the work of Curry and Post. This important and often neglected work is stressed a great deal and very well presented. The presentation of the traditional subject matter of an elementary course in logic, that is, propositional calculus, functional calculus, and the various systems of set theory, is often unusual. The reviewer would however criticize the author on several minor points.

First, we quote from page 28: " '$\sim p$' shall denote the proposition that it is false that p." This is not a good interpretation, for "$\sim p$" is a sentence of the object language, while "it is false that p" is a sentence of the syntax language. (Compare for instance Quine, *Mathematical logic*, p. 27.) In particular this use causes confusion when iterated and coupled with the interpretation of the word "true" in this book, for then "$\sim(\sim p)$" must be read: "it is not a theorem that it is not a theorem that p."

The second disagreement concerns the use of the word "categorical." This is defined on p. 44: " . . . In many deductive theories we wish the axioms to be categorical, that is, that the system should be adequate to decide the truth or falsehood of any proposition which can be formulated in the system. In the frame of A 1″–A 7″ (a formulation of the propositional calculus-rev.) we can give this demand the strong form that for every $p \in C$ (C is the class of wff's-rev.) either $\vdash p$ or $\vdash \sim p$" as this is contrary to the general usage of this word (cf. Fraenkel, *Einleitung i.d. Mengenlehre*, 3d ed., p. 349), the reviewer would in this case suggest the use of the word "complete." A similar objection applies to the use of the word "true." On page 94 we find the definition "A sentence q is said to be true if there is a proof of q." The reviewer would prefer the word "provable" in this connection. If "true" is used with Rosenbloom's meaning, every undecidable sentence is false. This contradicts the following statement on p. 179: "Thus any canonical language which is consistent and adequate for arithmetic will contain undecidable sentences expressing elementary arithmetical propositions. There will even be such sentences which we can prove to be true by an argument in the syntax language." The reviewer noted a few misprints, also a few misreferences (e.g. T9″ referred to on p. 44 could not be found, however T13 (p. 35) could be used here, also there is no Lemma 6 (cf. pp. 44–45), only Theorem 6 (on p. 22), also on p. 54, the ref. to A5″ seems incorrect).

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Espaços vetoriais topologicos 1. By L. Nachbin. (Notas de Matematica, no. 4.) Rio de Janeiro, Boffoni, 1948. 2+100 pp. 70 Cruzeiros.

This is intended as the first volume of a self-contained treatise on topological vector spaces. Of the 9 chapters it contains, chapters 1 to 4 are devoted to algebraic and topological preliminaries (topological spaces, fields, topological fields, vector spaces); in addition chapter 7 discusses mainly absolute values on fields and their generalizations, so that only 4 chapters remain for topological vector spaces proper.