THE ANNUAL MEETING OF THE SOCIETY

The fifty-eighth Annual Meeting of the American Mathematical Society was held at Brown University, Providence, Rhode Island, Wednesday through Friday, December 26–28, 1951, in conjunction with the annual meeting of the Mathematical Association of America.

The registered attendance at the meeting exceeded 525 and included the following 424 members of the Society.

C. R. Adams, R. B. Adams, Jose Adem Chahin, Shmuel Agmon, R. P. Agnew, L. V. Ahlfors, M. I. Aissen, E. J. Akutowicz, R. G. Albert, R. W. Allen, C. B. Allendoerfer, Warren Ambrose, R. D. Anderson, Joseph Andrushkiw, R. C. Archibald, H. E. Arnold, L. G. Arnold, Nachman Aronszajn, Maurice Auslander, W. E. Barnes, R. B. Barrar, Iacopo Barsotti, A. F. Bartholomay, R. G. Bartle, G. E. Bates, J. D. Baum, W. R. Baum, H. P. Beard, Ralph Beatley, E. G. Begle, R. E. Bellman, A. A. Bennett, Stefan Bergman, H. G. Bergmann, S. D. Bernardi, Lipman Bers, E. W. Beth, Kurt Bing, Garrett Birkhoff, D. W. Blackett, Jerome Blackman, A. L. Blakers, A. A. Blank, J. H. Blau, I. E. Block, H. W. Bode, Volodymyr Bohun-Chudyniv, Raoul Bott, S. G. Bourne, J. W. Bower, H. E. Bowie, Laura Brant, R. H. Breusch, H. W. Brinkmann, F. E. Browder, A. R. Brown, Bailey Brown, H. D. Brunk, G. S. Bruton, D. A. Buchsbaum, J. A. Bullard, R. S. Burington, L. J. Burton, Jewell H. Bushey, E. A. Cameron, M. E. Carlen, L. V. Carlton, Evelyn Carroll-Rusk, W. B. Carver, Lamberto Cesari, R. E. Chamberlin, Sarvadaman Chowla, K. L. Chung, R. A. Clark, A. B. Clarke, G. R. Clements, E. A. Coddington, L. W. Cohen, Harvey Cohn, Nancy Cole, A. H. Copeland, A. H. Copeland, Jr., Richard Courant, R. R. Coveyou, J. B. Crabtree, H. F. Cullen, Frederic Cunningham, J. M. Danskin, Norman Davids, Robert Davies, M. D. Davis, Philip Davis, R. B. Davis, W. M. Davis, C. R. DePrima, R. J. De Vogelaere, A. H. Diamond, J. B. Diaz, G. P. Dinneen, W. F. Donoghue, J. L. Doob, H. L. Dorwart, Jim Douglas, Avron Douglis, Melvin Dresher, F. G. Dressel, G. R. Dubé, Nelson Dunford, Albert Edrei, Samuel Eilenberg, B. J. Eisenstadt, Joanne Elliott, E. S. Elyash, L. N. Enequist, Carl Engelman, Benjamin Epstein, D. H. Erkiletian, Trevor Evans, M. I. Fauth, Herbert Federer, W. E. Ferguson, A. D. Fialkow, F. A. Ficken, R. S. Finn, C. D. Firestone, L. R. Ford, Tomlinson Fort, G. A. Foyle, J. S. Frame, F. N. Frenkiel, Bernard Friedman, H. D. Friedman, Orrin Frink, F. B. Fuller, R. E. Fullerton, J. W. Gaddum, I. S. Gál, A. S. Galbraith, David Gale, H. M. Gehman, B. R. Gelbaum, H. H. Germond, Irving Gerst, Murray Gerstenhaber, J. H. Giese, J. B. Giever, S. A. Gilbert, W. M. Gilbert, R. E. Gilman, R. D. Glauz, A. M. Gleason, Kurt Gödel, H. E. Goheen, Samuel Goldberg, Oscar Goldman, W. O. Gordon, Daniel Gorenstein, W. H. Gottschalk, Arthur Grad, J. W. Green, L. W. Green, T. N. E Greville, Emil Grosswald, V. B. Haas, Franklin Haimo, P. C. Hammer, E. E. Hammond, G. H. Handelman, Frank Harary, W. L. Hart, F. S. Hawthorne, E. V. Haynesworth, G. A. Hedlund, A. E. Heins, M. H. Heins, Melvin Henriksen, Manuel Herschdorfer, Aaron Herschfeld, I. R. Hershner, Fritz Herzog, F. B. Hildebrand, T. H. Hildebrandt, Einar Hille, W. M. Hirsch, G. P. Hochschild, S. P. Hoffman, D. L. Holl, L. A. Hostinsky, E. M. Hove, A. A. Howard, A. S. Howard, J. L. Howell, C. C. Hsiung, M. G. Humphreys, Witold Hurewicz, W. A. Hurwitz, L. C. Hutchinson, Shikao Ikehara, B. M. Ingersoll, H. G. Jacob, A. R. Jacoby, R. C. James, R. N. Johanson, L. W. Johnson, R. E. Johnson, R. F. Johnson, B. W. Jones, F. B. Jones, Bjarni Jónsson, R. V. Kadison, Shizuo Kakutani, G. K.

Kalisch, L. H. Kanter, Irving Kaplansky, S. N. Karp, R. E. Keirstead, M. E. Kellar, D. E. Kibbey, J. F. Kiefer, H. S. Kieval, J. R. Kinney, V. L. Klee, S. C. Kleene, J. S. Klein, J. R. Kline, Morris Kline, E. R. Kolchin, Marc Krasner, Wouter van der Kulk, H. W. Kuhn, Serge Lang, C. E. Langenhop, E. H. Larguier, J. A. Larrivee, P. D. Lax, E. H. Lee, B. A. Lengyel, Benjamin Lepson, F. W. Light, C. C. Lin, B. W. Lindgren, S. I. Lipsey, W. G. Lister, L. H. Loomis, Lee Lorch, Eugene Lukacs, R. C. Lyndon, V. O. McBrien, N. H. McCoy, L. A. MacColl, R. W. MacDowell, Saunders MacLane, H. M. MacNeille, H. P. Manning, Murry Mannos, R. J. Marcou, Morris Marden, L. F. Markus, M. H. Martin, W. T. Martin, A. E. Meder, Jr., D. F. Mela, Karl Menger, E. A. Michael, W. H. Mills, Don Mittleman, E. E. Moise, Deane Montgomery, T. W. Moore, R. K. Morley, G. D. Mostow, T. S. Motzkin, W. L. Murdock, G. G. Murray, E. D. Nering, P. P. Nesbeda, John von Neumann, C. V. Newson, E. N. Nilson, I. L. Novak, C. O. Oakley, G. G. O'Brien, L. F. Ollmann, Alex Orden, Oystein Ore, E. F. O'Shea, J. C. Oxtoby, D. K. Pease, W. H. Pell, J. L. Penez, A. J. Penico, P. M. Pepper, I. E. Perlin, H. P. Pettit, C. R. Phelps, H. L. Platzer, J. C. Polley, William Prager, Walter Prenowitz, G. B. Price, M. H. Protter, Tibor Radó, J. F. Randolph, G. E. Raynor, C. J. Rees, M. S. Rees, Eric Reissner, Helene Reschovsky, C. N. Reynolds, Harris Rice, H. G. Rice, D. E. Richmond, C. E. Rickart, G. de B. Robinson, S. L. Robinson, G. B. Robison, W. L. Root, I. H. Rose, Alex Rosenberg, P. C. Rosenbloom, Maxwell Rosenlicht, Arthur Rosenthal, J. C. Rothe, H. L. Royden, Herbert Ruderfer, Walter Rudin, Charles Saltzer, James Sanders, Arthur Sard, Leo Sario, S. S. Saslaw, J. A. Schatz, F. J. Scheid, M. A. Scheier, E. V. Schenkman, E. C. Schlesinger, E. R. Schneckenburger, I. J. Schoenberg, A. L. Schurrer, Abraham Schwartz, B. L. Schwartz, C. H. W. Sedgewick, I. E. Segal, Esther Seiden, Seymour Sherman, Harold Shulman, I. M. Singer, Maurice Sion, M. L. Slater, E. C. Smith, P. A. Smith, Andrew Sobczyk, E. S. Sokolnikoff, J. J. Sopka, Joseph Spear, D. E. Spencer, C. J. Standish, M. E. Stark, E. P. Starke, J. R. K. Stauffer, N. E. Steenrod, S. K. B. Stein, R. L. Sternberg, F. M. Stewart, R. W. Stokes, D. J. Struik, J. S. Stubbe, R. L. Swain, Otto Szász, Gabor Szegö, R. C. Taliaferro, J. T. Tate, Olga Taussky, J. J. Taylor, W. C. Taylor, G. B. Thomas, J. M. Thomas, G. L. Thompson, D. L. Thomsen, R. M. Thrall, W. J. Thron, D. E. van Tijn, John Todd, M. M. Torrey, J. I. Tracey, A. W. Tucker, Robert Ullman, E. P. Vance, H. E. Vansant, A. H. Van Tuyl, M. F. Vaudreuil, T. L. Wade, D. H. Wagner, R. W. Wagner, S. E. Walkley, J. L. Walsh, H. C. Wang, W. H. Warner, S. E. Warschawski, M. T. Wechsler, J. V. Wehausen, Herschel Weil, H. F. Weinberger, M. J. Weiss, B. A. Welch, David Wellinger, John Wermer, G. N. White, G. W. Whitehead, P. M. Whitman, G. T. Whyburn, D. V. Widder, A. B. Willcox, W. L. Williams, Henry Wolf, Jacob Wolfowitz, E. S. Wolk, Bertram Yood, D. M. Young, Oscar Zariski, Arthur Zeichner, Daniel Zelinsky, J. L. Zemmer, J. A. Zilber.

The twenty-fifth Josiah Willard Gibbs Lecture, entitled *Some basic theorems on the foundations of mathematics and their philosophical implications*, was delivered by Professor Kurt Gödel of the Institute for Advanced Study on Wednesday evening, December 26. Professor John von Neumann, President of the American Mathematical Society, presided.

Two addresses were presented at the invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings. On

Thursday afternoon, December 27, Professor Shizuo Kakutani of Yale University spoke on *Brownian motion*. Professor J. L. Doob presided. On Friday afternoon, December 28, Professor G. P. Hochschild of the University of Illinois and Yale University spoke on *Cohomology in algebraic number fields*. Professor Saunders MacLane, President of the Mathematical Association of America, presided.

The annual Business Meeting and Election of Officers was held at 3:15 P.M., Friday, December 28, with President John von Neumann in the chair. Details of the proceedings are reported below.

The members of the Society had the opportunity of visiting the new offices of the Society in the building at 80 Waterman Street, Providence.

The members of the Society and the Association were entertained at tea by the ladies of the Department of Mathematics, of the Graduate Division of Applied Mathematics, and the Department of the History of Mathematics on Wednesday afternoon, December 26.

On Thursday afternoon, December 27, there was a conducted tour of Pendleton House, a museum maintained by the Rhode Island School of Design. In the evening a film program was presented in the Faunce House Theatre.

The dinner for members of the Society, the Association, and their guests was held in the University Refectory on Friday evening, December 28. Professor C. R. Adams, toastmaster, introduced Dr. Henry Wriston, President of Brown University, who welcomed the organizations and made an address. He was followed by Professor John von Neumann, President of the American Mathematical Society, and Professor Jewell H. Bushey, Vice President of the Mathematical Association of America. The toastmaster recognized Professor C. B. Allendoerfer of the University of Washington who introduced a resolution expressing the appreciation and thanks of the Society and Association to Brown University, the Committee on Arrangements, and all who had helped in making the meetings successful and enjoyable. This resolution was adopted by a rising vote.

The Annual Business Meeting of the Society was held on Friday afternoon, December 28, 1951. The Secretary reported on the affairs of the Society.

The President announced that the Frank Nelson Cole Prize in the Theory of Numbers had been awarded "to Paul Erdös, for his many papers in the Theory of Numbers, and in particular for his paper *On a new method in elementary number theory which leads to an elementary proof of the prime number theorem*, Proceedings of the National Academy, vol. 35, pp. 374–385, July 1949, in which he makes important contributions to the new elementary theory of primes inaugurated by A. Selberg."

At the annual election, in which 639 votes were cast, the following officers were elected.

President Elect, Professor G. T. Whyburn.

Vice Presidents, Professors Richard Brauer and Deane Montgomery.

Associate Secretaries, Professors J. W. Green, W. M. Whyburn, and J. W. T. Youngs.

Member of the Editorial Committee of the Proceedings, Professor G. A. Hedlund.

Member of the Editorial Committee of the Transactions, Professor Saunders MacLane.

Member of the Editorial Committee of the Colloquium Publications, Professor Einar Hille.

Member of the Editorial Committee of Mathematical Reviews, Professor Hassler Whitney.

Member of the Editorial Committee of Mathematical Surveys, Professor R. J. Walker.

Members-at-large of the Council, Professors L. V. Ahlfors, C. B. Allendoerfer, R. H. Bing, E. R. Lorch, and J. C. Oxtoby.

Members of the Board of Trustees, Professors Einar Hille and P. A. Smith.

The Council met on Thursday evening, December 27, 1951.

The Secretary announced the election of the following twenty-three persons to ordinary membership in the Society.

Miss Lenora Yates Amos, Jarvis Christian College, Hawkins, Texas;
Mr. Randall Murray Conkling, Assistant, University of Florida;
Mr. John Edward Dutt, Columbia University;
Mr. Franz Edelman, Research Engineer, R.C.A. Victor Division, Indianapolis, Indiana;
Mr. Irving Leonard Glicksberg, Assistant Mathematician, Rand Corporation, Santa Monica, California;
Mr. Harold A. Goldberger, Brown University;
Mr. Louis Grossberg, Computer, New York University;
Mr. Isidore Heller, Research Associate, George Washington University;
Mr. Chaim Samuel Honig, Assistant, University of São Paulo, São Paulo, Brazil;
Assistant Professor Chen-Jung Hsu, Mathematical Institute, National Taiwan University, Taipei, Formosa;
Mr. Martin Morris Lipschutz, Evans Signal Laboratory, Belmar, New Jersey;
Mr. Geoffrey Stuart Stephen Ludford, University of Maryland;
Mr. LeRoy Frederick Meyers, Stanford University;
Mr. John David Neff, Purdue University;
Mr. Wallace Eldon Parr, Naval Ordnance Laboratory, White Oak, Maryland;
Mr. Robert Benedict Reisel, Northwestern University;
Mr. Wimberly C. Royster, University of Kentucky;

Mr. Douglas Howerth Shaffer, Carnegie Institute of Technology;
Associate Professor Jesse William Smith, Department of Physics, Roosevelt College of Chicago, Chicago, Illinois;
Mr. William Cady Stone, Union College, Schenectady, New York;
Mr. Nick Hampton Vaughan, Mathematician, Indianapolis Naval Ordnance Plant, Indianapolis, Indiana;
Mr. Robert Weinstock, Stanford University;
Mr. Emmet Finlay Whittlesey, Bates College.

It was reported that the following ten persons had been elected to membership on nomination of institutional members as indicated:

Lehigh University: Mr. Robert A. Chisholm Lane.
University of Maryland: Miss Ruth Margaret Davis and Mr. Donald Greenspan.
Michigan State College: Mr. Dale Marsh Mesner, Mr. Edward Stafford Northam, and Mr. James Henry Powell
University of Minnesota: Mr. Robert Edwin Zink.
Oregon State College: Mr. Verner Emil Hoggatt, Jr.
Swarthmore College: Mr. Louis Marshall Winer.
Wellesley College: Miss Jacqueline Pascal Evans.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: London Mathematical Society: Albert Edrei, Assistant Professor, University of Colorado, and John Arthur Jacobs, Associate Professor, University of Toronto; Matematisk Forening, Copenhagen: Dr. Thøger S. V. Bang, Amanuensis, Copenhagen University; Unione Matematica Italiana: Dr. Luigi Gatteschi, Visiting Research Associate, Stanford University; Wiskundig Genootschap te Amsterdam: Evert Willen Beth, Professor, University of California, Berkeley.

The Alabama Polytechnic Institute, Auburn, Alabama; Florida State University, Tallahassee, Florida; The University of Mississippi, Oxford, Mississippi; and the State College of Washington, Pullman, Washington, were elected to institutional membership.

The Secretary is pleased to report at this time that the ordinary membership of the Society is now 4458, including 353 nominees of institutional members and 44 life members. There are also 111 institutional members. The total attendance at all meetings in 1951 was 2165; the number of papers read was 636; there were 19 hour addresses, 1 Gibbs Lecture, 4 Colloquium Lectures, and 15 papers at the Applied Mathematics Symposium. The number of members attending at least one meeting was 1377.

The following appointments by the President were reported: as a committee to determine the facts concerning the loyalty oath requirements at the Oklahoma Agricultural and Mechanical College and the University of Oklahoma and their effects on mathematics and mathematicians: W. E. Duren, Chairman, G. M. Ewing, and J. F. Randolph; as a member of the Committee on Printing Contracts for the period 1952–1954: Professor J. R. Kline (committee now consists of Professors C. J. Rees, Chairman, R. A. Johnson, J. R. Kline, and H. M. MacNeille, ex-officio); as a member of the Committee on Places of Meetings for the period 1952–1954: Professor M. F. Smiley (committee now consists of Professor J. M. Thomas, Chairman, Orrin Frink, and M. F. Smiley); as members of the Committee on Applied Mathematics (for a period of three years beginning January 1, 1952): Professors F. J. Murray and Eric Reissner (committee now consists of Professors M. H. Martin, Chairman, R. V. Churchill, K. O. Friedrichs, John von Neumann, F. J. Murray, and Eric Reissner); as tellers for the 1951 annual election: Professors A. A. Bennett and Rohn Truell; as a member of the Committee to Select Hour Speakers for Summer and Annual Meetings for the period 1952–1953: Professor Tibor Radó (committee now consists of Professors E. G. Begle, Chairman, Deane Montgomery, and Tibor Radó); as a Committee to Select Hour Speakers for Eastern Sectional Meetings for the period 1952–1953: Professor R. D. Schafer (committee now consists of Professors L. W. Cohen, Chairman, M. H. Heins, and R. D. Schafer); as a Committee to Select Hour Speakers for Southeastern Sectional Meetings for the period 1952–1953. Professor Wallace Givens (committee now consists of Professors W. M. Whyburn, Chairman, B. J. Pettis, and Wallace Givens); as a Committee to Select Hour Speakers for Far Western Sectional Meetings for the period 1952–1953: Professor R. P. Dilworth (committee now consists of Professors W. T. Puckett, Chairman, A. P. Morse, and R. P. Dilworth); as Editor of Volume V of the Proceedings of the Symposium on Applied Mathematics: Professor A. E. Heins.

In an appendix to this report are excerpts from the report of the Treasurer for the fiscal year 1951 as verified by the auditors. A copy of the complete report will be sent, on request, to any member of the Society.

The Secretary reported that the Board of Trustees had appointed Dr. H. M. MacNeille (Chairman), Professor E. G. Begle, Professor Jekuthiel Ginsburg, and Dean A. E. Meder, Jr., a committee to advise the Council and the Board of Trustees concerning the problem of exchanges of the Society's publications. The Board of Trustees also called to the attention of the Council the fact that its action on September 4 in Minneapolis in placing the Executive Editor of Mathematical Reviews in charge of exchanges was in violation of Section 3 of Article 1 of the by-laws of the Society.

The Secretary reported that the following had accepted invitations to deliver addresses: Professor Alexander Weinstein at the meeting in New York City, February 23, 1952; Professors E. R. Kolchin and Marston Morse at the meeting in New York City on April 25–26, 1952; Professor S. M. Ulam at the meeting in Chicago on April 25–26, 1952; and Professor I. J. Schoenberg at the meeting in Fresno on May 3, 1952.

The Council voted to approve the following dates of meetings of the Society in 1952: October 25, 1952, at Yale University; November 29, 1952, at the University of Southern California.

After reviewing its actions concerning the University of California, the Council voted to request the President to appoint a committee of five to consider problems of controversial questions both as to procedures of the Council and as to the membership of the Society.

The Bulletin Editorial Committee reported that the journal used 520 of the 600 pages authorized in 1951, but that there had been a large influx of manuscripts of invited addresses at the end of 1951. The Council voted to recommend to the Board of Trustees that a total of 680 pages be authorized for the 1952 Bulletin.

The Transactions and Memoirs Editorial Committee reported that the Transactions published just a few pages less than the 1100 pages authorized for 1951. The interval between receipt of manuscripts and publications is approximately nine months. The Council voted to recommend to the Board of Trustees that 1100 pages be published in the Transactions for 1952.

The Mathematical Reviews Editorial Committee reported that 892 pages had been published in 1951. The subscription list as of November 1951 was 2125.

The Proceedings Editorial Committee reported that the interval between receipt of manuscripts and publication is now approximately one year. The Council voted to recommend to the Board of Trustees that 1000 pages be authorized for the 1952 Proceedings. Professors Bjarni Jónsson and A. E. Taylor were reported as new Assistant Editors for the Proceedings.

The Colloquium Editorial Committee recommended that Professor Antoni Zygmund be invited to deliver the Colloquium Lectures at the 1953 summer meeting. The Council voted to approve this recommendation.

The Council voted to invite Professor Marston Morse to deliver the Josiah Willard Gibbs Lecture at the annual meeting in 1952.

The Council voted to direct its Secretary to obtain, as a condition of holding a meeting, assurances that at any event scheduled in the program there will be no discrimination as to race, color, religion, or nationality, and that when accommodations and other facilities are provided these shall be provided to all attending the meeting.

The following resolution was read:

It is with great regret that the Council of the American Mathematical Society finds that the transfer of the office from New York to Providence involves the loss to the Society of the services of Miss Evelyn Hull. For seventeen years she has served the Society as Chief Clerk of the New York Office. In this capacity she has done extraordinary work in all branches of the Society's activity. Her work for the Librarian, for the Secretary, and for the Treasurer has been an invaluable aid in the performance of the duties of their positions; without her efficient and willing attention to details, no one of these officers could have performed the functions of his office in addition to his full-time academic duties. Her negotiations with the publishers of our journals and books have been carried on with great skill and never failing courtesy. She has given freely of her time and energy, devoting to the work of the Society many evening and week-end hours. She has won the respect and high regard of all with whom she was associated in the work of American mathematics.

The Council of the Society hereby expresses to Miss Hull its deep sense of loss over her resignation and its wholehearted appreciation for her work for the advancement of mathematics. The Council wishes her every success and happiness in her future career.

The Council unanimously voted to adopt this resolution.

Presiding officers at the sessions for contributed papers were Professor Gabor Szegö, Dr. R. S. Burington, Professors Morris Marden, J. S. Frame, Einar Hille, Samuel Eilenberg, W. A. Hurwitz, G. H. Handleman, F. J. Randolph, R. E. Johnson, C. E. Rickart, J. L. Walsh, J. W. Green.

Abstracts of the papers read follow. Those having the letter "*t*" after their numbers were read by title. Paper number 128 was presented by Professor Copeland, 131 by Dr. Harary, 150 by Dr. Klee, 152 by Professor Edrei, 157 by Professor Marden, 164 by Dr. Lindgren, 166 by Dr. Danskin, 185 and 186 by Professor Schoenberg, 205 by Mr. Rosenfeld, 206 by Professor Walsh, 209 by Professor Clark, 211 by Professor Diaz, 212 by Dr. Gerst, 232 by Dr. Hammer, 234 by Dr. Sobczyk.

Professor Finzi and Mr. Ankeny were introduced by Professor I. E. Segal, Professor Hopkins by Professor Rohn Truell, Miss Schuepbach by Professor Sarvadaman Chowla, Dr. Peters by Professor J. R. Kline, and Mr. Kokoris by Professor A. A. Albert.

Algebra and Theory of Numbers

117. N. C. Ankeny: *Euclidean algorithm in Abelian extensions of the rationals.*

H. Davenport has shown that there are only a finite number of Euclidean fields, i.e. which have Euclidean algorithm, which are quadratic, cubic which are not totally real, or totally imaginary quartic extensions of the rationals. H. Heilbronn extended this to normal cubic fields. These results are generalized to Abelian extensions of the rationals and to certain types of normal extensions of the rationals. Also analogous results can be obtained if ground fields other than the rationals are taken. (Received November 13, 1951.)

118t. H. W. Becker: *Words isomorphic with compositions.*

These are subsets of rhyme schemes and permutations of n letters enumerated by $c_n = 2^{n-1}$ (except $c_0 = 1$). Most of the breakdowns $c_{n,m} = (n-1, m-1)$, c_{n-m}, or $(n, 2m)$. Two of them involve the Fibonacci series with initial entries $\phi_0 = 1$, $\phi_1 = 0$ whose generating function is $y_0 = 1/(1-t\phi) = (1-t)/(1-t-t^2)$. If m is the number of rhyme scheme singletons, $c_{n+1,m} = c_{n,m} + c_{n-1,m} + c_{n,m-1} - c_{n-1,m-1} = \sum_0^m \phi_i \cdot c_{n-i,m-1}$, which has the generating function $y_m = 1/(1-tc_{n,m}) = t^m y_0^m + 1$. If m is the position of the last singleton, then $c_{n,m} = \phi_{n-m} \cdot c_{m-1}$. Interpreting these rhyme schemes as products, the sum of all c_{n+1} such products is $\Psi_{n+1} = \sum_0^n a^{i+1} \cdot \Psi'_{n-1} = \Psi_n * \sum_a^n k$. Here Ψ' denotes transformation of all letters $a \to b$, $b \to c$, etc., and $*$ a selective multiplication in which k multiplies only those terms of Ψ_n whose alphabetically last factors are powers of k or $k-1$. There is a subset of the exponential polynomial Y_n of E. T. Bell (Ann. of Math. vol. 35 (1934) pp. 258–277) in whose metrical interpretation there remains an unsolved problem: to find the number of n line stanzas wherein each line of m syllables is balanced by at least one other line of the same length. The inverse lexicon theorem, for finding the word of given rank, employs binary digits. A word of odd rank ends with a couplet. (Received November 13, 1951.)

119. Volodymyr Bohun-Chudyniv: *On a method and general scheme for solution of the Euler problem.*

We consider a problem Euler proposed in a letter to Lagrange (L. Euler, *Opera Posthuma*, Acad. Petrop. 1862, vol. I, p. 576). The scheme for solution offered in this paper depends on the product of four quaternions: $(x_0 + \sum_{\beta=1}^3 x_\beta i_\beta) k_\alpha (x' + \sum_{\beta=1}^3 x'_\beta i_\beta) = A'_{\alpha,1} + \sum_{\beta=1}^3 A'_{\alpha,\beta+1} i_\beta$; $(y_0 + \sum_{\beta=1}^3 y_\beta i_\beta) k'_\alpha (y'_0 + \sum_{\beta=1}^3 y'_\beta i_\beta) = A''_{\alpha,1} + \sum_{\beta=1}^3 A''_{\alpha,\beta+1} i_\beta$; $\alpha = 1, 2$. The quantities sought are the $A'_{\alpha,j}$, $A''_{\alpha,j}$ ($\alpha = 1, 2$; $j = 1, 2, 3, 4$). By suitable choices of the x_j, x'_j, y_j, y'_j ($j = 0, 1, 2, 3$) and k_α, k'_α two solution schemes due to Euler (Comm. Arith. I, pp. 427–443, Acad. Petrop. 1849), one due to C. Avery (Math. Miscellany, New York, p. 101, 1839), two due to G. K. Perkins (ibid. pp. 102–105) are obtained as well as a scheme previously given by the author (V. Chudyniv-Bohun, *Solution for Euler's problem,* Ukrainian Free Acad. of Sci., Regensburg, 1947). (Received November 13, 1951.)

120. Samuel Bourne: *On the homomorphism theorem for semirings.*

A semiring is a system consisting of a set S together with two binary operations, called addition and multiplication, which forms a semigroup relative to addition, a semigroup relative to multiplication, and in which the right and left distributive laws hold. A semiring S is said to be semi-isomorphic to the semiring S', if S is homomorphic to S' and the kernel of this homomorphism is (0). *If the semiring S is homomorphic to the semiring S', then the difference semiring $S-I$ is semi-isomorphic to S', where I is the ideal of elements mapped onto $0'$.* The correspondence $s \to s'$, where s' is the image of an element of the coset s, is a semi-isomorphism of $S = S - I$ into S'. This correspondence is certainly a homomorphism, for if $s = (s_1, s_2, \cdots)$, then $s_1 + i_1 = s_2 + i_2$, i_1 and i_2 in I, and $(s_1 + i_1)' = s'_1 = (s_2 + i_2)' = s'_2$. $\bar{s} + \bar{t} \to s' + t'$ and $\bar{s}\bar{t} \to s't'$ follow from coset addition and multiplication. If $\bar{s} \rightarrow 0'$, then $s_1 + i_1 = s_2 + i_2$, i_1 and i_2 in I, and $s_1' = s_2' = 0'$. Thus s_1 and s_2 are contained in I. This implies that the kernel of the homomorphism $s \rightarrow s'$ is $\bar{0}$ and this homomorphism is a semi-isomorphism. A previous result of the author (Proc. Nat. Acad. Sci. U.S.A. vol. 37 (1951) p. 164) is the converse of this theorem. (Received November 2, 1951.)

121t. Leonard Carlitz: *Congruences for the coefficients of hyperelliptic functions*. II.

We use the notation of the abstract of the first part of this paper (Bull. Amer. Math. Soc. Abstract 58-1-37). The main new result is the following. Put $(x/g(x))^\lambda = \sum_0^\infty \beta_m^{(\lambda)} x^m/m!$. Then $\sum_{i=0}^r (-1)^{r-i} C_{r,i} C_p^{r-1} \beta_{m+i(p-1)}^{(\lambda)} \equiv 0 \pmod{p^s}$, where $\lambda \geqq 2$; $m \geqq r \geqq r_0(p, \lambda)$, $s = [2^{-1}(r+1)] - \lambda_1$ and $1 \leqq \lambda_1 < \lambda$. In particular this result holds for the Bernoulli numbers of higher order as well as for the coefficients in $(x/\text{sn } x)^\lambda$, where sn x is the Jacobi elliptic function. (Received October 31, 1951.)

122t. Leonard Carlitz: *Some congruences for the Bernoulli numbers*.

The following theorem is proved. Let $(p-1)p^{e-1} | b$, $c = (p-1)u > re$; then $B^c(B^b - 1)^r \equiv 0 \pmod{p^{re-h}}$, where $h = e$ for $r \leqq p-1$ or $r = p-1$, $e > 1$; $h = 2$ for $r = p-1$, $e = 1$; $h =$ least integer $\geqq (re+1)/p$ for $r \geqq p$. A similar result is proved for $\sigma^c(\sigma^b - 1)^r$, where $\sigma_m = (1/m)(B_m + 1/p - 1)$ for $p-1 | m$. (Received November 13, 1951.)

123t. Leonard Carlitz: *Some theorems on the Schur derivative*.

Schur (Preuss. Akad. Wiss. Sitzungsber. (1933) pp. 145–151) defined the derivative of a sequence $\{a_m\}$ by means of $\Delta a_m = (a_{m+1} - a_m)/p^{m+1}$; higher derivatives are defined in the obvious way. He proved that if $p \nmid a$, then the derivatives $\Delta^2 a^{p^m}, \cdots, \Delta^{p-1} a^{p^m}$ are all integral, while $\Delta^p a^{p^m}$ has the denominator p. A. Brauer gave another proof of these results. M. Zorn proved Schur's results by p-adic methods (Ann. of Math. (2) vol. 38 (1937) pp. 451–464) and indeed found the residue of $\Delta^r X_m \pmod{p^m}$, where $X_m = (x^{p^m} - 1)/p^{m+1}$, $x \equiv 1 \pmod{p}$. In the present paper a simple elementary proof is given of Zorn's results as well as a number of similar results. For example, the residue of $\Delta^r a^{p^m}$ is determined. In addition several generalizations of Schur's theorem are proved, one being a generalization to algebraic numbers. Finally some applications are indicated to the Euler and Bernoulli numbers; for example, $\Delta^r E_{k+p^m}$ is integral for $p > 2$, $r < p$, $r \leqq m$. (Received October 31, 1951.)

124t. K. T. Chen: *Iterated integrals of paths and exponential mappings*.

In a euclidean space E with coordinates $(x_1, \cdots, x_n)$, all piecewise smooth paths initiating from the origin form a semigroup Ω. Introduce α^{-1} and the relations $\alpha\alpha^{-1} \equiv 1$, $\alpha \in \Omega$; a group G is thus obtained from Ω. Let α be given by $\{x_1(t), \cdots, x_n(t)\}$, $0 \leqq t \leqq 1$. Define $I_{i_1 \ldots i_p}(\alpha) = \int_0^1 \int_0^{t_p} \cdots \int_0^{t_2} dx_{i_1}(t_1) \cdots dx_{i_{p-1}}(t_{p-1}) dx_{i_p}(t_p)$. Then $I_{i_1 \ldots i_p}(\beta\alpha\alpha^{-1}\gamma) = I_{i_1 \ldots i_p}(\beta\gamma)$, α, β, $\gamma \in \Omega$. Consequently $I_{i_1 \ldots i_p}(\alpha)$ is defined for α taken as an element of G. Let $X_1, \cdots, X_n$ be nonabelian indeterminates. The exponential mapping θ is a homomorphism with $\theta(\alpha) = 1 + \sum_{p=1}^\infty \psi_p(\alpha)$, $\psi_p(\alpha) = \sum_{i_1 \ldots i_p} I_{i_1 \ldots i_p}(\alpha) X_{i_p} \cdots X_{i_p}$. Let G_d consist of all $\alpha \in G$ with $\psi_1(\alpha) = \cdots = \psi_{d-1}(\alpha) = 0$, i.e. $I_{i_1 \ldots i_p}(\alpha) = 0$, $p < d$. Then G_d/G_{d+1}, $d \geqq 1$, is a vector space over the reals. All straight paths along the coordinate axes generate a subgroup H in G. H is a free product $R_1 * \cdots * R_n$, where each R_i is the additive group of the reals. For $a \in R_i$, $\theta(a) = 1 + aX_i + (aX_i)^2/2! + \cdots$. Moreover $\bigcap_{d=1}^\infty H_d$

$=\{1\}$ ($H_d=H\cap G_d$), and each H_d/H_{d+1} is also a vector space over the reals. There is a free group $F\subset G$. By using results of Magnus and Witt, $F_d=F\cap G_d$ is identified with the dth lower central commutator subgroup of F. (Received November 13, 1951.)

125. Sarvadaman Chowla: *The rational points on a cubic curve.*

This paper contains remarks on the problem of determining the "rank" of a given cubic curve (see A. Weil, Bull. Soc. Math. France vol. 54 (1930) p. 182). (Received November 13, 1951.)

126*t*. Eckford Cohen: *Rings of arithmetic functions.*

An arithmetic function f relative to a positive integer r and a field F of characteristic 0 is a single-valued function such that $f(a)\in F$ if a is an integer and $f(a)=f(a')$ if $a\equiv a'$ (mod r). Under ordinary addition and Cauchy multiplication the set of all such functions forms a commutative algebra over F. Structure properties of this algebra are determined and a principle relating to additive representations is deduced. Corresponding results for $GF[p^n, x]$ are also obtained. (Received November 14, 1951.)

127*t*. Harvey Cohn: *Some remarks on fields of small discriminant.* Preliminary report.

It has been conjectured that for totally real fields of degree $n=(p-1)/2$ (p prime), the field of smallest discriminant is $R(\cos 2\pi/p)$. This is known to be true for $n=2, 3$. It is here shown that the conjecture is false for $n=6$ and $n=18$, the counterexamples being $R(\cos 2\pi/7, 5^{1/2})$ and $R((6-6\cos 2\pi/19)^{1/2})$ $R((7+2(7+2\cdot 5^{1/2})^{1/2})^{1/2})$. (Received October 10, 1951.)

128. A. H. Copeland and Frank Harary: *The extension of an arbitrary Boolean algebra to an implicative Boolean algebra.*

An implicative Boolean algebra is one which contains an implication or conditional which is appropriate to the theory of probability. The conditional has been defined as an inverse of a cross product operation which is associative, noncommutative, distributive with respect to the binary Boolean operations, satisfies left cancellation, and has the unit element as its unit. It is known that not all Boolean algebras are implicative, in fact it is shown here that any implicative Boolean algebra is atomless. The question then arises whether there is an essential restriction imposed by introducing such a conditional into an arbitrary Boolean algebra. The question is answered in the negative by the fact that the Boolean algebra can be enlarged so as to include the additional elements produced by the operations of the cross product and the conditional. To accomplish the extension, the Stone representation is modified to one in which all nonzero elements have the same cardinality. The cross product operation and its inverse are then defined in terms of mappings with respect to the points of the representation space. These mappings produce the desired new elements. (Received November 13, 1951.)

129. Trevor Evans: *Embedding theorems for multiplicative systems and projective geometries.*

The following theorems are proved. Any countable loop (quasigroup, groupoid) can be embedded in a loop (quasigroup, groupoid) generated by one element. Any countable semigroup can be embedded in a semigroup generated by two elements. Any finitely generated projective plane can be embedded in a projective plane generated by four points. These results correspond to recent theorems for groups, Graham Higman, B. H. Neumann, and Hanna Neumann, *Embedding theorems for groups*, J. London Math. Soc. vol. 24 (1949) pp. 247–254, and for nonassociative algebras, A. I. Zukov, *Reduced systems of defining relations in non-associative algebras*, Mat. Sbornik N.S. vol. 27 (1950) pp. 267–280. For all such theorems the author has the following interpretation, which, for simplicity, is stated for the semigroup case. Let F be the free semigroup on two generators. A suitable free subsemigroup on a countable set of generators is chosen and on these generators are imposed the defining relations of the semigroup to be embedded. Then these relations considered as relations in F do not imply any new relations in the subsemigroup and so F with these relations added is the required containing semigroup. (Received November 13, 1951.)

130. Franklin Haimo: *The FC-chain of a group.* Preliminary report.

Let G be a group and let subgroups $H(i)$ be defined, $i=0, 1, 2, \cdots$, where $H(0) = (e)$ and if $H(i)$ is defined then $H(i+1)$ is the complete inverse image of that subgroup in $G/H(i)$ which consists of the elements with only a finite number of conjugates. (See Baer, Duke Math. J. vol. 15 (1948) pp. 1021–1032.) The $H(i)$ turn out to be an ascending chain of strictly characteristic subgroups. A recent result due to B. H. Neumann (Proc. London, Math. Soc. (3) vol. 1 (1951) pp. 178–187) is extended to this chain. Questions involving nilpotency are also discussed. (Received November 13, 1951.)

131. Frank Harary and G. E. Uhlenbeck: *On some generalizations of rooted trees.*

The problem of counting linear graphs satisfying certain conditions has come up in certain problems in statistical mechanics, especially connected to the theory of condensation. The appropriate kind of graph is a "generalized Husimi tree." A partial solution of this problem has been secured. A Husimi tree of type n_2, n_3 is a connected linear graph with n_2 segments (cycles of length 2), n_3 triangles (cycles of length 3), and no other cycles, in which two different cycles have at most one common point. Husimi trees of type $n_2, n_3, \cdots, n_k$ are defined similarly. A rooted Husimi tree is one in which there is a preferred point. If $H(n_2, n_3)$ is the number of different rooted Husimi trees of type n_2, n_3 and $h(x, y) = \sum_{n_2,n_3=0}^{\infty} H(n_2, n_3)x^{n_2}y^{n_3}$, then an explicit functional equation for $h(x, y)$ has been found. An analogous functional equation has also been obtained for rooted Husimi trees of type $n_2, n_3, \cdots, n_k$. The methods used are due to Pólya, Acta Math. vol. 68 (1937) pp. 145–254. A Cayley tree is a linear graph with no cycles. A Cayley tree with n points is said to have r roots, if r of the points are distinguished from the remaining points and also from each other. Functional equations for these multiply rooted Cayley trees have been found which are generalizations of Cayley's equation for the number of singly rooted trees. (Received November 13, 1951.)

132. Melvin Henriksen: *On the free ideals of the ring of entire functions.* Preliminary report.

Let R denote the ring of entire functions. An ideal of R is called *fixed* if all the functions in it vanish at at least one common point. Otherwise it is called *free*. The maximal free ideals and their residue class fields have previously been investigaged by the author. (See Bull. Amer. Math. Soc. Abstract 51-4-325.—The contents of that abstract will appear in a subsequent issue of the Pacific Journal of Mathematics.) Further properties of free ideals are obtained. In particular it is shown that if f is any function with simple zeros only in a maximal free ideal M, then M contains uncountably many proper divisors of f with simple zeros only. The principal tools used are the two-valued measure functions of S. Ulam (Fund. Math. vol. 14 (1929) pp. 231–233) and A. Tarski (Fund. Math. vol. 15 (1930) pp. 42–50). (Received November 15, 1951.)

133. B. W. Jones: Automorphs of symmetric matrices.

The following theorem is proved: If S is an automorph of a nonsingular symmetric matrix A such that the matrices $S-I$ and $S+I$ are both singular and have ranks whose sum is not greater than n, the order of A, then $S^2=I$ and, if r is the rank of $S-I$, there is an n by r matrix U of rank r such that U^TAU is nonsingular and $S=I-2U(U^TAU)^{-1}U^TA$. This theorem holds in any field in which $2\neq 0$. For any n by r matrix U for which U^TAU is nonsingular, an S defined as above is an automorph. (Received November 13, 1951.)

134. Irving Kaplansky: *Representations of separable algebras.*

The author presents two results, the first of which is a new proof of a theorem of Johnson and Kiokemeister (Trans. Amer. Math. Soc. vol. 62 (1947) pp. 404–430), while the second is the Hilbert space analogue. (1) Let A be the ring of all linear transformations on a vector space of countable dimension. Then any representation of A on a vector space of countable dimension is faithful, a direct sum of irreducible representations, and continuous in the weak topology. (2) Let B be the algebra of all bounded operators on a separable Hilbert space. Then any *-representation of A on a separable Hilbert space is faithful, a direct sum of irreducible representations, and continuous in the strongest topology. (Received November 7, 1951.)

135. L. A. Kakoris: *New results on power-associative algebras.*

We call a commutative algebra A over F power-associative if the algebra $K[x]$ generated by every x of A_K is associative for every scalar extension K of F. With this definition the known structure theory for commutative power-associative algebras of characteristic prime to 30 (see A. A. Albert, *A theory of power-associative commutative algebras*, Trans. Amer. Math. Soc. vol. 69 (1950) pp. 503–527) is extended to include the cases of algebras of characteristic 3 and 5. Proofs of the results require the associativity of fourth and fifth powers in the case of characteristic 3 and the associativity of fourth and sixth powers in case the characteristic is 5. Albert's results indicated the possibility of the existence of simple commutative power-associative algebras of degree two not Jordan algebras. We construct two types of such algebras. The algebras have characteristic p. One of them has dimension $4p$ and is stable. The other has dimension $3p$ and provides the first known example of a simple nonstable commutative power-associative algebra. (Received December 24, 1951.)

136. W. G. Lister: *A structure theory of Lie triple systems.*

In a recent paper (Trans. Amer. Math. Soc. vol. 70 (1951) pp. 141–169) Jacobson has characterized subspaces of Lie algebras closed under triple products, called them Lie triple systems, and shown that every Jordan algebra can be regarded as a Lie triple system. In the present paper a structure theory similar to that of Lie algebras is developed for Lie triple systems. A radical is defined, the relations between it and the radical of an enveloping Lie algebra given, and the existence of a Levi decomposition is established. The class of simple Lie triple systems is shown to determine in the usual way all semi-simple systems, and the simple systems are classified. Finally, results similar to those of Jacobson for Lie algebras (Proceedings of the American Mathematical Society vol. 2 (1951) pp. 105–113) on the complete reducibility of Lie triple systems of linear transformations are obtained. (Received November 13, 1951.)

137. W. H. Mills: *On the nonisomorphism of certain holomorphs.*

Let G be a finite abelian group, H the holomorph of G, and N an arbitrary group with holomorph isomorphic to H. Write G as the direct product of its Sylow subgroups: $G=P_1\times P_2\times \cdots \times P_t$. Then N can be written as a direct product $N=J_1\times J_2\times \cdots \times J_t$, where the holomorphs of P_i and J_i are isomorphic. It is shown that a finite abelian group P_i of prime power order and a group that is not abelian cannot have isomorphic holomorphs. Hence N is abelian. In a previous paper (*Multiple holomorphs of finitely generated abelian groups*, Trans. Amer. Math. Soc. vol. 71 (1951) pp. 379–392) it was shown that two finitely generated abelian groups have isomorphic holomorphs if and only if they are isomorphic. It follows that N is isomorphic to G. Thus a finite abelian group and an arbitrary group have isomorphic holomorphs if and only if they are isomorphic. (Received October 25, 1951.)

138. P. M. Pepper: *An expected value of the number of pairs of twin primes* $\leqq x$.

For a given integer k, at least one of the integers $6k\pm 1$ is composite if and only if there exist a prime $p\geqq 5$ satisfying $p^2\leqq 6k+1$, an integer $b\geqq [(p+1)/6]$, and a unit $e=\pm 1$, such that $k=bp+e[(p+1)/6]$. This characterization describes a generalized sieve of Eratosthenes, G, which screens from the class of all positive integers precisely those integers k for which both of the numbers $6k\pm 1$ are primes $\geqq 5$. A study of sieves formed by superposing several general periodic sieves having relatively prime minimal periods leads to a generalization of the Euler ϕ function, which (a), under suitable specialization, shows that the Euler ϕ function has a much broader scope of applicability than is usually attributed to it, and (b) leads to certain invariants of composite sieves which implement the definition of a probability function by means of which is derived an expected value $E(x)=3+\sum_{i=1}^{m-1} D_iP_i+P_m(x-p_m^2)/6$ for the number of prime pairs $\leqq x$ wherein p_i is the ith prime $\geqq 5$, $D_i=(p_{i+1}^2-p_i^2)/6$, $P_i=\prod_{j=1}^{i} (1-(2/p_j))$, and m is the greatest integer for which $p_m^2\leqq x$. Moreover, $E(p_{a+1}^2)-E(p_i^2)\geqq p_{i+1}-p_i$, whereas, for any real N, there exist infinitely many values of i for which $p_{i+1}-p_i>N$. Finally, $E(p)_m^2\geqq p_m-2$, whence $E(x)>p_m-2$ for $x>p_m^2$. (Received December 26, 1951.)

139. H. J. Reiter: *Investigations in harmonic analysis.* I. *A generalization of Wiener's theorem.*

Let G be a locally compact abelian group, with a denumerable fundamental system of neighborhoods of the identity, and let $\hat{G}$ be the dual group. Let I be a closed ideal in L^1 and $\hat{Z}_I$ the set of elements of $\hat{G}$, where all Fourier transforms of functions in I are zero. The following is proved: Let $k(x)$ be in L^1 and $\hat{Z}_k$ the set of zeros of its Fourier transform. If (i) $\hat{Z}_I\subset\hat{Z}_k$ and (ii) the intersection of the frontiers of $\hat{Z}_I$ and $\hat{Z}_k$ is *denumerable*, then $k(x)$ belongs to I. The proof is based on the theorem of Plancherel-Weil and the theory of bounded linear functionals on L^1; it is connected with that given by S. Mandelbrojt and S. Agmon (*Une généralisation du théorème* *tauberien de Wiener*, Acta Math. Szeged. vol. 12 (1950) part B, pp. 167–176) in the case of the group of the real numbers. (Received November 13, 1951.)

140*t*. H. J. Reiter: *Investigations in harmonic analysis*. II. *On functions orthogonal to closed ideals in* L^1.

Let G, I, $\hat{Z}_I$ be defined as in part I. A bounded, measurable function ϕ is called orthogonal to I if $\int f(xy)\overline{\phi(y)}dy=0$ for all $f(x)$ in I. The following results are proved: Let ϕ be orthogonal to I. If $\hat{Z}_I$ is finite, ϕ is a linear combination, with constant coefficients, of characters of G. If $\hat{Z}_I$ is discrete, any uniformly continuous ϕ is almost periodic; moreover, *any* ϕ has a "Fourier series." The uniqueness theorem and "Bessel's inequality" hold. "Parseval's equality" is proved under certain restrictions on the "density" of $\hat{Z}_I$. If G is the group of real numbers and $\hat{Z}_I$ is discrete, any ϕ is the limit (almost everywhere) of uniformly bounded trigonometric polynomials. If the distance between any two elements of $\hat{Z}_I$ is larger than some positive constant (or if $\hat{Z}_I$ is the union of a finite number of sets satisfying that condition), then any ϕ is almost periodic B^2 (Besicovitch). The proofs are based mainly on the results in part I. (Received November 13, 1951.)

141*t*. H. J. Reiter: *Investigations in harmonic analysis*. III. *Closed ideals in* L^1 *and their homomorphisms*.

Let G be a locally compact abelian group. For any function $f(x)$ in L^1 the following is proved: $\inf \int |f(x)-\sum_1^N \lambda_n f(xy_n)|dx=|\int f(x)dx|$, where N ranges over all positive integers, $\lambda_1, \lambda_2, \cdots, \lambda_N$ range over all complex numbers satisfying $\sum_1^N \lambda_n=0$, and $y_1, y_2, \cdots, y_N$ over the elements of G. This leads to the proof of the following theorem: Let I be any closed ideal in L^1. Any continuous homomorphism of I upon C, the field of complex numbers, is given by a Fourier transform; moreover, the isomorphism $I/I_0 \cong C$ is isometric (I_0 is the kernel of the given homomorphism and the Banach algebra I/I_0 is normed in the usual way). Another result is the following: If $\bar{G}$ is a homomorphic image of G, then the L^1-algebra on $\bar{G}$ is a homomorphic image of the L^1-algebra on G and the isomorphism $L^1(G)/I_0 \cong L^1(\bar{G})$ is isometric. The proofs are based on the Hahn-Banach theorem (complex case) and several lemmas, e.g.: If $\int f(x)dx=0$, then the integral equation $\int f(xy)\phi(x)dx=1$ (for all y in G) has no bounded, measurable solution ϕ. (Received November 13, 1951.)

142. G. de B. Robinson: *On a conjecture by J. H. Chung*.

The modular representation theory of the symmetric group is far more explicit than in the general case as a result of its relationship with the Young diagrams. Recently (Canadian Journal of Mathematics vol. 3 (1951) pp. 309–327), J. H. Chung conjectured that the number of ordinary (modular) irreducible representations of S_n belonging to a given p-block is independent of the p-core, and is determined only by the weight of the block for a given prime p. A proof of this statement can be given which utilizes the notion of *integration* (Robinson, Canadian Journal of Mathematics vol. 2 (1950) pp. 334–343; Murnaghan, Proc. Nat. Acad. Sci. U.S.A. vol. 37 (1951) pp. 55–58) as applied to the star diagram (Proc. Nat. Acad. Sci. U.S.A. vol. 37 (1951) pp. 694–696). (Received November 13, 1951.)

143. Eugene Schenkman: *On infinite Lie algebras*.

The object of this paper is to study Lie algebras whose underlying vector spaces need not be finite-dimensional. We assume, however, that the Lie algebra L under consideration has a nilpotent ideal M (i.e. $M^k=0$ for some k) such that (a) L/M is finite-dimensional and (b) if M' stands for $[M, M]$, then M/M' is a direct sum of ideals $P_{\alpha/M'}$ of L/M' each of dimension less than j. Our results are then as follows: (1) an extension of Engel's theorem is true, namely, that L is nilpotent if all its elements are nilpotent. (2) L is nilpotent if each of its maximal subalgebras is an ideal or equivalently if the meet of all the maximal subalgebras contains $L'=[L, L]$. (3) An extension of the tower theorem is valid as follows: Let L have zero center and let Z be the center of $L^\omega=\bigcap_{i=1}^\infty L^i$. Let D_i be any algebra of the tower of derivation algebras $L=D_0, D_1, \cdots, D_n, \cdots$ of L, then $D_{i/Z}$ is isomorphic to a subalgebra of $D(L^\omega)$ the algebra of derivations of L^ω. Our result (2) is analogous to a result of Hirsch for infinite solvable groups in Proc. London Math. Soc. vol. 49 (1945) and (3) is an extension of our earlier result in Amer. J. Math. vol. 73 (1951). (Received December 26, 1951.)

144t. Dorothy Schuepbach: *Integer solutions of* $y^2=(x+a)(x+b)\cdot(x+c)$.

Nagell and Delaunay have found that all of the integer solutions of (1) $y^2=x^3+17$ are $(x, y)=(-2, \pm 3), (-1, \pm 4), (2, \pm 5), (4, \pm 9), (8, \pm 23), (43, \pm 282), (52, \pm 375)$, $(5234, \pm 378{,}661)$. Hence, there are 16 solutions in integers. The author has found an equation of the type mentioned in the title, namely (2) $y^2=(x+9)(x-3)(x-6)$ which has at least 19 solutions in integers. They are: $(x, y)=(-9, 0), (-6, \pm 18)$, $(-3, \pm 18), (1, \pm 10), (3, 0), (6, 0), (7, \pm 8), (9, \pm 18), (21, \pm 90), (51, \pm 360), (153, \pm 1890)$. Mordell has shown that (2) has a finite number of solutions in integers. (Received November 13, 1951.)

145t. Alfred Tarski: *On representable relation algebras*. Preliminary report.

Lyndon showed (Ann. of Math. vol. 51 (1950) pp. 707–729) that the class $\boldsymbol{R}$ of all relation algebras which are representable, i.e., isomorphic to proper relation algebras, cannot be axiomatically characterized by means of any—finite or infinite—system of algebraic identities. He raised the problem whether this class can be characterized by means of any system of arithmetical axioms, i.e., statements formalized within the lower predicate calculus; in other words, whether $\boldsymbol{R}$ is an arithmetical class or an intersection of arithmetical classes (cf. Bull. Amer. Math. Soc. Abstract 55-1-74). The solution is negative. This is an obvious consequence of the following results: (I) Let $\boldsymbol{K}$ be any class of algebras which is an intersection of arithmetical classes. If every finite set of elements of a given algebra $\mathfrak{A}$ can be (isomorphically) imbedded in an algebra belonging to $\boldsymbol{K}$, then $\mathfrak{A}$ itself can be imbedded in an algebra belonging to $\boldsymbol{K}$. (II) There is a non-representable relation algebra all finitely-generated subalgebras of which are representable. (III) Every subalgebra of a representable relation algebra is representable. (I) is a somewhat stronger formulation of a result contained in the Princeton University doctoral dissertation of Henkin (1947); (II) was found by Lyndon (op. cit.); (III) is obvious. (Received November 14, 1951.)

146t. N. A. Wiegmann: *Pairs of normal matrices with property* L.

A generalization of a theorem (due to Motzkin and Taussky) concerning pairs of hermitian matrices with property L is obtained. The theorem states that if two normal matrices A and B have property L, then they commute. The proof depends on a theorem in the earlier paper, and on the fact that if a normal matrix has its characteristic roots down the main diagonal, then it must be in diagonal form. (Received October 29, 1951.)

147. Daniel Zelinsky: *Operator-compact groups*. Preliminary report.

The concept of linear compactness for topological vector spaces [Lefschetz, *Algebraic topology*, p. 78] can be generalized to a concept of operator-compactness for groups with operators. Theorems analogous to the standard elementary theorems for compact spaces can be proved. These ideas are applied to prove a conjecture of the author's [Duke Math. J. vol. 18 (1951) p. 440]. In terms of these same ideas, a sufficient condition is given for the possibility of raising infinite collections of orthogonal idempotents from a ring modulo its radical to the ring itself. An example is given to show that some condition like operator-compactness is needed and that Wedderburn's theorem $A=S+N$ is not always extendible to infinite-dimensional algebras. (Received November 13, 1951.)

148. J. L. Zemmer: *Ordered algebras which contain divisors of zero*. Preliminary report.

Using the definition of ordered ring given by G. Birkhoff (*Lattice theory*, rev. ed., Amer. Math. Soc. Colloquium Publications, vol. 25, New York, 1948) one can construct ordered rings containing proper divisors of zero. In this paper ordered linear algebras are investigated. The main results are concerned with irreducible algebras of finite dimension which are neither nilpotent nor fields. Using a result of A. A. Albert (*On ordered algebras*, Bull. Amer. Math. Soc. vol. 46 (1940) pp. 521–522) and the Wedderburn principal theorem it is shown that if such an algebra A is ordered then (i) $A=B+eNe+U$, where B is a subfield of A, e the identity of B, N the radical of A, and U is either the set of all $x\in A$ such that $ex=0$ or the set of all $x\in A$ such that $xe=0$, (ii) one, but not both, of the subalgebras eNe, U may be zero, (iii) if $eU\neq 0$ ($Ue\neq 0$) then e is a left (right) identity for U, (iv) U has either a right or a left basis over B, (v) eNe has both a right and a left basis over B, (vi) if x, y, z are nonzero elements of B, eNe, U respectively, then $|x|>|y|>|z|$, (vii) if the base field is a subfield of the real field then the elements of B commute with those of eNe. (Received November 13, 1951.)

Analysis

149. Shmuel Agmon: *Complex variable Tauberians*. Preliminary report.

Let $f(s)=\int_0^\infty e^{-su}d\alpha(u)$ with $\alpha(u)\uparrow$ converge for Re $(s)>0$. Suppose that $f(s)$ has $s=0$ as its only singularity on the imaginary axis. Let $\phi(x)=\sum a_nx^n$ be a Taylor series converging for $|x|<1$ and having $x=1$ as its only singularity on the unit circle. Suppose, furthermore, that $f(s)-\phi(e^{-s})$ is regular at $s=0$. Then, if the a_n's satisfy a very general regularity condition, the following holds: $\alpha(u)=\sum_{n\leq u}a_n+o(a_{[u]})$. In the above the condition that the origin is the only singularity of $f(s)$ on the imaginary axis can be replaced by: $s=0$ is the dominant singularity on the imaginary axis in a certain sense. The condition on $\alpha(u)$ to be increasing can also be relaxed. The general result contains Ikehara's Tauberian as a very particular case. (Received November 9, 1951.)

150. R. D. Anderson and V. L. Klee: *Convex functions and upper semi-continuous collections.*

A real-valued function f is called *convex* if its domain D_f is an open convex subset of E^n and $f(tx+(1-t)y) \leqq tf(x)+(1-t)f(y)$ whenever $0 \leqq t \leqq 1$ and $\{x, y\} \subset D_f$. A *C-collection* is an upper semicontinuous collection of compact convex sets in E^n. A theorem is proved concerning the dimensionality of certain subsets of *C*-collections, and with a result of Wilder [Bull. Amer. Math. Soc. Abstract 54-11-551] it shows that *no open set in* E^n *can be filled by a nontrivial* continuous *C-collection.* With each positive convex f there is associated a *C*-collection which is used to prove "smoothness" theorems for f. For each $x \in D_f$ there is a unique linear manifold $L_x \ni x$ maximal relative to having $f | D_f \cap L_x$ differentiable at x. (The dimension of L_x is k if and only if the hyperplanes supporting f at $(x, f(x))$ have $n-k$ "degrees of freedom.") It is proved that if $0 \leqq k \leqq n$ and S_k is the set of all $x \in D_f$ for which dim $L_x \leqq k$, then S_k *is the union of countably many closed sets of finite k-dimensional Hausdorff measure.* (Carathéodory [p. 83 in Blaschke's *Kreis und Kugel*] and Reidemeister [Math. Ann. vol. 83 (1921) pp. 116–118] showed that if $n=2$ and $k=1$, then S_k has zero 2-dimensional measure.) Also included are some remarks on antipodal points of convex bodies. (Received November 13, 1951.)

151. Joseph Andrushkiw: *The reality interval of a polynomial.*

Let $f(z)$ be a polynomial with real coefficients. The problem is: To find necessary and sufficient conditions that all the zeros of $F(z)=f(z)+xz^{n+1}$, $x \neq 0$, be real. Without loss of generality let $f(z)=1+a_1z+ \cdots +a_nz^n$. Denoting by $f_1(z)$ and $F_1(z)$ the reciprocal polynomials of $f(z)$ and $F(z)$ respectively, one obtains the equivalent problem: To find necessary and sufficient conditions that $F_1(z)=zf_1(z)+x$, $x \neq 0$, have all its zeros real. They are: 1. The derivative $(zf_1(z))'$ must have all its zeros real. 2. Maximum minimorum of $zf_1(z)$, N, must be not greater than the minimum maximorum of $zf_1(z)$, P, and, if $(zf_1(z))'$ has some multiple zeros $r_1, r_2, \cdots, r_k$, also 3. $r_1f_1(r_1)=r_2f_1(r_2)= \cdots =r_kf_1(r_k)$. For any $x \neq 0$ of the open interval $(-P, -N)$, which is termed "the reality interval of $f(z)$," the polynomial $F(z)$ has all its zeros real and distinct. If $x=-P$ or $x=-N$, $F(z)$ has some multiple zeros. These conditions are satisfied if $f(z)$ has all its zeros real and distinct. In this case $N<0$, $P>0$ and it can be shown that $-N$ is the smallest positive and $-P$ the absolutely smallest negative zero of the discriminant of $F(z)$. (Received November 6, 1951.)

152. R. J. Arms and Albert Edrei: *On the Padé table and continued fraction associated with certain mermorphic functions.*

The Padé table of the exponential function has been thoroughly investigated; its convergence properties are remarkably simple but it does not seem to have been noticed that other functions display a similar behaviour. The authors show that this is the case for functions of the form $\phi(z)=\prod_{\nu=1}^{\infty} \{(1+\alpha_\nu z)/(1-\beta_\nu z)\}$ $[\alpha_\nu \geqq 0, \beta_\nu \geqq 0, \sum(\alpha_\nu+\beta_\nu)<+\infty]$. For functions of the more general form $f(z)=\exp(\gamma z)\,\phi(z)$ $[\gamma \geqq 0]$, their assertions are slightly less precise but of the same general character. Their methods also yield information on the asymptotic behaviour of the elements k_n [which are necessarily positive] and l_n of the continued fraction associated with $f(z)$ [Perron, *Die Lehre von den Kettenbrüchen*, p. 322]. For instance, in the special case $\gamma=0$, they show that: $\sum l_\nu$ converges; $k_\nu \to 0$; $(k_1k_2 \cdots k_n)^{1/n}=o(1/n)$. (Received November 8, 1951.)

153. Nachman Aronszajn: *Reproducing kernels in Banach spaces.*

Until now reproducing kernels were considered only for functional Hilbert spaces. An attempt is made here to extend this concept to Banach spaces. (Received November 23, 1951.)

154. R. G. Bartle: *Singular points of functional equations.* Preliminary report.

Let $\mathfrak{X}$ and $\mathfrak{Y}$ be real or complex Banach spaces and Φ be a continuous function on a neighborhood of the origin in $\mathfrak{X}\times\mathfrak{Y}$ to $\mathfrak{X}$. If $\Phi(0, 0)=0$ and if the partial differential $d_x\Phi(0, 0; dx)$ has a continuous inverse, Hildebrandt and Graves have shown that the equation $\Phi(x, y)=0$ can be solved explicitly for x in a neighborhood of the origin in $\mathfrak{Y}$. The author studies the case that $\Phi(x, y)$ has the form $x-K(x)+F(x, y)$ where K is a completely continuous linear transformation and F satisfies a Lipschitz-type condition assuring that it contains no linear terms in x. The Banach space problem is reduced to a system of implicit equations in y and n scalar variables, where n is the number of linearly independent eigenfunctions of K corresponding to the eigenvalue 1. This is similar to some recent work of J. Cronin. With the additional hypothesis that F can be written in a finite Taylor expansion with remainder and that $n=1$, it is possible to examine the existence, uniqueness, and reality of the solutions. This generalizes and extends some work of Erhard Schmidt and L. Lichtenstein on nonlinear integral equations. (Received November 13, 1951.)

155. Lipman Bers: *Mildly nonlinear partial difference equations of elliptic type.*

Let $F(x, y, z, p, q)$ be a smooth function, $|F_p|$, $|F_q|\leqq A<+\infty$, $F_z\geqq 0$, and set $\mathcal{L}[\phi]=\phi_{xx}+\phi_{yy}-F(x, y, \phi, \phi_x, \phi_y)$. A difference operator $\mathcal{L}_h$ is obtained by replacing the partial derivatives in $\mathcal{L}$ by differences. It is shown that the first boundary value problem for the equation $\mathcal{L}_h[\phi]=0$ has a unique solution, that this solution can be obtained by a Liebmann iteration method, and that it approximates the solution of the boundary value problem for $\mathcal{L}[\phi]=0$. These results are extended to the operator $\mathcal{L}[\phi]=A(x, y)\phi_{xx}+2B(x, y)\phi_{xy}+C(x, y)\phi_{yy}-F(x, y, \phi, \phi_x, \phi_y)$, $AC-B^2>0$. (Received August 23, 1951.)

156*t*. R. P. Boas: *Growth of analytic functions along a line.* Preliminary report.

If $\{\lambda_n\}$ is an increasing sequence of positive numbers of unit density, $|\lambda_n-n| \leqq o(n)$. We may characterize the distribution of the sequence more precisely by requiring that $|\lambda_n-n|\leqq\epsilon(n)$ with a given $\epsilon(n)$. It is shown that if $f(z)$ is regular and of exponential type in the right half-plane, and if $\limsup_{n\to\infty}\{\log|f(\lambda_n)|\}/\epsilon(\lambda_n)<\infty$, then $\limsup_{x\to\infty}\{\log|f(x)|\}/\epsilon(x)<\infty$ provided that $\lambda_n-\lambda_{n-1}\geqq\delta>0$, $\log x=o(\epsilon(x))$, $\int^\infty t^{-1}\epsilon(t)dt$ converges, and $\epsilon(t)$ satisfies some monotonicity and regularity conditions (for example, $\epsilon(t)=t^a$, $0<a<1$). This result is intermediate between the cases $\limsup \lambda_n^{-1}\log|f(\lambda_n)|<\infty$ (V. Bernstein) and $f(\lambda_n)$ bounded (Duffin and Schaeffer), neither of which is included. Some applications will be given later. (Received November 9, 1951.)

157. F. F. Bonsall and Morris Marden: *Zeros of rational functions with self-inversive polynomial factors.*

The authors prove the following theorem concerning the rational function $\phi(z) = (fgh)/(FGH)$ where f, g, h, F, G, H are polynomials of degrees m, n, p, M, N, P respectively. Let C denote the unit circle $|z| = 1$; let γ and Γ be the circles of radii r and R tangent internally to C at a point ζ, and let δ and Δ be the circles of radii s and S tangent externally to C at ζ. Let f and F each have its zeros symmetric in C; let all the zeros of g lie inside γ, those of h outside δ, those of G outside Γ, and those of H inside Δ. Let $m+(n/r)-(p/s)>M+(N/R)-(P/S)$. Then $\phi'(\zeta)=0$ if and only if ζ is a multiple zero of $\phi(z)$. The inequality may be replaced by an equality in certain cases. The use of this theorem and continuity leads to the following result. Let the above f and F have respectively q and Q zeros inside C and let all the zeros of g lie inside C and all those of G outside C. Let F have no multiple zeros and FG no zeros common with fg. Then, if $M+N<m+n$ and $Q \leqq n+q$, the derivative of $\phi(z) = (fg)/(FG)$ has exactly $m+n-q+Q-1$ zeros inside or on C. When $n=M=Q=0$, this result reduces to a theorem due to A. Cohn. (Received November 13, 1951.)

158*t*. F. E. Browder: *On the existence, uniqueness, and multiplicity of solutions of nonlinear functional equations.* I.

Let X, Y be Banach spaces, T a mapping of X into Y such that $T=J+C+S$ where J is an isometry of X with Y, C completely continuous, and S satisfies a Lipschitz condition with constant less than one. Suppose that T is locally one-to-one and satisfies the following generalized a priori bound: For every pair y_1, y_2 in Y there exists a constant N such that if $Tx=ty_1+(1-t)y_2$ for any t, $0 \leqq t \leqq 1$, then $\|x\| \leqq N$. Then T is a homeomorphism of X onto Y. More generally if T is a local homeomorphism of the connected topological space X into a locally convex linear metric space and if the inverse image of every line segment in the image space is compact, then T is a homeomorphism onto. The a priori bound of the first result may be replaced by a local a priori inequality. These results are a generalization of local results obtained by Schauder to theorems in the large. (Received November 13, 1951.)

159*t*. F. E. Browder: *On the existence, uniqueness, and multiplicity of solutions of nonlinear functional equations.* II.

Let X, Y be Banach spaces, T a mapping of X into Y satisfying the conditions of I including the a priori bound except that T need be no longer locally one-to-one. Let R be the set of points in X in some neighborhood of which T is locally one-to-one, $S=Y-T(X-R)$. Suppose that $T(R)-T(X-R)$ is nonempty and either (a) R is connected or (b) S is connected. Then T is onto and a finite covering mapping of $R \cap T^{-1}(S)$ onto S. In particular the hypotheses on R, S are satisfied if $X-R$ is compact, dim $(X)=\infty$. Analogous results can be formulated for locally convex linear metric spaces as in I. These theorems are applied to obtain results on the multiplicity of solutions of functional equations when local uniqueness is no longer present. (Received November 13, 1951.)

160*t*. F. E. Browder: *On the existence, uniqueness, and multiplicity of solutions of nonlinear functional equations.* III. *The generalized Dirichlet problem.*

Let $f(x_1, x_2, \cdots, x_n; z; \partial z/\partial \phi_1, \cdots, \partial z/\partial \phi_n; \partial^2 z/\partial \phi_1^2, \partial^2 z/\partial \phi_1 \partial \phi_2, \cdots, \partial^2 z/\partial \phi_n^2)$ be the general nonlinear elliptic partial differential operator of the second order on a domain D of E^n. Using the theorems of I and II, results are established on the existence, uniqueness, and multiplicity of solutions of the first boundary value problem for $f=0$. Assumptions are made on a priori bounds for the Hölder norms of second derivatives of solutions as well as on the local uniqueness of solutions. Theorems on the actual multiplicity of solutions are established in cases for which ramification may occur. The assumed a priori bounds are shown to hold for solutions of a general class of quasi-linear equations in the plane including the homogeneous quasi-linear equations treated by Leray and Schauder. (Received November 13, 1951.)

161. F. E. Browder: *The Dirichlet problem for self-adjoint linear elliptic equations of arbitrary even order with variable coefficients.*

Let L be a self-adjoint linear elliptic partial differential operator of order $2m$, $m \geqq 1$, on a bounded domain D in E^n. By the Dirichlet problem for L on D is meant the following: Given $g \in C^m(D)$, all of whose mth derivatives are square-integrable of D, to find $u \in C^{2m}(D)$, a solution of the equation $Lu=0$, such that the derivatives of $g-u$ of order less than m tend to zero in a suitable generalized sense on the boundary of D. It is shown that if L satisfies a certain positive definiteness condition and if the coefficients of the differential operators in L of order j have continuous derivatives up to orders $2m+j+2$, then the Dirichlet problem always has a solution and the solution is unique. The class of equations considered includes as a special case the equations with constant coefficients treated by L. Gårding. The methods used in the proof are an extension of the method of orthogonal projection as applied by Vischik and Gårding. (Received November 13, 1951.)

162t. F. E. Browder: *Weak and strong solutions of linear elliptic equations with variable coefficients.*

Let L be a linear elliptic differential operator defined on an arbitrary domain D in E^n. The function $v\,\mathcal{L}^2(D)$ is said to be a weak solution of the equation $Lu=0$ if $\int v(x)\bar{L}(f(x))dx=0$ for all $f \in C^{2m}(D)$ ($2m=$the order of L) such that f vanishes outside a compact subset of D and $\bar{L}$ is the adjoint differential operator of L. It is shown that if the coefficients of the jth differential operators in L have continuous partial derivatives of all orders up to $j+2m+2$, then every weak solution v of $Lu=0$ is equal almost everywhere to a strong solution $u \in C^{2m}(D)$. This result is a generalization of the corresponding theorem for the case of constant coefficients proved by L. Gårding and L. Schwartz. Extensive use is made of the results of F. John on the fundamental solution of equations with analytic coefficients. (Received November 13, 1951.)

163. H. D. Brunk: *A convergence property of certain generalizations of the interpolatory cardinal series.*

The cardinal series as an interpolatory function may readily be generalized to give a function which assumes given values at equally spaced points on a line, and which satisfies one of a wide class of partial differential equations. The role of the Fourier transform of the cardinal series in obtaining conditions for approximation to a solution of the partial differential equation coinciding with a given function on a line is studied, and applications are made to special partial differential equations. (Received November 14, 1951.)

164. R. H. Cameron, B. W. Lindgren, and W. T. Martin: *Linearization of certain nonlinear functional equations.*

Relating to papers of Cameron and Martin (Trans. Amer. Math. Soc. vol. 66

(1949) pp. 253–283, and Ann. of Math. vol. 51 (1950) pp. 385–392) the present theorem shows that a minimizing process involving solutions of linear algebraic equations yields approximate solution of the equation $y(t) = x(t) + \Lambda(x|t)$, where $\Lambda(x|t)$ is in general a nonlinear functional satisfying certain smoothness conditions. These approximations can be made arbitrarily close to the true solution in the $L_1(C)$ sense. (Received September 27, 1951.)

165. Lamberto Cesari: *On the characterization of Fréchet surfaces of finite Lebesgue area.*

Let $S \subset E_3$ be any continuous Fréchet surface, $T: x = T(w)$, $w \in Q$, any representation of S (Q simple Jordan region of the w-plane), $L(S)$ the Lebesgue area of S, T_1, T_2, T_3 the three plane mapping projections of T on the coordinate planes E_{21}, E_{22}, E_{23}, $W(T_s)$ the total variation and $N(x, y; T_s)$, $(x, y) \in E_{2s}$, the characteristic function of T_s, $s = 1, 2, 3$, $[S]$ the set of the points of S, C the boundary curve of S, R the set of the coordinate axes X, Y, Z. The inequality (A) $W(T_s) \leqq L(S) \leqq W(T_1) + W(T_2) + W(T_3)$, $s = 1, 2, 3$, was proved by the author first with elementary methods for regular surfaces with $|[S]| = 0$ (Mem. Acad. Italia vol. 12 (1941) pp. 1305–1397), then for all surfaces S (Annali della Scuola Normale Superiore di Pisa vol. 2 (1942p) p. 10–11, 253–294, 1–42) by using a lemma of combinatorial topology (L. Cesari, Rend. Ist. Lomb. vol. 75 (1941) pp. 267–291; S. Eilenberg, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 1192–1195. The author has given now a new proof of (A) based on the following lemma: Let I_s be the set of points $(x, y) \in E_{2s}$ such that $N(x, y; T_s) = 0$ and such that the components of $T^{-1}(x, y)$ are continua of constancy for T. If $CR = 0$, if $(0, 0) \in I_s$ and $(0, 0)$ is a point of density for I_s, $s = 1, 2, 3$, then C is nullhomotop in $E_3 - R$. This lemma can be proved in an elementary way. (Received November 13, 1951.)

166. J. M. Danskin and Leonard Gillman: *Explicit solution of a game over function space.*

The authors consider the zero-sum two-person game determined by the payoff $\pi(x, y) = \int_0^\infty Q_y(t) dQ_x(t)$ where $Q_x(t) = \exp\left[-\int_t^\infty x(\tau)\xi(\tau)d\tau\right]$ and $Q_y(\tau) = \exp\left[-\int_t^\infty y(\tau)\eta(\tau)d\tau\right]$, ξ and η being fixed continuous, strictly decreasing, positive, summable functions of t. The x-strategies are chosen from the space $\mathfrak{X}$ of measurable functions satisfying for a given positive X the conditions (a) $0 \leqq x(t) \leqq 1$ and (b) $\int_0^\infty x(t)dt \leqq X$. The y-strategies are chosen from the space $\mathfrak{Y}$ of measurable functions satisfying for a given positive Y the conditions (c) $0 \leqq y(t) \leqq 1$ and (d) $\int_0^\infty y(t)dt \leqq Y$. They first prove the existence of, and then obtain an explicit formula for, a (unique) saddle-point—i.e., a pair (x_0, y_0) with $x_0 \in \mathfrak{X}$ and $y_0 \in \mathfrak{Y}$ satisfying the condition (e) $\pi(x_0, y) \geqq \pi(x_0, y_0) \geqq \pi(x, y_0)$ for all $x \in \mathfrak{X}$ and $y \in \mathfrak{Y}$. (Received November 8, 1951.)

167. Melvin Dresher: *Moment spaces and inequalities.*

An integral inequality is interpreted geometrically as a condition that a given point lie in a space defined by the convex hull of a given curve. By characterizing the boundaries of various dimensions, integral inequalities are derived from the requirement that points of the space lie within elements of the boundaries. The Hölder, Minkowski, and Jensen inequalities are examples of the application of "lower boundaries" of a 3-dimensional general moment space. Using higher-dimensional moment spaces, addition inequalities are obtained, such as $\left[\int_0^1 (f+g)^p d\phi / \int_0^1 (f+g)^r d\phi\right]^{1/(p-r)} \leqq \left[\int_0^1 f^p d\phi / \int_0^1 f^r d\phi\right]^{1/(p-r)} + \left[\int_0^1 g^p d\phi / \int_0^1 g^r d\phi\right]^{1/(p-r)}$ where $p \geqq 1$, $1 > r > 0$, f and g are continuous functions, and ϕ is a distribution function over the interval (0, 1). The "upper boundaries" of the space yield such inequalities as the following closure of the Liapounoff inequality $(1-t^a)(1-\int_0^1 x^c d\phi) \leqq (1-t^c)(1-\int_0^1 x^a d\phi)$ where $a>b>c$ and t satisfies the equation $(1-t^b)(1-\int_0^1 x^c d\phi) = (1-t^c)(1-\int_0^1 x^b d\phi)$. (Received November 13, 1951.)

168. Joanne Elliott: *On a class of integral equations.*

The author discusses the integral equation $g(x) = \text{P.V. } \pi^{-1}\int_{-b}^{+b} f(t)(t-x)^{-1}dt + \lambda\int_{-b}^{+b} h(x, t)f(t)dt$, where g and h are given functions and f is unknown. The first integral is taken in the sense of a Cauchy principal value, and b may be infinite. It is shown that if the equation is modified formally by introducing a suitable weight function, it may be reduced in a natural way to a Fredholm equation. If b is finite, the transformation $Tg(x) = \text{P.V. } \pi^{-1}\int_{-b}^{+b} g(t)(b-t)^{-1/2}(b+t)^{1/2}(t-x)^{-1}dt$ maps the Hilbert space of functions which are square integrable with respect to $w(t) = (b-t)^{-1/2}(b+t)^{1/2}$ isometrically onto the Hilbert space of functions square integrable with respect to $1/w(x)$. With the aid of this transformation and expansions in Jacobi polynomials the desired reduction is obtained. In the case $b = \infty$, the weight function is identically 1, the transformation Tg becomes the Hilbert transform of g, and the polynomial expansions are replaced by the Fourier transform in $L_2(-\infty, +\infty)$. (Received November 5, 1951.)

169t. William Feller: *On positivity preserving semigroups of transformations on $C[r_1, r_2]$.*

The purpose of the paper is to characterize differential operators which generate positivity preserving semigroups on $C[r_1, r_2]$. It is shown that such an operator coincides (in a certain sense) almost everywhere with $a(x)d^2/dx^2 + b(x)d/dx + c(x)$, but on an exceptional set the operator may be of quite different form. The theorem serves to characterize the parabolic differential equation and represents a new justification of the Fokker-Planck equation. [The paper is to appear in Annales Soc. Polonaise Mathématiques.] (Received November 13, 1951.)

170t. William Feller: *The parabolic differential equation and the associated semigroups of transformations.*

It is shown that an operator Φ can give rise to infinitely many semigroups of transformations in the same Banach space $\mathfrak{X}$. Each infinitesimal generator is a contraction of Φ to a set Σ dense in $\mathfrak{X}$. For a given Φ one can derive all sets Σ leading to continuous semigroups. This theory leads to an algebrization of the initial value problem for differential equations and to a definition of boundary conditions in terms of functional analysis. For the parabolic equation $u_t = \Phi(u)$ with $\Phi = a(x)d^2/dx^2 + b(x)d/dx + c(x)$ in $C[r_1, r_2]$ $(-\infty \leqq r_1 < r_2 \leqq \infty)$ n.a.s. conditions are given for uniqueness ("natural boundaries") and the totality of all possible homogeneous boundary conditions are derived in all other cases. They involve global functionals and differential operators of second order. In each case the true adjoint is derived in the conjugate space. In special cases this leads to the classical "adjoint equation" $v_t = \Phi^*(v)$ with $\Phi^* = d[ad/dx - b]/dx + c$, but in general this differential equation is replaced by a more general functional equation. All possible boundary conditions are again derived and they are found to be of quite different a form for Φ and Φ^*. The theory applies without distinction to so-called singular equations and answers several outstanding problems of diffusion theory. (Received November 13, 1951.)

171t. D. T. Finkbeiner: *On linear closures of convex sets.* Preliminary report.

This note investigates the relation between two notions of linear closure for convex sets in a linear space $\mathfrak{L}$ for which no topology is assumed. Definition 1 (Berg-Nikodým). A convex set A is *linearly closed* if and only if each line of $\mathfrak{L}$ intersects A in a set which is closed in the natural topology of the line. The *linear closure* (cl A) of A is the intersection of all linearly closed sets containing A. Definition 2 (Klee). The *weak linear closure* (lin A) of A is the set of all points x such that $[y, x)\subseteq A$ for some $y=y(x)\in A$. *Theorem.* $A\subseteq \text{lin } A\subseteq \text{lin}(\text{lin } A)\subseteq \cdots \subseteq \text{lin}^p A\subseteq \cdots \subseteq \text{cl } A$, and if equality holds somewhere in the chain, it holds thereafter. *Corollary.* A is linearly closed if and only if A is weakly linearly closed. *Theorem.* The following are equivalent: (1) $\mathfrak{L}$ is finite-dimensional, (2) lin $A=\text{lin}^2 A$ for every convex $A\subseteq\mathfrak{L}$, (3) lin $A=$ cl A for every convex $A\subseteq\mathfrak{L}$, (4) lin hull $(A, B)=$ hull (lin A, lin B) for all convex A, $B\subseteq\mathfrak{L}$, (5) lin $(A+B)=$ lin $A+$lin B for all convex A, $B\subseteq\mathfrak{L}$. Klee proved (Duke Math. J. vol. 18, pp. 443–466) the equivalence of (1) and (2). (Received November 13, 1951.)

172. R. E. Fullerton: *On the subdivision of surfaces into pieces with rectifiable boundaries.*

Let S be a nondegenerate Fréchet surface of finite Lebesgue area defined over a closed 2-cell. By utilizing a recent result of Cesari [Bull. Amer. Math. Soc. Abstract 57-3-188] it is shown that there exists a representation T of S defined over a unit square Q with the following property. There exists a nested sequence of partitions $\{P_k\}$ of Q into rectangles R_{ik} such that (i) the ratios of the shorter to the longer sides of the R_{ik} are bounded away from zero, (ii) for each $\epsilon>0$ there exists a k_ϵ such that the mesh of the partition P_k is less than ϵ for $k\geqq k_\epsilon$, (iii) $T(R_{ik}^*)$ is a rectifiable continuous curve for each R_{ik}. (Received November 13, 1951.)

173. I. S. Gál: *On the resonance-method and its application to the principle of uniform boundedness.*

A great many results in various fields of analysis can be obtained from the well known principle of uniform boundedness of linear operations in Banach spaces. The usual proofs of these results are based on a powerful method due to H. Lebesgue (*resonance-method*) [Annales de Toulouse (3) vol. 1 (1909) p. 25], which can also be used to prove theorems which do not follow from the principle of uniform boundedness. This fact suggests the application of the resonance-method directly to the sequence of linear operations instead of using the category principle. This gives the following result: An operation $u(x)$ defined over a Banach space E to a normed vector-space E' is called *bounded* if $\|u(x)\|\leqq M\|x\|$, *homogeneous* if $\|u(\lambda x)\|=|\lambda|\cdot\|u(x)\|$ for every $x\in E$ and λ real. Thus the norm of the operation $u(x)$ can be defined as $|u|=\sup_{\|x\|\leqq 1}\|u(x)\|$. A sequence of bounded, homogeneous operations $u_n(x)$; $n=1, 2, \cdots$, is said to be *asymptotically subadditive*, if $\|u_n(x+y)\|\leqq\|u_n(x)\|+O(|u_n|\cdot\|y\|)$ uniformly in x, $y\in E$ and if $\inf_{y\in E}(\|u_n(x+y)\|+\|u_n(x)\|-\|u_n(y)\|)=o(|u_n|)$ for every fixed $x\in E$, but not necessarily uniformly in $x\in E$. Using the method of resonance one obtains the following: If the sequence $\{u_n(x)\}$, $n=1, 2, \cdots$, is asymptotically subadditive (hence E is supposed complete) and if $\limsup_{n\to\infty}\|u_n(x)\|<\infty$ for every $x\in E$, then the sequence of norms $|u_n|$ is bounded. (Received November 13, 1951.)

174. H. H. Germond: *Properties of trinomial coefficients.*

Certain trinomial coefficients are analogous to Bessel functions and satisfy corresponding recurrence relationships, difference equations, expansions, and summations. (Received November 13, 1951.)

175. Samuel Goldberg: *A singular diffusion equation.*

The Fokker-Planck partial differential equation $u_t=(au)_{xx}-(bu)_x$, $0<x<1$, $t>0$, for a probability density function $u=u(t, x)$ is studied, where $a=x(1-x)$, $b=\beta-(\alpha+\beta)x$, α and β are positive constants. This equation arises in the study of the diffusion of a gene through a population undergoing random Mendelian mating in the presence of reversible mutation (S. Wright, *Statistical genetics in relation to evolution*, Paris, 1939). Using a separation of variables technique, explicit solutions as infinite series of Jacobi polynomials are obtained corresponding to the usual conditions that the boundary be an absorbing or reflecting barrier. If α and β are both greater than one, the solution obtained is unique. Let $L(t)$ and $R(t)$ be the accumulated mass, at time t, at the boundaries $x=0$ and $x=1$, respectively. Boundary conditions, developed by W. Feller, relating these functions to the density $u(t, x)$ are applied and corresponding solutions obtained. In the case where α or β is less than one, it is shown that the stationary solutions may have a positive accumulation of mass at the boundaries. (Received November 13, 1951.)

176. J. W. Green: *On the level surfaces of potentials of masses with fixed center of gravity.*

Admissible distributions are those distributions of positive mass which lie in the closed unit sphere about the origin in three dimensions, have total mass 1, and have their centers of gravity at the origin. At a point P at distance a greater than 1 from the origin, the potential $u(P)$ of an admissible distribution satisfies the inequality $1/(1+a^2)^{1/2}\leqq u(P)\leqq a/(a^2-1)$. If a level surface, $u=u_0$, has minimum and maximum distances r and R, respectively, from the origin, where $r>1$, then $R\leqq(r^2+1)^{1/2}+(r^2+5)^{1/2}$. These bounds are sharp and are attained by the distribution consisting of diametrically opposed equal point masses on the surface of the sphere. The proofs depend only on the decreasing and convex nature of the function $1/r$, and so analogous bounds exist for the potential based on any decreasing convex function; for example, the logarithmic potential. (Received November 7, 1951.)

177*t*. C. A. Hayes: *Differentiation with respect to ϕ-pseudo-strong blankets and related problems.*

In his paper, *A theory of covering and differentiation*, Trans. Amer. Math. Soc. vol. 55 (1944) pp. 205–235, A. P. Morse considered the class of ϕ-strong blankets, whose covering families are disjointed, and showed, among other things, that these blankets may be used for the purpose of differentiating any given member of his class $\mathcal{V}$ with respect to ϕ. In the present paper we prove that these differentiability properties still hold for a weaker class of blankets, called ϕ-pseudo-strong, whose covering families are allowed to overlap somewhat, the measure of the overlapping being expressed by a certain integral. That Morse's ϕ-strong blankets form a proper subclass of these blankets is shown by a concrete example. A further example is constructed to show that the condition on the overlapping cannot be relaxed much further without a serious loss in differentiability properties. (Received December 4, 1951.)

178. R. V. Kadison: *A generalized Schwarz inequality and algebraic invariants for operator algebras.*

The pure state space of a C^*-algebra together with its set of representing functions on this space is shown to be a complete set of algebraic invariants for the Jordan algebra of self-adjoint elements in the C^*-algebra (Jordan C^*-algebra). The principal tool is a generalized Schwarz inequality which states: if ϕ is a linear order-preserving map of one C^*-algebra into another then $\phi(A^2) \geqq \phi(A)^2$ for A self-adjoint. It follows that a linear isomorphism between two C^*-algebras which preserves the identity and, together with its inverse, preserves order, is a C^*-(Jordan) isomorphism. It is then shown that an isometry between two Jordan C^*-algebras deviates from a C^*-isomorphism at most by multiplication with a self-adjoint unitary in the center of the image algebra. The function representation of a C^*-algebra on its pure state space yields a function algebra under pointwise multiplication if and only if the representation is an algebraic isomorphism with all continuous functions on the space of pure states. It is proved that a linear map of one C^*-algebra into another which preserves the identity operator and preserves absolute values (of self-adjoint operators) is a C^*- (Jordan) homomorphism. Extensions to the non-unit situation are treated. (Received September 25, 1951.)

179t. M. S. Klamkin: *On the real and imaginary parts of a positive real function.*

Bode has shown that if $Z(p)$ is a positive real function having no poles on the imaginary axis, and if $R(w)$ and $X(w)$ are the attenuation and phase characteristics respectively (i.e., $Z(iw)=R(w)+iX(w)$), then $R(w)-R(\infty)$ or $R(w)-R(0)$ can be expressed as a function of the phase characteristic $X(w)$ provided that $R(\infty)$ or $R(0)$ exist. These results are extended such that $R(w)-R(w_0)$ is expressed as a function of $X(w)$ but subject to the condition that $|Z(iw)/w^2| \to 0$ as $|w| \to \infty$. This latter result includes both of Bode's results. $R(w)-R(\infty)$ is obtained by setting $w_0=\infty$ provided that $|X(w_0)/w_0| \to 0$ as $|w_0| \to \infty$. $R(w)-R(0)$ is obtained by setting $w_0=0$ provided that $|w_0X(w_0)| \to 0$ as $|w_0| \to 0$. The expression for $R(w)-R(w_0)$ is established by evaluating a certain line integral along a suitable contour. (Received November 13, 1951.)

180. P. D. Lax: *Operator theoretic treatment of hyperbolic equations.* Preliminary report.

According to a theorem of Hille-Yosida the operator equation $dU/dt=AU$ has a solution tending to I as $t\to 0$ if and only if $\|(\lambda-A)^{-1}\| \leqq 1/\lambda+O(1/\lambda^2)$ for large real positive λ. This is applied to the mixed initial-boundary value problem for second order hyperbolic equations $u_{tt}=a_{ij}(x)u_{x^ix^j}+b_i(x)u_{x^i}+c(x)u+d(x)u_t=Mu+du_t$; u, u_t prescribed at $t=0$ in D, $u=0$ for all t on the boundary of D. The equation is reduced to first order by introducing $u_t=v$; the metric $\|[u; v]\|^2=\int a_{ij}u_{x^i}u_{x^j}+\int v^2$ is chosen and the Hille-Yosida condition is verified. The solution thus constructed is twice differentiable provided the coefficients of M and the initial data have enough derivatives. Mixed initial-boundary problems for hyperbolic systems are also handled; the operators A that figure here have no finite spectrum. (Received August 27, 1951.)

181t. Walter Leighton: *On self-adjoint differential equations of second order.*

This paper contains a number of theorems dealing with the behavior of solutions of the differential equation (1): $[r(x)y']'+p(x)y=0$. Typical results are a generalization of the Sturm separation theorem and the following: Let $r(x)\equiv 1$ and suppose

$g^2(x)=x^2p(x)-1/4$ is a continuous positive monotone function of x on the interval $a\leqq x<\infty$. Then a necessary condition that solutions of (1) be oscillatory on this interval is that $\int_a^\infty x^{-1}g(x)dx=\infty$. (Received December 5, 1951.)

182. Benjamin Lepson: *Note on non-monotone Dirichlet series.*

Let $\{\lambda_n\}$ be a sequence of non-negative real numbers. Consider the series (1) $\sum_{n=0}^\infty a_n e^{-\lambda_n z}$, where z is a complex variable and $\{a_n\}$ a sequence of complex numbers. The set E of points in the plane at which (1) converges is investigated, and the following theorem is proved: *The necessary and sufficient condition on the sequnece $\{\lambda_n\}$ in order that E be a half-plane (or the whole plane or the empty set) plus a portion of its boundary for every sequence $\{a_n\}$ is that $\sum_{n=0}^\infty |e^{-\lambda_{n+1}\alpha}-e^{-\lambda_n\alpha}|<\infty$ for all positive α.* Let D be the interior of E. The determination of D is sufficient for many purposes. It is conjectured that *D is always a half-plane* (in the above sense). (Received November 13, 1951.)

183. Lee Lorch: *Asymptotic expressions for some integrals which include certain Lebesgue and Fejêr constants.*

All integrals in this abstract are definite integrals over the interval $(0, \pi/2)$. Asymptotic expressions are found for the following integrals: $L(x, r)=(2/\pi)\int|\sin(2x+1)t|^r/\sin t dt$, $L_E(x, r)=(2/\pi)\int \cos^x t|\sin(x+1)t|^r/\sin t dt$, and $L_B(x,r)=(2/\pi)\int \exp(-2x\sin^2 t)|\sin(x\sin 2t+t)|^r/\sin t dt, r\geqq 1$. For $r=1$, these are the respective (extended) Lebesgue constants arising in the theory of Fourier series from summation by ordinary convergence, Euler's $(E, 1)$-means, and Borel's exponential method. For $r=2$, they are the respective (extended) "Fejér constants," which play the same role in the divergence theory of Fourier series as do the Lebesgue constants. It is shown that $L(x, r)=(2/\pi)m_r \log x+\alpha+O(1/x)$, $L_E(x, r)=(1/\pi)m_r \log x+\delta+O(1/x^{1/2})=L_B(x, r)$, as x becomes infinite, where $m_r=(2/\pi)\int \sin^r t dt=(2^r/\pi)\cdot\{\Gamma(r/2+1/2)\}^2/\Gamma(r+1)$, and α, δ are certain (rather complicated) constants. (Received November 13, 1951.)

184*t*. G. G. Lorentz: *Hahn-Banach theorem for semi-groups and multiply subadditive functions.*

Let S be a semi-group with commutative and associative addition and with a set of real numbers a as operators. For any $x_0\in S$ and any real α_0, $q(x_0)\leqq\alpha_0\leqq p(x_0)$, there is a linear functional on S with $f(x_0)=\alpha_0$, $q(x)\leqq f(x)\leqq p(x)$. Here p is subadditive, q superadditive, $q(x+y)\leqq p(x)+q(y)\leqq p(x+y)$, and p, q are homogeneous. Necessary and sufficient conditions for set-functions $\Phi(e)$, $\Psi(e)$ are obtained under which $\Phi(e)$ is the least upper and $\Psi(e)$ is the greatest lower bound of $\phi(e)$ for a class of additive set-functions ϕ. Special cases are results of the author on multiply subadditive set-functions. Another application concerns the existence of positive functionals on S invariant under a semi-group of linear operators which leave invariant a cofinal sub-semigroup $S_0\subset S$ (this contains a theorem of Krein and Rutman). (Received November 7, 1951.)

185. T. S. Motzkin and I. J. Schoenberg: *On lineal entire functions of n complex variables.*

A polynomial $P(z_1, \cdots, z_n)$ with real or complex coefficients is called *lineal* if it splits into a product of linear functions of the variables in the complex field of coefficients. A lineal polynomial $P(z_1, \cdots, z_n)$ is called *really lineal*, or *positively* *lineal*, provided the coefficients of its linear factors are all real, or all real and non-negative, respectively. A sequence of entire functions $\{f_k(z_1, \cdots, z_n)\}$ is said to converge regularly to a function $f(z_1, \cdots, z_n)$, provided the convergence holds uniformly in every bounded portion of space. We denote by $L^{(n)}$, $L_r^{(n)}$, $L_p^{(n)}$, the three classes of entire functions $f(z_1, \cdots, z_n)$ which are regular limits of a sequence of polynomials $\{P_k(z_1, \cdots, z_n)\}$ whose elements are all lineal, really lineal, or positively lineal, respectively. For $n=1$ the class $L^{(1)}$ comprises all entire functions $f(z_1)$, while the classes $L_r^{(1)}$, $L_p^{(1)}$ have been determined by Laguerre and Pólya [Pólya, Rendiconti di Palermo vol. 36 (1913) pp. 1–17]. In this paper a complete characterization of the lineal classes $L^{(n)}$, $L_r^{(n)}$, $L_p^{(n)}$ is obtained for every $n \geqq 1$, in terms of their Weierstrass product representations. (Received November 13, 1951.)

186. T. S. Motzkin and I. J. Schoenberg: *On quadral entire functions of n complex variables.*

Following an oral suggestion of E. G. Straus, the problems treated in the preceding abstract are generalized as follows. A polynomial $P(z_1, \cdots, z_n)$ is called *quadral* if it splits into a product of quadratic (or linear) functions in the complex field of coefficients. Similarly a quadral polynomial $P(z_1, \cdots, z_n)$ is called *really quadral*, or *positively quadral*, provided the coefficients of its quadratic factors are all real, or all real and non-negative, respectively. Let $Q^{(n)}$, $Q_r^{(n)}$, $Q_p^{(n)}$ denote the classes of entire functions $f(z_1, \cdots, z_n)$ which are regular limits of polynomials which themselves are quadral, really quadral, or positively quadral, respectively. These quadral classes are here determined for all $n=1, 2, \cdots$. Similar results are obtained for *cubal* and generally for *N-al* functions. (Received November 13, 1951.)

187*t*. E. J. Moulton: *Characterization of the solutions of certain differential equations.*

For an ordinary differential equation of the first order it is sometimes possible to determine the general features of the solution curves without an integration or numerical calculations. This paper discusses available methods, with illustrations in the solution of one type of differential equation: $y' = A \exp Q(x, y)$, where $Q(x, y)$ is quadratic in (x, y). (Received November 15, 1951.)

188. Jacqueline L. Penez: *Approximation by boundary values of analytic functions.* Preliminary report.

Let D be a finite domain of the z-plane bounded by n-simple analytic curves C. Let $L_p(C)$ be the class of all complex-valued functions f defined on C which have a finite Lebesgue integral, $(\|f\|_p)^p = \int_C |f(z)|^p |dz|$ where $p>1$. The problem considered is to minimize $(\int_C |f(z)-g(z)|^p |dz|)^{1/p}$ where f is a given function in $L_p(C)$, and g ranges through the boundary functions on C, of functions analytic in D, which are in $L_p(C)$. F. Riesz, Kakeya, Doob, Golusin, P. R. Garabedian, Schiffer, Spitzbart, MacIntyre, and Rogosinski (unpublished) are among those who have considered problems of this type. The author shows that there exists a function, g_0, uniquely determined, neglecting sets of measure zero, which minimizes the above integral. The difference, $f-g_0$, is called a minimal function. A characterization formula for a minimal function is obtained. Results of J. L. Doob for the case where f is a rational function and D is a simply-connected domain have been generalized in part. Also, this extremal problem is shown to be related to the problem of maximizing $|L(f)|$ where $L(f) = \int_C f(z)\omega(z)dz$,

f is in $L_p(C)$ and is the boundary function of an analytic function in D and $\|f\|_p \leqq 1$, ω is in $L_q(C)$, and $p+q=pq$. (Received November 8, 1951.)

189. G. O. Peters: *Binomial expansion of negative factorials.*

The conditions for the binomial expansion, $(a+b)^n = \sum_{i=0}^{\infty} {}_nC_i a^{n-i}b^i$, n real, to hold are well known. Defining the factorials of x by the relations $x^{(n)} = x(x-1) \cdots (x-n+1)$ and $x^{(-n)} = 1/(x+1)(x+2) \cdots (x+n) = 1/(x+n)^{(n)}$, we can readily show, by induction or by use of Newton's interpolation formula, that the binomial expansion $(a+b)^{(n)} = \sum_{i=0}^{n} {}_nC_i a^{(n-i)}b^{(i)}$ holds for $n=0, 1, 2, \cdots$. The author proves that the binomial expansion $(a+b)^{(-n)} = \sum_{i=0}^{\infty} {}_{-n}C_i a^{(-n-i)}b^{(i)}$, $n=1, 2, 3, \cdots$, holds provided the real part of $a+b$ is greater than $n-2$, i.e. $\Re[a+b] > n-2$. (Received November 14, 1951.)

190. M. H. Protter: *Uniqueness of the problem of Tricomi.*

Let $K(y)$ be a monotone increasing function with one continuous derivative; suppose $K(0)=0$. Consider the equation (*) $K(y)U_{xx} + U_{yy} = 0$. Let D_1 be the domain bounded by a rectifiable arc α lying in the upper half of the xy-plane with end points on the x-axis at $(a, 0)$ and $(b, 0)$, and the segment of the x-axis from a to b. Let β and γ be the characteristic curves of (*) emanating from $(a, 0)$ and $(b, 0)$ respectively which meet at the point $((a+b)/2, c)$. Let D_2 be the domain bounded by β, γ and the segment of the x-axis from a to b. Suppose values are prescribed along the arcs α and β. Then it is shown that there is at most one solution of (*) in the domain $D = D_1 + D_2$ assuming the prescribed values on α and β. (Received September 25, 1951.)

191t. E. D. Rainville: *Generating functions for Bessel and related polynomials.*

Krall and Frink (Trans. Amer. Math. Soc. vol. 65 (1949) pp. 100–115) studied generalized Bessel polynomials in some detail. Their work was without benefit of generating functions, except in the special case of the simple Bessel polynomials. Burchnall (Canadian Journal of Mathematics vol. 3 (1951) pp. 62–68) obtained a generating function for generalized Bessel polynomials. This paper obtains two generalized hypergeometric generating functions which contain as special cases two generating functions, one of them Burchnall's, for the generalized Bessel polynomials. The methods used are distinct from Burchnall's method. (Received November 13, 1951.)

192. P. C. Rosenbloom: *A sufficient condition for uniform convexity.*

A sufficient condition for uniform convexity is obtained in terms of the second differential of the norm of a Banach space. This yields a simple proof of the uniform convexity of the L_p spaces for $1 < p \leqq 2$. A simple proof of Clarkson's inequalities for $2 \leqq p < +\infty$ is also given. A different proof of the uniform convexity of these spaces, which overlaps in generality with the present ones, but which gives the best possible estimates for the quantities involved, was found by Beurling (unpublished work) in 1947. (Received November 16, 1951.)

193. H. L. Royden: *The interpolation problem for schlicht functions.*

Let $z_0, z_1, \cdots, z_n$ be $n+1$ points in the interior of the circle $|z|<1$. The interpolation problem for schlicht functions is the problem of determining the possible sets of values $w_2, w_3, \cdots, w_n$ which can be taken at $z_2, z_3, \cdots, z_n$ by a function $f(z)$

which is regular and schlicht in the circle $|z|<1$ and normalized by $f(z_0)=0$, $f(z_1)=1$. By minimizing the maximum modulus of the functions taking a given a set of values, it is shown that whenever such a set of values is taken by a schlicht function, this set of values is also taken by a schlicht function satisfying a hyperelliptic differential equation. In the particular case $n=2$, the region of variability of w_2 is a region whose image in the elliptic modular plane is a circle. The parameters of the circle can be expressed in terms of certain complete hyperelliptic integrals involving the points z_0, z_1, and z_2. These results hold true if the problem is refined by considering the different homotopy classes to which the functions $f(z)$ may belong. (In fact for $n=2$ the different fundamental regions in the elliptic modular plane correspond to the different homotopy classes of the function $f(z)$.) (Received November 5, 1951.)

194. Walter Rudin: *L^2-approximation by partial sums of orthogonal developments.*

Let $\|f\|=\{\int_0^1 f^2(x)dx\}^{1/2}$. For any set $\{\phi_n\}$, orthonormal on $[0, 1]$, put $s_n(f; x) = \sum_1^n \phi_k(x)\int_0^1 f(t)\phi_k(t)dt$. Let $V(f)$ be the total variation of f on $[0, 1]$, and put $N(f)$ $=\text{l.u.b.}\ |(f(x)-f(t))/(x-t)|$ $(0\leqq x\leqq 1,\ 0\leqq t\leqq 1)$. Define $\mu_n=\text{l.u.b.}\ \|f-s_n\|/V(f)$, $\lambda_n=\text{l.u.b.}\ \|f-s_n\|/N(f)$, the l.u.b. being taken over all nonconstant $f\in L^2$. The main result of the paper is Theorem I: There exist absolute positive constants A and B such that $\mu_n>An^{-1/2}$ and $\lambda_n>Bn^{-1}$ for every complete orthonormal set $\{\phi_n\}$. The proof of Theorem I yields Theorem II: For every orthonormal set $\{\phi_n\}$, the sequences $\{V(\phi_n)\}$, $\{N(\phi_n)\}$ are unbounded. If the functions are arranged so that $\{V(\phi_n)\}$ is nondecreasing, then $V(\phi_n)>Cn^{1/2}$. If $\{N(\phi_n)\}$ is nondecreasing, then $N(\phi_n)>Dn$. Here C and D are absolute positive constants. In all the above cases, consideration of the trigonometric set shows that the orders of magnitude obtained for the lower bounds cannot be improved. (Received November 13, 1951.)

195. Charles Saltzer: *Ring conjugate functions.*

If $f(z)$ is a function which is single-valued and analytic in the region $\lambda<|z|<1$, and $f(z)=u_1+iv_1$ on $|z|=\lambda$ and $f(z)=u_2+iv_2$ on $|z|=1$ where u_1, v_1, u_2, and v_2 are real functions, then v_1 and v_2 are said to be the inner and outer Villat λ-ring conjugates of u_1 and u_2. Since there is no loss of generality, the case where u_1 is a constant is considered and representations of the λ-ring conjugates are discussed. It is shown that if the conjugate of u_2 exists in the usual sense, then the λ-ring conjugate also exists, and more generally, if the partial sums of the Fourier series of the conjugate (in the usual sense) of u_2 are bounded, then the Fourier series of the λ-ring conjugate of u_2 converges and represents u_2. In addition, inequalities comparing the integral means of v_1 and v_2 with the integral mean of u_2 are derived together with approximation theorems. Applications of these results to certain conformal mapping problems are given. (Received November 13, 1951.)

196. James Sanders: *Uniqueness theorems for some classes of fourth-order partial differential equations.*

The system of equations (1) $\mathcal{L}(u)=((\sigma(x)/\tau(y))u_x)_x+((\sigma(x)/\tau(y))u_y)_y= -(k/\tau(y))\phi_x$, (2) $\overline{\mathcal{L}}(v)=((\tau/\sigma)v_x)_x+((\tau/\sigma)v_y)_y=-(k/\sigma)\phi_y$, (3) $\phi=\sigma u_x+\tau v_y$, (4) $\mathcal{L}'(\phi)$ $=((1/\sigma\tau)\phi_x)_x+((1/\sigma\tau)\phi_y)_y=0$, where $\sigma\tau>0$ and k is a constant, is considered. This system is a generalization of the equations for the displacements in the theory of elasticity. If $k\neq -1$, (4) is a consequence of (1), (2), (3). If u and v are given on the boundary C of a domain D, then the solution of the system is shown to be unique provided $k>-1$. The method used consists of applying Green's theorem to

$\int_D u\mathcal{L}(u)dxdy$ and $\int_D v\overline{\mathcal{L}}(v)d(x)dy$. The method can be extended to equations in n variables. If $k=-1$, uniqueness is shown by proving that $\int_D(1/\tau\sigma)\phi^2dxdy=0$. In the case $k=-1$, the above system is equivalent to the fourth-order equation $\mathcal{L}'[\sigma\tau\mathcal{L}'(\psi)]=0$, and so it has been proved that the solution of the boundary-value problem $\mathcal{L}'[\sigma\tau\mathcal{L}'(\psi)]=g(x, y)$ in D, $\psi=f(s)$, $\partial\psi/\partial n=g(s)$ on C, is unique. Finally a direct proof of the uniqueness of the boundary-value problem $L[f(x, y)L(\psi)]=g(x, y)$ in D, $\psi=f(s)$, $\partial\psi/\partial n=g(s)$ on C, is given. Here, $L(u)=(\sigma_1(x, y)u_x)_x+(\sigma_2(x, y)u_y)_y$, $\sigma_1\sigma_2>0$ and $f(x, y)>0$ in D. The method used here consists of applying Green's theorem to $\int_D\psi L[f(x, y)L(\psi)]dxdy$. All domains considered in the above theorems are bounded. (Received November 13, 1951.)

197. Arthur Sard: *Remainders as integrals of partial derivatives.*

The kernel theorem of an earlier paper (Acta Math. vol. 84 (1951) pp. 319–346) is applied to a number of particular functionals. The variety of integral formulas possible in each case is indicated. A lemma is established which simplifies the calculation of kernels. Among the functionals studied is $Rf=\pi f(0, 0)-\iint_{x^2+y^2\leq 1} f(x, y)dxdy$. Kernels k, k^* exist such that $Rf=\int_{-1}^{1}f_{2,0}(x, 0)k^*(x)dx+\iint_{x^2+y^2\leq 1}f_{1,1}(x, y)k(x, y)dxdy+\int_{-1}^{1}f_{0,2}(0, y)k^*(y)dy$ whenever f is a function with partial derivatives $f_{2,0}(x, 0)$, $f_{1,1}(x, y)$, $f_{0,2}(0, y)$ that are continuous in x, (x, y), y, respectively. The kernels k, k^* are given explicitly. (Received November 9, 1951.)

198. Leo Sario: *On the construction of mappings onto slits domains.*

The mapping of a planar Riemann surface R onto a horizontal slits domain, the existence of which follows by Dirichlet's principle, can be performed in a purely constructive way by the use of the linear operator method. Take the initial function $s\equiv\text{Re}\,(1/z)$ in a parameter disc S_0, $s\equiv 0$ in $S_1=R-\overline{S}_0$. The condition $\int d\bar{s}=0$ is then fulfilled. There is a normal linear operator L_0 in $S=S_0+S_1$, associating with any harmonic function v on $1=R-S$ a harmonic function $u=L_0v$ on S such that $u=v$ on 1, $D_s[u]=\int_{I+_\beta I^-}v\,d\bar{u}=\min$. This function can be directly constructed, using an exhaustion of S_1. The corresponding principal function p_0, characterized by $p_0-s\equiv L_0(p_0-s)$ in S, is the real part of the desired mapping function. The vanishing of the total area of the slits is implied by $D_{s_1}[u]=\int_I v\,d\bar{u}$. For simply-connected surfaces with an arbitrary boundary, this provides a new proof of the Riemann mapping theorem. (Received November 13, 1951.)

199*t*. Seymour Sherman: *On a conjecture of Kakutani concerning doubly-stochastic matrices.*

A doubly-stochastic (d.s.) matrix is a real $n\times n$ matrix $P=(p_{ij})$ such that $p_{ij}\geq 0$, $1\leq i\leq n$, $i\leq j\leq n$; $\sum_j p_{ij}=1$, $1\leq i\leq n$, and $\sum_i p_{ij}=1$, $1\leq j\leq n$. Introduce a partial order among d.s. matrices (henceforth designated by P's with superscripts) by defining $P^1\prec P^3$ to mean there exists a d.s. matrix P^2 such that $P^1=P^2P^3$. Introduce a partial order among real vectors $a=(a_1, \cdots, a_n)$ of real n-dimensional space E by defining $a\prec b$ to mean for each real convex ϕ, $\sum_i\phi(a_i)\leq\sum_i\phi(b_i)$. It is known that for each real n-vector a, $P^1\prec P^3\rightarrow P^1a\prec P^3a$. Kakutani has raised the following conjecture: If, for each real n-vector a, $P^1a\prec P^3a$, then $P^1\prec P^3$. This conjecture is settled in the affirmative. (Received November 5, 1951.)

200*t*. Seymour Sherman: *On a paper by B. O. Koopman concerning a probabilistic generalization of matric Banach algebras.*

In a recent note Koopman [*A probabilistic generalization of matric Banach algebras*, Proceedings of the American Mathematical Society vol. 2 (1951) pp. 404–413], whose notation and definitions are followed here, showed that a certain set of functions $\phi(x, E)$ with appropriate operations and norm constitute a Banach algebra. Additional information as well as motivation for his theorem and a simplification of his proof is provided by observing with Yosida and Kakutani [*Operator-theoretical treatment of Markoff's process and mean ergodic theorem*, Ann. of Math. vol. 42 (1941) pp. 188–228]: THEOREM. Let (M^*) be the Banach space of bounded complex-valued measurable functions on X with $\|f\| = \sup_{x \in X} \{|f(x)|\}$. Then each $\phi(x, E)$ represents a bounded linear operator $\overline{T}_\phi$: $(M^*) \to (M^*)$ given by $\overline{T}_\phi f = g$ where $g(y) = \int_X \phi(y, dx) f(x)$, i.e., for fixed y integrate f with respect to the measure $\phi(y, E)$. In this representation the bound and algebraic operations involving operators go into Koopman's norm and operations. From the theorem above and the well known fact that complete algebras of bounded linear operators on a Banach space when normed by bound form Banach algebras, Koopman's result follows. (Received November 5, 1951.)

201. Seymour Sherman: *On a theorem of Hardy, Littlewood, Pólya, and Blackwell.*

Let $U = \{u\}$ be a real vector space. Let $a(b)$ be a measure on U with a finite spectrum, $S_a = \{u_i \mid 1 \leqq i \leqq n_a\}$ $(S_b = \{u^j \mid 1 \leqq j \leqq n_b\})$. Define $b \prec_1 a$ to mean for each real convex $\phi \mid U$, $\sum_i a(u_i)\phi(u_i) \geqq \sum_j b(u^j)\phi(u^j)$. Define $b \prec_2 a$ to mean $b(u^j)u^j = \sum_i p_{ij} a(u_i) u_i$, where $p_{ij} \geqq 0$, $\sum_i p_{ij} a(u_i) = b(u^j)$, and $\sum_j p_{ij} = 1$. From the definition of convex function and the properties of p_{ij} it follows immediately that $b \prec_2 a \to b \prec_1 a$. This note is devoted to the proof of: THEOREM. $b \prec_1 a \to b \prec_2 a$. (Received November 5, 1951.)

202. M. L. Slater: *On a duality property.* Preliminary report.

Let $\alpha(x)$ and $\beta(x)$ be fixed functions $\in C[0, 1]$. Let F be the set of non-negative $f \in C'[0, 1]$ which satisfy $-f'(x) \leqq \alpha(x)$ for all $x \in [0, 1]$. Let G be the set of non-negative $g \in C'[0, 1]$ which satisfy $g'(x) \geqq \beta(x)$ for all $x \in [0, 1]$ and also $g(0) = g(1) = 0$. The pair (α, β) will be said to have property I if F is nonvoid and $\operatorname{Sup}_{f \in F} \int_0^1 f\beta dx < \infty$; (α, β) will be said to have property II if G is nonvoid and $\operatorname{Inf}_{g \in G} \int_0^1 g\alpha dx > -\infty$. Theorem: (α, β) *has property* I *if and only if it has property* II. *Furthermore, if* (α, β) *has either property, then* $\operatorname{Sup}_{f \in F} \int_0^1 f\beta dx = \operatorname{Inf}_{g \in G} \int_0^1 g\alpha dx$. This theorem is an analogue of a discrete problem solved by Gale, Kuhn, and Tucker (*Linear programming and the theory of games*, Cowles Commission Monograph no. 13). Cf. also Hardy, Littlewood, and Pólya, *Inequalities*, Theorem 399. Remark: In the definition of property I, "*F* is nonvoid" may be deleted since F is never empty. To conform with further problems now under study, the phrase has been retained. (Received November 13, 1951.)

203*t*. R. L. Sternberg: *Non-oscillation theorems for systems of differential equations.*

Consider the system of differential equations (*) $[R(x)y' + Q(x)y + \phi^*(x)\mu]' - [Q^*(x)y' + P(x)y + \theta^*(x)\mu] = 0$, $\phi(x)y' + \theta(x)y = 0$ with R, Q, P given $n \times n$ matrices, ϕ, θ given $m \times n$ matrices, $0 \leqq m < n$, y, μ vectors, R, P symmetric, R, Q, ϕ of class C' and P, θ of class C, and with R satisfying the strengthened Clebsch condition and, finally, with a suitable $(n+m)$-rowed square matrix in R and ϕ nonsingular on $[a, \infty)$.

Call (*) non-oscillatory if x_1, x_2 large and $y(x_1)=0=y(x_2)$ imply $y\equiv 0$ on $[x_1, x_2]$. The reduced form of (*) is $[G(x)u'+\psi^*(x)\nu]'+F(x)u=0$, $\psi(x)u'=0$ where u is related to y by a nonsingular transformation. If $\pi^*G\pi \geqq \pi^*\pi$ when $\psi\pi=0$ and if there exists a symmetric constant matrix H such that, for large x, $|\eta^*[\int_a^x tF(t)dt-H]\eta| < \eta^*\eta/2$, then (*) is non-oscillatory. If $m=0$, $\eta^*G\eta \leqq \eta^*\eta$, and $\eta^*F\eta \geqq 0$, then (*) is non-oscillatory if and only if $\int_a^\infty x^r F(x)dx$ exists for $0 \leqq r < 1$ and there exists an $n \times n$ symmetric matrix W of class C' satisfying $W(x)=\int_x^\infty W(t)G^{-1}(t)W(t)dt+\int_x^\infty F(t)dt$ for large x. These and other results which are analogues of theorems of E. Hille and A. Wintner for a single differential equation are proved in the paper using a theorem from the calculus of variations of W. T. Reid. (Received November 7, 1951.)

204. Otto Szász: *On the product of two summability methods.*

We denote by $T_1 \cdot T_2$ the iteration product of two summability methods T_1 and T_2, that is, the transform T_1 of the transform T_2 of a sequence (or a function). It is clear that summability T_2 always implies summability $T_1 \cdot T_2$, when T_1 is regular; however the relationship of the methods T_1 and $T_1 \cdot T_2$ is quite complicated. We have shown in a previous paper that T_1 implies $T_1 \cdot T_2$ in the following cases for T_1 and T_2 respectively: (1) Abel and Cesàro, (2) Borel and Cesàro, (3) Borel and Euler. We now show that the same is true for the following more general methods: (a) Abel and Hausdorff, (b) Borel and Hausdorff, (c) Abel and the circle method. Obviously (a) and (b) include the previous results (1), (2), and (3), as Cesàro and Euler are special Hausdorff means. (Received November 9, 1951.)

205. J. L. Walsh and L. Rosenfeld: *On the boundary behavior of a conformal map.* Preliminary report.

Carathéodory's theory of conformal mapping of variable regions is applied to the study of a conformal map in the vicinity of a zero angle of the boundary. A function $\phi(u)$, not necessarily single-valued, is said to possess *property* B at $u=1$ if uniformly in every interval I: $|U| \leqq U_0$, we have $\lim_{u\to 1-}\phi[U\phi(u)+u]/\phi(u)=1$. This condition is weaker than the requirement for an L-tangent and is sufficient for many applications. If a region R_w contains the interval $0 \leqq w < 1$ and if its boundary near $w=1$ consists of two parts respectively represented by functions which possess property B at $u=1$, then most of the well known (Ostrowski, Warschawski) results concerning the map of R_w onto a half-plane can be established. The analogue of property B is sufficient to obtain most of the known results on the mapping of an infinite strip. (Received December 21, 1951.)

206. J. L. Walsh and D. M. Young: *An upper bound for the moduli of continuity of harmonic functions.*

Let G denote a closed simply-connected region with interior R whose boundary S is a closed Jordan curve with the following property: there exist constants $r>0$ and $A \geqq 0$ such that for every point P of S there exists a circular sector with vertex at P with radius r and included angle A containing no point of R. Let $u(x, y)$ be harmonic in R, continuous in G, and with modulus of continuity $\omega(\delta)$ on S. If $\delta/D \leqq (r/D)^{(\pi+B)/\pi}$, then the modulus of continuity of $u(x, y)$ in G satisfies the inequality $\omega^*(\delta) \leqq \omega[D(\delta/D)^{\pi/(\pi+B)}]+(8/\pi)M(\delta/D)^{\pi/(\pi+B)}$ where $B=\text{Max }(2\pi-A;\ \pi)$, M is the oscillation of $u(x, y)$ on S, and D is any positive constant. The proof is based on inequalities for the harmonic measure of the bounding radii of a circular sector at interior points near the vertex of the sector. The result is used in connection with the determination of upper bounds for the error in the solution of the Dirichlet problem by finite differences for rectangles and for regions consisting of overlapping rectangles (see *On the accuracy of the numerical solution of the Dirichlet problem by finite differences*, Bull. Amer. Math. Soc. Abstract 57-6-500). (Received November 13, 1951.)

207. H. F. Weinberger: *An optimum problem in the method of Weinstein.*

The method of Weinstein gives upper bounds for the eigenvalues of the projection L' into a subspace $\mathfrak{S}$ of a completely continuous positive symmetric operator L in a Hilbert space $\mathfrak{H}$. These bounds are the eigenvalues of the projections of L into a subspace $\mathfrak{H}\ominus\{p_1, \cdots, p_m\}$ of finite index m. They can be explicitly computed in terms of the eigenvalues and eigenvectors of L. The vectors p_i are restricted to lie in the space $\mathfrak{H}\ominus\mathfrak{S}$, but are otherwise arbitrary. The problem is to determine the best (lowest) upper bound which can be obtained for the kth eigenvalue λ_k' of L' by an appropriate choice of the vectors $p_1, p_2, \cdots, p_m$, the index k being fixed. It is shown that there is always a choice of the vectors p_i so that the upper bound for λ_k' is either λ_k' itself or λ_{m+1}, the $(m+1)$st eigenvalue of L. It is also shown that this is the best upper bound which can be obtained when m vectors p_i are used. (Received November 13, 1951.)

208t. Albert Wilansky: *On the convergence fields of row-finite and row-infinite reversible matrices.*

Erdös and Piranian (Proceedings of the American Mathematical Society (1950)) displayed a row-infinite regular matrix with no stronger normal matrix. In this paper the nonexistence of a reversible such matrix is shown. They also showed a row-finite regular matrix with no stronger row-infinite matrix. In this paper it is shown that to each row-finite reversible matrix corresponds an equipotent consistent row-infinite matrix. It is known (Banach: *Théorie des opérations linéaires*, p. 50) that the inverse transformation to a reversible matrix A is given by a sequence and a matrix. It is pointed out here that the sequence vanishes if A is row-finite. A non-reversible matrix with a two-sided inverse is shown. (Received November 7, 1951.)

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209. R. A. Clark and Eric Reissner: *A problem of finite bending of toroidal shells.*

The nonlinear differential equations of finite axi-symmetrical bending of thin shells of revolution as given recently [E. Reissner, Proceedings of Symposia in Applied Mathematics, vol. 3, 1950, pp. 27–52] are considered for a toroidal shell with circular cross section of radius b, with constant wall thickness h and radius a of the center line of the torus. It is shown that when $b/a \ll 1$ and for edge loading in the direction of the axis of revolution these differential equations may be reduced, if terms up to the second degree are retained, to the following form: $f'' + (\mu \sin x)g = \mu[\Omega(\cos x + f \sin x) + fg \cos x]$; $g'' - (\mu \sin x)g = -2^{-1}\mu f^2 \cos x$. The constant parameter Ω is proportional to the external load and $\mu = (12(1-\nu^2))^{1/2}b^2/ah$. A representative set of boundary conditions is: $f(\pm\pi/2) = g(\pm\pi/2) = 0$. When $\mu = O(1)$ integration is possible by developing the solutions in powers of $\mu\Omega$. The range of applicability of the linearized theory is given by the condition $\mu\Omega \ll 1$. When $\mu \gg 1$ a method of asymptotic integration is used which extends to the nonlinear range earlier results of the linear theory [R. A. Clark, Journal of Mathematics and Physics vol. 29 (1950) pp. 146–178]. The range of applicability of the linearized theory is now given by the condition $\mu^{2/3}\Omega \ll 1$. (Received November 13, 1951.)

210. R. J. De Vogelaere: *On the symmetric periodic orbits in the cosmic rays problem.*

The movement in the meridian plane following the particle in the cosmic rays problem leads to a system of two differential equations of second order, depending on a parameter γ_1, which is nonintegrable and symmetric with respect to the x-axis. It is shown here how the notion of surface of section can be used to find, for a given value of γ_1, all symmetric periodic orbits, proving that this problem is equivalent to finding the intersections of some lines $\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_n, \cdots$, of a new diagram $(x, \dot{x})$, with $\dot{x}=0$. As application, the line $\mathcal{E}_1$ was calculated for the case $\gamma_1=0.97$, using integrations of Störmer and an equivalent number of integrations that were performed to complete the line. From those calculations and from some properties of the $(x, \dot{x})$ diagram, new symmetric periodic orbits were found; one of them has only two points on the x-axis and two symmetric double points (in which it differs from the oval); ten others have on the x-axis, besides the two points where they are perpendicular to this axis, one other double point. Conjectures are made on the form of the $\mathcal{E}_1$ line for values of γ_1 different from 0.97. (Received November 8, 1951.)

211. J. B. Diaz and M. H. Martin: *Riemann's method and the problem of Cauchy.* II. *The wave equation in n dimensions.*

Riemann's method for the solution of Cauchy's problem for a linear second order partial differential equation $L(u)=0$, $u=u(x, y)$, employs the well known Lagrange's differential identity in order to obtain a line integral $I_1=\int\{B(u, v)dx-A(u, v)dy\}$ vanishing on closed paths whenever u and v are solutions of $L(u)=0$ and of its adjoint equation, respectively. Recently, one of the authors (Bull. Amer. Math. Soc. vol. 57 (1951) pp. 238–249) modified Riemann's method by using a different line integral vanishing on closed paths (whenever u and v are solutions of $L(u)=0$ and of an "associate" equation $M(v)=0$, this last equation replacing the adjoint equation in Riemann's method) and employing a differential identity other than Lagrange's. This modification permitted the extension of Riemann's method to the Cauchy problem (data given on $t=0$) for the equation $u_{xx}+u_{yy}-u_{tt}=0$, the line integral I_1 being replaced by a surface integral I_2 which vanishes on closed surfaces. The authors now extend this method to the Cauchy problem (data given on $t=0$) for the wave equation $u_{x_1x_1}+u_{x_2x_2}+\cdots+u_{x_nx_n}-u_{tt}=0$, $n\geqq 2$, using an integral I_n vanishing on closed n-dimensional surfaces. The present method employs differential identities other than Lagrange's and treats simultaneously n even and n odd. (Received October 25, 1951.)

212. A. D. Fialkow and Irving Gerst: *Transfer functions of networks without mutual reactance.* I.

The networks under consideration contain resistance, capacitance, and (self) inductance, but no mutual inductance. The following conditions are necessary and sufficient that a real rational function $A(p)$ given in simplest form by $A(p)=KN/D$ $=K(p^n+a_1p^{n-1}+\cdots+a_n)/(p^m+b_1p^{m-1}+\cdots+b_m)$ be the transfer function of a passive three terminal network of the above kind: (1) The zeros of D are anywhere in the left half-plane or on the imginary axis with the origin excluded. (2) At pure imaginary zeros of D, $A(p)$ has a simple pole with pure imaginary residue. (3) The zeros of N may not be positive real but are otherwise arbitrary. (4) $m \geqq n$. (5) The number K satisfies the inequalities $0<K<K_0$, where K_0 is the least of the three quantities K_g, b_m/a_n, 1 if $m=n$ and of the first two quantities if $m>n$. If $K_0 \neq K_d$, then K may equal K_0. Here K_d is the least positive value of K (if it exists) for which the equation $D-KN=0$ has a positive multiple root. A synthesis procedure is given which depends in part on methods used in an earlier paper on networks containing only two kinds of elements. (To appear in Quarterly of Applied Mathematics.) (Received November 13, 1951.)

213t. A. D. Fialkow and Irving Gerst: *Transfer functions of networks without mutual reactance.* II.

In the case of a passive four terminal network of the above type, the necessary and sufficient conditions that $A(p)$ be a transfer function are identical with those stated in the preceding abstract except that condition (3) is lacking (i.e., the zeros of N are arbitrary) and condition (5) must be modified to read: (5′) The number K satisfies the inequalities $-K_0<K<K_0$, where K_0 is the least of the three quantities $|K_d|$, $|b_m/a_n|$, 1 if $m=n$, and of the first two quantities if $m>n$. If $K_0 \neq |K_d|$ then K may actually equal $\pm K_0$. Here K_d is that real value of K of smallest absolute value (if it exists) for which the equation $D-KN=0$ has a positive multiple root. An algorithm for the synthesis of the network corresponding to $A(p)$ is given. (Received November 13, 1951.)

214. F. A. Ficken: *Uniqueness theorems for certain parabolic problems.*

On the domain D $(0 \leqq x \leqq 1, 0 \leqq t)$, let the problem P consist of the differential equation $u_{xx}-u_t=N(x, t, u, u_x)$, the boundary conditions $u_x-a_0u_t=n_0(u)+f_0(t)$ $(x=0)$ and $u_x+a_1u_t=n_1(u)+f_1(t)$ $(x=1)$, and the data $u(x, 0+)=g(x)$; here the constants a_i $(i=0, 1)$ are $\geqq 0$, the functions f_i are continuous, N and n_i have continuous derivatives, and N_u and N_{u_x} are bounded uniformly in t. Let the class S' consist of functions $u(x, t)$ defined on D so that u, u_t, u_x, and u_{xx} are continuous for $t>0$, $|u|$ and $|u_x|$ are uniformly bounded, and $u(x, 0+)=u(x, 0)$. If u and v solve P, set $w=v-u$, let μ be a real constant, and set $\zeta(t)=e^{-\mu t}(a_0w_0^2+a_1w_1^2+\int_0^1 w^2 d\xi)$. An elementary argument, based on showing that $\dot{\zeta} \leqq 0$ for sufficiently large μ, proves that P can have at most one solution in S' if either $a_0>0$ or ($a_0=0$ and) $n_0(u)$ does not decrease with u *and* either $a_1>0$ or ($a_1=0$ and) $n_1(u)$ does not increase with u. A second theorem gives similar sufficient conditions for uniqueness of the solution of a more special problem in a larger class of functions. The second theorem applies to a nonlinear diffusion problem of some practical interest. (Received November 5, 1951.)

215. H. E. Goheen: *On certain boundary value problems for ordinary differential equations.*

The author uses the method developed in Bull. Amer. Math. Soc. Abstract 57-3-215 for the most general linear two-point boundary value problem for the equation there discussed. A similar technique is applied to simplify H. Weyl's solution of the problem of Blasius in his paper, in Proc. Nat. Acad. Sci. U.S.A., *Concerning the differential equations of some boundary layer problems*, vol. 27 (1941) pp. 578–583. (Received November 9, 1951.)

216t. F. B. Hildebrand: *On the convergence of numerical solutions of the heat-flow equation.*

The problem $u_{xx}(x, t) = u_t(x, t)$, $u(0, t) = u(1, t) = 0$, $u(x, 0) = f(x)$ is replaced by the problem in which the difference equation $[u(x+h_x, t) - 2u(x, t) + u(x-h_x, t)]/h_x^2 = [u(x, t+h_t) - u(x,t)]/h_t$ is to be satisfied over the net $x_m = mh_x \equiv m/M$ $(m = 1, 2, \cdots, M-1)$, $y_n = nh_t$ $(n = 0, 1, 2, \cdots)$, and where $u(0, nh_t) = u(1, nh_t) = 0$ $(n = 1, 2, \cdots)$ and $u(mh_x, 0) = f(mh_x)$ $(m = 1, 2, \cdots, M-1)$. If $f(x)$ is continuous in $(0, 1)$ except for a finite number of finite jumps, and is of bounded variation in $(0, 1)$, it is proved that the solution of the approximate problem converges to that of the exact problem as h_x and h_t tend to zero for any fixed $h_t/h_x^2 < 1/2$. The convergence is uniform over $(0 \leqq x \leqq 1, t \geqq t_0)$ for any $t_0 > 0$. (Received November 13, 1951.)

217. H. G. Hopkins: *The bending of an elastic plate supported on an elastic foundation.*

Hankel transforms are used to solve the general problem of an infinite elastic plate, supported on an elastic foundation, and subjected to an arbitrarily prescribed distribution of pressure, the displacements and stresses being zero at infinity. The problem is statical; thin isotropic plate theory is used, and it is assumed that at any point the foundation exerts a normal stress only, proportional to the local normal displacement, on the plate. The solution of the general problem is new; it is specialized to the problem in which there is a uniform pressure distribution over a circular area (which includes, as a limiting case, a concentrated load), and simple formulas are found for the displacements and stresses of most practical importance. (Received November 5, 1951.)

218t. Edward Kasner and John De Cicco: *Potential theory in space of n dimensions.*

Potential theory is extended to a Euclidean space of n dimensions by means of the following adoption of Newton's Law of Universal Gravitation. The magnitude of the force of attraction between two particles is directly proportional to the product of the masses and inversely proportional to the $(n-1)$th power of the distance between them. For $n = 3$, this reduces to Newton's well known inverse square law, and for $n = 2$, this leads to the theory of the logarithmic potential. The force of attraction F upon a particle P exerted by a material Riemannian manifold V_N of N dimensions where $1 \leqq N \leqq n$, with a continuous density function μ, is obtained. The field of force is conservative and its potential U is evaluated. As this field is solenoidal, it is found that U is a harmonic function. Applications are given to hyperspheres of N dimensions. When $N = n$, the well known theorems concerning the force exerted by a homogeneous material spherical surface or volume are extended to n dimensions. However, when $1 \leqq N < n$, the situation becomes complicated and the discussion involves hypergeometric functions. (Received December 6, 1951.)

219. E. H. Lee: *An unloading wave problem in dynamic plasticity.*

The question of the relation between elastic-plastic theory and plastic-rigid theory cannot at present be treated in general. The discussion of specific solutions therefore must be used to assess the validity of a plastic-rigid model of an actual material. Such a solution is presented and discussed. A uniformly stretching rod is considered to fracture at the section $x = 0$, at the time $t = 0$. The stress there is instantaneously reduced from the yield stress to zero. An unloading wave of stress discontinuity travels from the section with elastic wave velocity c_0 causing the cessation of plastic flow behind it. It is absorbed at $x=\alpha$, and the subsequent plastic-elastic boundary moves with constant velocities through regions of length α, with velocity magnitudes c_0/n, $n=3, 5, 7, \cdots$. The corresponding plastic-rigid analysis smooths out this polygonal plastic-unloading boundary into a parabola in the x, t plane through the vertices of the polygon. The conditions under which the plastic-rigid analysis provides a good approximation to the elastic-plastic solution are discussed in terms of strain magnitudes and energy magnitudes. (Received November 9, 1951.)

220. C. C. Lin: *Instability of the boundary layer over curved surfaces.* Preliminary report.

A study is made of the instability of the boundary layer at the wall of a circular pipe produced by the combined effect of an axial flow and the rotation of the pipe about its axis. This includes the usual theories for boundary layers over straight and curved walls. It is shown that the combined effect of axial flow and rotation produces a new kind of instability not caused by either one separately. The investigation is carried out for the inviscid case, as this is usually found to be indicative for disturbances of the cellular type found in the present analysis. The general discussion of the eigenvalue problems is based on oscillation theorems, and some simple examples are given for illustration. (Received November 13, 1951.)

221. G. G. O'Brien: *Theorems on stability and convergence in numerical solutions of partial differential equations.*

The use of numerical methods in the solution of partial differential equations has increased noticeably during and since world war II. The undertaking of such solutions has become feasible due to the availability of modern, high-speed, electronic computing machines. However, there is a great need for developing the mathematical theory related to these numerical solutions. In this paper a study is made relative to stability and convergence of the one-dimensional heat flow problem with given boundary conditions. In one section the initial temperature is taken to be constant everywhere except at the end points, where there is a discontinuity. In another section the initial conditions are more general. In both cases it is shown that if a suitable difference equation with satisfactory auxiliary conditions is used to approximate the differential equation with its auxiliary conditions, that choosing the ratio within a certain range of values was sufficient to insure convergence of the difference solution to that of the differential equation. On the other hand an unstable choice of the ratio would lead to a lack of convergence of the difference solution to that of the differential equation for that particular solution. (Received October 26, 1951.)

222*t*. L. E. Payne and Alexander Weinstein: *Capacity, virtual mass and generalized symmetrization.*

Let B be a body of revolution in n-dimensional space. Its capacity $C\{n\}$, virtual mass $M\{n\}$, and volume $V\{n\}$ are defined as generalizations of the classical definitions of $C\{3\}$, $M\{3\}$, and $V\{3\}$. Let xy be the meridian plane of B, x being the axis of symmetry. A line $x=$constant, $y\geqq 0$, cuts the boundary of the meridian section of B in m points $y_1(x)>y_2(x)>\cdots>y_m(x)>0$. Consider $y_*(x)$ defined by $y_*^q(x) = \sum_{k=1}^{m}(-1)^{k-1}y_k^q(x)$, $q>0$. The meridian section of a new body B, bounded by the curve $y_*(x)$, is said to be obtained by a generalized symmetrization S_q. The following results are obtained. (1) $M\{n\}+V\{n\}=\pi^{n/2}(n-1)^{-1}\Gamma^{-1}(n/2+1)C\{n+2\}$. (2) $C\{n\}$ decreases under $S_{n-1}, S_{n-2}, \cdots, S_1$. (3) $M\{n\}$ decreases under S_{n-1}, S_n, and S_{n+1}. These results generalize the following theorems established for $n=2$ and $n=3$: $C\{n\}$ and $M\{n\}$ decrease under S_{n-1}. (See G. Pólya and G. Szegö, *Isoperimetric inequalities in mathematical physics*, Princeton University Press, 1951. P. R. Garabedian and D. C. Spencer, *Extremal methods in cavitational flows*, Report No. 3, Office of Naval Research, 1951.) (Received November 26, 1951.)

223. W. H. Pell: *Limit design of plates: The upper bound for the safety factor.*

Following methods initiated by W. Prager and the author (*Limit design of plates*, forthcoming in the Proceedings of the First National Congress of Applied Mechanics, Chicago, Ill, June 11–16, 1951), limit design of a thin plate, simply supported, under uniform transverse pressure, made of perfectly plastic material obeying the Prandtl-Reuss stress-strain law, is discussed. Methods for determining upper and lower bounds of the safety factor are given. The upper bound is given in terms of a kinematically admissible rate of deflection $w=w(x_1, x_2)$. If $w(x_1, x_2)$ is smooth, the internal rate of energy dissipation, E, is represented by a double integral impossible of evaluation, in general. If discontinuities in $\partial w/\partial x_1$ and $\partial w/\partial x_2$ are allowed along curves, E is modified by the adjunction of line integrals along the curves of discontinuity. If w is polyhedral, and convex on the side opposite that of loading, then $E=-\oint_{(C)}(\partial w/\partial n)ds$, where (C) is the curve bounding the plate, and n the external normal thereto. Contrary to expectation, it is not possible to calculate E for a smooth w surface as the limit of E for a polyhedral approximation to w when the approximating surface tends to the smooth surface as a limit. (Received November 13, 1951.)

224. Eric Reissner: *A problem of finite bending of circular ring plates.*

Differential equations obtained recently (E. Reissner, Proceedings of Symposia in Applied Mathematics, vol. 1, 1949, pp. 213–219) are applied to the problem of finite bending by transverse edge forces of circular ring plates. If b is the width of the plate and a the radius of the inner edge, then within the range of applicability of linearized theory this plate problem can be treated by beam theory for sufficiently small values of b/a. It is shown here that this is no longer the case in the nonlinear range. The problem is governed by two parameters ϕ_0 and μ, where ϕ_0 is the average slope of the deflected plate according to linear theory and $\mu=(12(1-\nu^2))^{1/2}b^2/ah$ where ν is Poisson's ratio and h is the thickness of the plate. The condition for beam theory to be applicable is found to be $\mu\phi_0 \ll 1$. When $\mu\phi_0=O(1)$, nonlinear plate theory must be used. When $\mu\phi_0 \gg 1$, a boundary layer phenomenon is found. The boundary layer equations are $f''=-1+fg$, $g''=-2^{-1}f^2$ with the boundary conditions $f(0)=g(0)=0$, $f'(\infty)=g'(\infty)=0$. (Received November 9, 1951.)

225t. James Sanders: *Some axially-symmetric problems in hydrodynamics and the theory of elasticity.*

A number of general methods of constructing solutions of (1) $\Delta_n\Delta_n\phi=0$ are given ($\Delta_n u=u_{xx}+u_{yy}+(n/y)u_y$). The case $n=-1$ occurs in the theory of the slow axially-symmetric flow of a viscous fluid while the case $n=1$ occurs in the theory of the axially-symmetric deformation of an elastic body. It is first shown that if $\Delta_n\rho=0$, then $\Delta_{n+2}\Delta_{n+2}\rho=0$. Next, if $\Delta_n\Delta_n(\phi)=0$, then $\Delta_{n+2}\Delta_{n+2}((1/y)\phi_y)=0$. Finally, if $\Delta_n(A)=0$, $\Delta_n(B)=0$, then (2) $\phi=xA+B$ is a solution of (1). In fact, every solution of (1) in a simply-connected domain is of the form (2). This last fact leads to a method of "correspondence" for solving boundary-value problems connected with (1), similar to the correspondence method proposed by Bers and Gelbart (Quarterly of Applied Mathematics vol. 1 (1943) no. 2) for solving equations of the second order. If a boundary-value problem has been solved for the biharmonic equation $\Delta\Delta u=0$, then a slightly different boundary-value problem can be solved for equation (1) by this method. (Received November 13, 1951.)

226t. Seymour Sherman: *Stability of flow in a bipropellant rocket motor.*

In a recent paper by Gunder and Friant, *Stability of flow in a rocket motor*, Journal of Applied Mechanics vol. 17 (1950) pp. 327–333, a mathematical model is proposed for examining the stability of flow in a bipropellant rocket motor. The stability computations following from this model involve constructing a Cauchy-Nyquist diagram. The ideas of Ansoff and Krumhansl, *A general stability criterion for linear oscillating systems with constant time lag*, Quarterly of Applied Mathematics vol. 6 (1948) pp. 337–341, heretofore applied to other stability phenomena involving time lag, are extended so as to simplify the Gunder-Friant calculations. In particular, one can avoid the construction of a Cauchy-Nyquist diagram. (Received November 6, 1951.)

227. Domina E. Spencer: *TEM-waves, metric coefficients, and the scalar quasi-potential.*

A rigorous proof is given that a TEM-wave is possible for any waveguide consisting of a pair of metal cylinders which conform to coordinate surfaces $u^1=\text{const.}$ of any cylindrical coordinate system. The $\boldsymbol{E}$-vector is always in the u^1-direction and its magnitude is simply $E_1=(F_1(u^1)/(q_{11})^{1/2})e^{i\omega(t-(\epsilon\mu)^{1/2}z)}$. The $\boldsymbol{H}$-vector is always in the u^2-direction and its magnitude is $H_2=E_1(u/\epsilon)^{1/2}$. It is rigorously shown that, while a scalar potential cannot be used to describe the electric field in a TEM-wave, a scalar quasi-potential can always be introduced such that $\boldsymbol{E}=-(1/\zeta(u^1, u^2, u^3, t))\nabla\phi^*$. This contribution gives a comprehensive view of TEM waves and the various waveguides that may be used. (Received November 5, 1951.)

GEOMETRY

228t. D. O. Ellis: *On metric representations of groups.*

Any finite or countable group is (isomorphic to) a group of motions on the metric Baire space formed on the elements of the group. (Received November 5, 1951.)

229. Arrigo Finzi: *On Liouville's theorem concerning conformal transformations.*

According to Liouville's theorem a conformal transformation of the n-dimensional space ($n\geqq 3$) transforms spheres into spheres. The usual demonstration requires the existence of the derivatives of the first three orders; on the other hand the definition of the conformal transformations requires only the existence of the first derivatives. Is it possible to demonstrate the theorem under these more general conditions? Basically, it should be necessary to demonstrate, for this purpose, that a transformation which transforms infinitesimal spheres into infinitesimal spheres and leaves infinitesimal angles invariant transforms every sphere into a sphere. The following proposition (which here is given for $n=3$) could perhaps lead to a demonstration of the theorem. Let us consider a system of six spheres. Let us suppose each of the six spheres to be externally tangent to four of the others and all the six spheres to be externally tangent to a seventh sphere. Then the six spheres are internally tangent to an eighth sphere. Note that the proposition given above can be generalised to every $n>2$. There is no corresponding proposition for $n=2$. (Received November 13, 1951.)

230. J. W. Gaddum: *Metric methods in differential geometry.*

Let A be an arc (homeomorph of a segment) in euclidean three-dimensional space. Define $K(p_1, p_2, p_3) = [-D(p_1, p_2, p_3)]^{1/2}/(p_1p_2 \cdot p_2p_3 \cdot p_1p_3)$, and, if no triple of p_1, p_2, p_3, p_4 is linear, define $T(p_1, p_2, p_3, p_4) = 18^{1/2}|D(p_1, p_2, p_3, p_4)|^{1/2} \cdot [D(p_1, p_2, p_3)D(p_1, p_2, p_4) \cdot D(p_1, p_3, p_4)D(p_2, p_3, p_4)]^{-1/4}$, where D is the Cayley determinant of the points $p_1, \cdots, p_k$. For a point p of A, let $K(p) = \lim_{p_i \to p} K(p_1, p_2, p_3)$ and $T(p) = \lim_{p_i \to p} T(p_1, p_2, p_3, p_4)$, if these limits exist. These are the metric curvature (Menger) and the metric torsion (Blumenthal). If both are different from zero, A is called *bi-regular.* Using a theorem of Pauc it is shown that A is rectifiable. A metric proof is given of the first fundamental theorem of curve theory, stated as follows: If the points of two bi-regular arcs of E_3 can be put into a one-to-one correspondence in such a way that arc length, curvature, and torsion are preserved, then the arcs are congruent. Using a result in the Missouri dissertation of J. W. Sawyer it is proved that a bi-regular arc possesses a moving trihedral at each point and that the Frenet-Serret formulas hold. (Received November 13, 1951.)

231*t*. P. C. Hammer: *Associated convex bodies.*

This paper gives an extension to n dimensions of some results previously obtained by the author for the plane. Let C be a closed bounded convex body. Let $C_b(r)$ be the convex body obtained from C by a similarity with ratio $r>0$ and center point b on the boundary of C. Define associated bodies $C(r)$ as the intersection of all $C_b(r)$ for $r \leqq 1$ and $C(r)$ as the union of all $C_b(r)$ for $r>1$. Then the family of nonvacuous $C(r)$ are convex sets the boundaries of which simply cover the space. Furthermore, for $r<1$, $C(r)$ is the set of points dividing chords of C in a maximum ratio at most r. For $r>1$, $C(r)$ is the minimal convex body such that points of C divide chords in a maximum ratio no greater than $r/(2r-1)$. For the ratio in which a point divides a chord through it take the ratio of the larger or equal part to the whole chord. For $r \leqq 1$ let $B=C(r)$. Then if the convex body $B(r/(2r-1))=C$ and $C(r)$ is the smallest body for which this is true, C is said to be reducible to $C(r)$. For every convex body C without a center point there exists a unique body $C(r_i)$, $r_i \leqq 1$, to which C is reducible. In a sense this body may be considered the "center" of C. (Received October 31, 1951.)

232. P. C. Hammer and Andrew Sobczyk: *Symmetrization of convex regions.*

For a convex region C in n-dimensional linear space E_n, a convex region C is defined as the limiting region, as $r \to \infty$, of the similar regions with ratio $1/r$ to the exterior associated regions with respect to an $n-k$ dimensional linear subspace E_{n-k}, $C_r(E_{n-k})$. (See another abstract entitled, *Convex regions associated in subspaces.*) Sections of the region C in flats π_{n-k} parallel to E_{n-k} are symmetric; the region $\overline{C}$ obtained by translating each π_{n-k} to center the symmetric section of C in a complementary subspace E_k to E_{n-k} is also convex (and symmetric in sections parallel to E_{n-k}). In the case $k=0$, $\overline{C}(E_{n-k})$ is the region obtained by centering translated diameters of C on a single point. This region is called the *symmetroid* of C. In case $n=2$, the symmetroid has the same circumference as C. The process of constructing the region C with respect to E_{n-k} and E_k is a generalization of Steiner's symmetrization, with which it coincides when $k=(n-1)$. (Received November 9, 1951.)

233t. Andrew Sobczyk and P. C. Hammer: *Adjoint symmetrization of convex regions.*

If $H(u)$ is the support function for a convex region C in a linear space L (Bonnesen-Fenchel, *Theorie der konvexen Körper*, pp. 23–25), the adjoint region C^* in the conjugate space L^* is the set of u for which $H(u) \leqq 1$. It is shown that the region D^* in L^* defined by $K(u) \leqq 1$, where $K(u)=[H(u)+H(-u)]/2$, is the adjoint region of a symmetric region D, obtained from C by centering the midpoints of any subfamily of diameters of C which has the property that the extended diameters simply cover the exterior of C. In particular, if C is a constant diameter region, D^* and D are spheres. Various methods of symmetrization are introduced, for example that corresponding to $J(u)=\sup [H(u), H(-u)]$, and their adjoint methods are determined. (Received October 31, 1951.)

234. Andrew Sobczyk and P. C. Hammer: *Convex regions associated in subspaces.*

The union C_K of the translates of a convex set C according to a convex set K, and the set of positions $p+z$ of a point p fixed in an arbitrary set D, when all translations $D+z$ with $(D+z) \subset C$ are allowed, are convex sets. In particular, the interior and exterior associated regions of Hammer (Proceedings of the American Mathematical Society (1951)) are similar respectively to the intersection and to the union of the same set of translates. For a convex region C in n-dimensional linear space E_n, interior and exterior associated regions with respect to any $(n-k)$-dimensional linear subspace E_{n-k} are similarly defined in terms of sections and translates; it is shown that the exterior region is always convex, and by examples that for $k>0$ the interior region need not be convex. Let π_{n-k} denote a flat parallel to E_{n-k}. Then any function $y''=t(x')$, defined on the projection $P(C)$ parallel to E_{n-k} of C onto E_k, with range in E_{n-k}, defines a set of translations of the sections $C\cdot\pi_{n-k}$; it is shown that the set T of t for which the correspondingly sliced and reassembled region is convex is a convex set in the linear space of continuous functions on $P(C)$ to E_{n-k}. (Received November 9, 1951.)

Logic and Foundations

235. S. C. Kleene: *Finite axiomatizability in the predicate calculus using additional predicate symbols.*

Let the formulas constructible in the logical symbolism of the (first order) predicate calculus with a finite list $P_1, \cdots, P_s$ of predicate symbols be called *P-formulas.* Let a class C of P-formulas be called *finitely axiomatizable in the predicate calculus using additional predicate symbols*, if there is a formal system S, having as its formulas those constructible in the symbolism of the predicate calculus using besides $P_1, \cdots, P_s$ a finite list $Q_1, \cdots, Q_r$ of additional predicate symbols, and as its postulates those of the predicate calculus and a finite class of non-logical axioms, such that in S a P-formula is provable if and only if it belongs to C. Sufficient (and obviously necessary) conditions for this are that (1) C be recursively enumerable and (2) C be closed under deduction in the predicate calculus. The result holds for both the classical and the intuitionistic versions of the predicate calculus; and also when instead of predicate symbols only also individual and function symbols (finitely many) are allowed in the construction of P-formulas. (Received November 9, 1951.)

236t. H. G. Rice: *Classes of recursively enumerable sets and their decision problems.*

The partial recursive functions of one variable can be generated in a systematic manner which recursively enumerates them. The set enumerated by a partial recursive function $f(x)$ is defined as the set $\{f(0), f(1), \cdots\}$ up to the least argument, if any, for which $f(x)$ is not defined. Classes whose members are recursively enumerable sets of non-negative integers are considered, and the decision problem for them is shown to be negative, in the following sense. For a class A, we let θ_A be the set of all partial recursive functions enumerating sets of A. Then no such θ_A is a recursive set. There are classes for which θ_A is recursively enumerable, and a (possibly exhaustive) method for generating them is displayed. Among the methods used in reaching the main result are diagonal constructions and transformations by recursive functions of the partial recursive functions into themselves such that every class has a unique transform. (Received November 9, 1951.)

237. H. G. Rice: *Recursively enumerable and recursive orders.* Preliminary report.

We consider sequences (without repetitions) of non-negative integers. Equivalence classes are defined by the following relation: two sequences are equivalent if the same permutation of their terms arranges each in order of size. A sequence containing all non-negative integers is distinguished as the principal sequence of its class. We call an order (an equivalence class) recursively enumerable (r.e.) if at least one of its sequences is produced by a recursive function, recursive if its principal sequence is so produced. (The algebra of infinite sets permits an analogous development.) With iteration of permutations as the operation, the group of permutations on the non-negative integers is obtained. The recursive orders form a subgroup. The inverse of a r.e. but not recursive order is not r.e. A recursive set cannot be recursively enumerated in a r.e. but not recursive order. All the recursive enumerations of a r.e. set are obtained by applying all recursive permutations to a given recursive enumeration. Among the problems mentioned is whether or not the class of r.e. but not recursive orders forms a single left coset of the subgroup of recursive orders. This, if true, implies that there is only one degree of undecidability relative to general reducibility (Post, Bull. Amer. Math. Soc. vol. 50 (1944) p. 311). (Received November 9, 1951.)

Statistics and Probability

238. K. L. Chung: *Contributions to the theory of Markov chains.*

In a denumerable Markov chain with stationary transition probabilities the probability, starting from i, of hitting j before k is studied. In a positive class the mean first passage times (mfpt), starting from i, to the "union" and the "intersection" of j and k are defined. Relations between these quantities and the aforesaid probabilities are obtained. In particular Doblin's ratio ergodic theorem and central limit theorem are made more precise and recent interesting results due to T. E. Harris (unpublished) are proved. For the mfpt's extended to a finite number of states a Poincaré-like formula is obtained. Another simple result expresses the mfpt's directly in terms of the transition probabilities as follows: $\sum_{n=1}^{n=\infty} (P_{ij}^{(n)} - P_{ik}^{(n)}) = (m_{jk} - m_{ik})/m_{kk}$. (Received October 24, 1951.)

239. Jim Douglas: *Two equivalence methods for sequences of random variables.*

Two methods of generalizing Khintchine equivalence for sequences of random variables are given. Let $\{x_k\}$ and $\{y_k\}$ be sequences of independent random variables, and let $\{F_k(t)\}$ and $\{G_k(t)\}$ be the corresponding sequences of distribution functions. Let $\delta_k = \text{l.u.b.}\ |F_k(t) - G_k(t)|$. If $\sum_{k=1}^{\infty} \delta_k < \infty$ and $\{x_k\}$ satisfies a sufficient condition of Brunk (Duke Math. J. vol. 15 (1948) pp. 181–195) for the strong law of large numbers, $\{y_k\}$ satisfies the strong law. Applications are made for independent variables to the convergence of series, the central limit law, and the generalized law of the iterated logarithm. The second method utilizes a quasi-metric of Kakutani (Ann. of Math. vol. 49 (1948) pp. 215–224). Let m and μ be the measures generated on an infinite product space of real lines by (dependent) $\{x_k\}$ and $\{y_k\}$, respectively. Let $m^{(n)}$ and $\mu^{(n)}$ be the measures generated by $\{x_{n+1}, x_{n+2}, \cdots\}$ and $\{y_{n+1}, y_{n+2}, \cdots\}$. If $\lim\sup \rho(m^{(n)}, \mu^{(n)}) = 1$, m and μ are equal on the class of all measurable sets which are $(n+1, n+2, \cdots)$-cylinder sets for every n. Hence, the sequences of random variables obey or disobey together the strong law of large numbers, etc. Brunk (Proceedings of the American Mathematical Society vol. 1 (1950) pp. 409–414) previously considered the second method. The two methods are independent. (Received November 5, 1951.)

240. W. M. Hirsch: *On the maximum cumulative sum of independent random variables.* Preliminary report.

Let $X_1, X_2, \cdots$ be a sequence of independent random variables with mean 0. Put $S_n = X_1 + X_2 + \cdots + X_n$, $\overline{S}_n = \max_{1 \leq j \leq n} S_j$, and $S_n^* = \max_{1 \leq j \leq n} |S_j|$. In 1944 W. Feller called attention to the desirability of studying the random variables $\overline{S}_n$ and S_n^*. Since then counterparts of the Berry-Esseen theorem and of Feller's generalized law of the iterated logarithm have been obtained by K. L. Chung for S_n^*. Theorems like these referring to $\overline{S}_n$ are considered here. The results are as follows: If α_n is an arbitrary sequence of positive numbers, and $\Gamma_n = \max \{|X_j|^3 | 1 \leq j \leq n\}/\min \{X_j^2 | 1 \leq j \leq n\}$ satisfies a mild regularity condition, then $P\{\overline{S}_n < \alpha_n s_n\} = (2/\pi)^{1/2} \int_0^{\alpha_n} e^{-t^2/2} dt + O(s_n^{-\theta})$, where $s_n^2 = \text{var}\ (S_n)$ and $\theta > 0$ depends on the order of magnitude of Γ_n. A similar result referring to the "truncated" probability, $P\{\overline{S}_n < \alpha_n s_n;\ S_n > -s_n\}$, is also obtained. With the aid of these estimates it is shown that if $\psi(t) \uparrow \infty$ is arbitrary, then the probability, $P\{\overline{S}_n < s_n/\psi(s_n^2) \text{ i.o.}\}$, is equal to zero or one according as the series $\sum_{k=1}^{\infty} [\psi(2^k)]^{-1}$ is convergent or divergent. (Received November 5, 1951.)

241. Eugene Lukacs: *An essential property of the Fourier transforms of distribution functions.*

Three important theorems are valid for Fourier transforms of distribution functions, the uniqueness theorem, the convolution theorem, and the continuity theorem. These theorems make Fourier transforms an important tool in the theory of probability. The question is investigated whether any other integral transforms applicable to distribution functions have these properties. It is shown that the uniqueness theorem and the convolution theorem—without using the continuity theorem—determine the kernel. The kernel is given by $K(s, t) = e^{itA(s)}$ where $A(s)$ is a real-valued function which assumes all values of a set which is dense between $-\infty$ and $+\infty$. (Received October 1, 1951.)

242. Herschel Weil: *On a combination of correlated random variables.*

The probability density $p(r)$ of $r=(x^2+y^2)^{1/2}$ is obtained when x and y are random variables satisfying a general two-dimensional Gaussian distribution. The result is given in the form of a constant times $r \exp(-ar^2) f(r, \mu_{ij})$ where f is a series of products of Bessel functions whose arguments are functions of r and the moments μ_{ij} of the distribution of x and y and a is a function of the moments. For the special case of zero means, $\mu_{10}=\mu_{01}=0$, and for the case of a common variance $\mu_{02}=\mu_{20}$ combined with a zero correlation $\mu_{12}=0$, the series f reduces to a Bessel function of zero order. The latter special case has been worked out previously and applied to various physical problems such as the study of noise and signal in a linear detector and to the problem of random aiming errors, although the restrictions on the moments are not always valid in these applications. Except in the special cases, the integrals involved in obtaining the moments of $p(r)$ by direct integration of $r^n p(r)$ seem prohibitively difficult, but the even order moments may be obtained by direct differentiation of the generating function for the probability distribution of r^2 which is obtainable in closed form. (Received November 19, 1951.)

Topology

243. R. D. Anderson: *Continuous collections of continuous curves in the plane.* Preliminary report.

Let G denote a continuous collection of mutually exclusive compact continuous curves in the plane. If the elements of G are all nondegenerate and G^* is compact and connected, then G with respect to its elements as points is a hereditary continuous curve such that the closure of the set of emanation points of G is totally disconnected in G. If J is such a hereditary continuous curve which is also a subset of the plane, then there exists a collection G all of whose elements are arcs such that G is homeomorphic to J. If a collection G fills up the plane, then either all of the elements of G are degenerate or at most one element of G is degenerate and in the latter case G is a ray. If a collection G all of whose elements are nondegenerate fills up a 2-cell, then G is an arc. (Received November 13, 1951.)

244t. B. J. Ball: *Concerning continuous and equicontinuous collections of arcs.*

It is proved that if G is a continuous and equicontinuous collection of mutually exclusive arcs in the plane and G^*, the sum of the arcs of G, is closed and compact, then there exists a reversibly continuous transformation of the plane into itself which carries every arc of G into a straight line interval. Furthermore, if G^* is connected, it is either a simple closed curve plus its interior or the sum of two mutually exclusive simple closed curves and the connected domain bounded by them. (Received December 10, 1951.)

245. A. F. Bartholomay: *On type-invariance and h-retraction.* I.

A transitive (Borsuk) class Rh of mappings is introduced; viz., continuous mappings on a separable, metric space which have right homotopic inverses. A stratification of type-invariance called $\mathfrak{R}h$-type-invariance is then obtained. In an attempt to relate this notion to retraction theory, the author defines "h-retraction": Given a mapping $\rho: A \supset B$, where $B \subset A$ (separable, metric space) and $\rho | B \simeq$ identity of B, then B is called an "h-retract" of A and ρ is an "h-retraction." The following theorem (and its converse) is established: *If B is homotopically equivalent to an h-retract of A, there exists an rh-continuous mapping of A onto B.* Corollary: *Type-invariance of h-retracts $\subset$ Rh-type-invariance.* The following definitions are then introduced: (1) Spaces A, B will be called "Rh-equivalent" if and only if $\mathfrak{R}h$ $(A) = \mathfrak{R}h$ (B), where $\mathfrak{R}h$ (A) is the class of all spaces obtained as ranges of all elements of $\mathfrak{R}h$ using A as domain. (2) Sets for which $\mathfrak{R}h$-equivalence is identical with homotopic equivalence are called "$\mathfrak{R}h$-determinable." The relation of this latter notion to Borsuk's notion of "$\mathfrak{R}$-determinability" is then discussed. A study of $\mathfrak{R}h$-determinability of spaces is planned for the next part. (Received November 7, 1951.)

246. W. R. Baum: *Characterization of a factor-group of the n-dimensional homology group.*

H. Hopf has shown that the n-dimensional homology group $\mathfrak{H}^n(K)$ of a connected complex K contains a subgroup $\mathfrak{P}^n(K)$ of homology classes represented by the n-cycles which are (simplicial) images of n-dimensional "handle-manifolds" and that the factor-group $\mathfrak{H}^n(K)/\mathfrak{P}^n(K)$ is isomorphic to the factor-group of two subgroups of of the Hurewicz homotopy group $\pi_{n-1}(K^{n-1})$ of the $(n-1)$-skeleton K^{n-1} of K. (Cf. H. Hopf, Comment. Math. Helv. vol. 17 (1945) pp. 307–326.) The group $\mathfrak{H}^n(K)/\mathfrak{P}^n(K)$ can, on the other hand, be characterized by the group $\Omega_P^{n-1}(K)$ of the reduced $(n-1)$-dimensional nullspherods which are nullhomotopic in K in the ordinary sense of homotopy: $\mathfrak{H}^n(K)/\mathfrak{P}^n(K) \cong \Omega_P^{n-1}(K)$. The group $\Omega_P^{n-1}(K)$ is a subgroup of the group $\Omega^{n-1}(K)$ of the reduced $(n-1)$-nullspherods. (Cf. Bull. Amer. Math. Soc. Abstract 57-3-200.) This fact and an earlier result about $\Omega^{n-1}(K)$ (contained in the mentioned abstract) allow furthermore an interpretation of the Hopfian group $\Delta^{n-1}(K)$ (cf. H. Hopf, loc. cit.) by such reduced $(n-1)$-nullspherods which can not be deformed on K into a point in the sense of ordinary homotopy; one shows that: $\Delta^{n-1}(K) \cong \Omega^{n-1}(K)/\Omega_P^{n-1}(K)$. (Received November 13, 1951.)

247. A. L. Blakers: *Obstruction theorems for pairs.*

Let $A = \{A^r, \delta_A\}$, $B = \{B^r, \delta_B\}$, $C = \{C^r, \delta_C\}$ be Mayer cochain complexes. (See J. L. Kelley and Everett Pitcher, Ann. of Math. vol. 48 (1947) p. 686.) Assume given cochain transformations $\alpha: A \rightarrow C$, $\beta: B \rightarrow C$. We define a new cochain complex $K = \{K^r, \delta_K\}$ as follows: $K^r = \{(a^r, b^r) | \alpha a^r = \beta b^r\}$, and $\delta_K(a^r, b^r) = (\delta_A a^r, \delta_B b^r)$. We readily verify that $\delta_K(K^r) \subset K^{r+1}$, and that $\delta_K \delta_K = 0$. Some special cases are considered and some relations shown between the cohomology groups of K and those of A, B, and C. In particularly, consider the following extension problem: $(L; L_0, M)$ is a simplicial triad; $L_0 \cap M = M_0$. Assume that we are given a mapping $f: (L_0, M_0) \rightarrow (X, A)$, where X and A are arcwise connected. Consider the problem of extending the mapping f to a map $f': (L, M) \rightarrow (X, A)$. If $A^r = \mathfrak{C}^r(M, M_0, \pi_{r-1}(A))$, $B^r = \mathfrak{C}^r(L, L_0, \pi_{r-1}(X))$, $C^r = \mathfrak{C}^r(M, M_0, \pi_{r-1}(X))$ and α, β are the cochain transformations induced by the natural homomorphism of $\pi_{r-1}(A) \rightarrow \pi_{r-1}(X)$, and the inclusion mapping $(M, M_0) \rightarrow (L, L_0)$ respectively, then it is shown that the obstructions to the extension of f lie in the groups $H^r(K)$ as defined above, and that the usual obstruction theorems hold. Similar constructions may be defined to take care of obstruction problems for more complex systems. (Received November 6, 1951.)

248t. Raoul Bott: *A note on Whitney's duality theorem.*

The join operation between sphere bundles B and B', originally introduced by

Whitney and modified by Wu, is extended to apply to general bundles, yielding a bundle $B\wedge B'$ over the product of the respective base spaces. The basic lemma of the paper shows that the bundle space and projection of $B\wedge B'$ can be defined in the large in terms of the mapping cylinders of the projections of B and B'. It then follows practically from the definition of the objects involved that the homotopy classes of partial cross sections in B and B' are paired to the homotopy classes of partial cross sections in $B\wedge B'$, and that the induced pairing of obstructions to extending these cross sections is the cross product pairing of cohomology classes under a natural pairing of the local coefficient groups. This result, which generalizes the first formula of Whitney's duality theorem, is then shown to imply the whole duality theorem (which applies only to sphere bundles) by two different methods. One of these consists of the observation that the Whitney theorem now is a rewriting of the Cartan expansion of the Steenrod squares of cross products, due to the recent results of Thom and Wu. (Received November 13, 1951.)

249. Raoul Bott: *Periodic involutions and the Steenrod squares.* I.

In this paper it is shown by a refinement of the P. A. Smith and M. Richardson theory of "periodic involutions" and "symmetric product" that: (1) Every periodic involution $t:K\rightarrow K$ of a finite complex K onto itself, which is of prime period p and satisfies a certain axiom of "separation," induces dimension raising homomorphisms $\phi_t^\alpha:H^i(L)\rightarrow H^{i+\alpha}(L)$ in the cohomology groups mod p of L, where L is the fixed point set of t. (2) A certain class of involutions, which includes the cyclic permutations $T_{(p)}$ of $K^{(p)}=K\times K\times \cdots \times K$ (p terms), satisfies the axiom of separation. In a subsequent paper the ϕ_{T_p} will be identified as the "reduced pth powers" of Steenrod. (Received November 13, 1951.)

250*t*. F. E. Browder: *Local homeomorphisms and covering mappings.*

Let f be a local homeomorphism of the topological space X into the connected space Y. Suppose that one of the following three conditions holds: (a) Y is locally connected and has a denumerable basis of neighborhoods at each point: (b) Y is locally connected, X is paracompact; (c) Y is locally pathwise connected and semilocally simply connected. Then f is a covering mapping of X onto Y if and only if for each point of Y there exists a neighborhood V such that f is a closed mapping of each component of $f^{-1}(V)$ into V. In particular if f is a closed mapping, then f is a covering mapping and, if (a) holds, a finite covering mapping. (Received November 13, 1951.)

251*t*. Bailey Brown: *On imbedding in product spaces.*

All spaces considered are topological. A space R is homeomorphic to a subspace of the product of a family of spaces R_h indexed by a set H if and only if there exists a family of maps $f_h:R\rightarrow R_h$ such that (1) the collection $\{f_h^{-1}(U_h);\ U_h$ open in R_h, $h\in H\}$ is a subbase for the open sets of R, and (2) if $a\neq b$ in R, then $f_h(a)\neq f_h(b)$ for some h. This is used to characterize to within homeomorphism: trivial spaces as subspaces of products of two-element trivial spaces, T_0-spaces as subspaces of products of two-element Sierpinski spaces, zero-dimensional spaces (T_0 with a subbase of open and closed sets) as subspaces of products of two-element discrete spaces. Moreover, every space R can be topologically imbedded in the product of its quotient T_0-space and the trivial space whose elements are the elements of R. It follows that every space can be topologically imbedded in a product of two-element spaces which have the trivial or the Sierpinski topology. (Received November 5, 1951.)

252t. M. L. Curtis: *Deformation-free continua.*

Let M be a continuum separating S^n, and let A be a domain of S^n-M. M is said to be deformation free into A if there exists a deformation $h: MxI\to\bar{A}$ such that $h(x, t) \in A$ for $t>0$. It is known that in this case there is only one other domain B of S^n-M, both A and B are ulc^n (rationals) and M is an $(n-1)$-gcm. This same result is obtained in this paper when M is imbedded in an LC^{n-1} n-gcm S with $p^1(S)=0$. Various consequences of M being deformation free into A are obtained (M now in S^n). For example, $\bar{A}$ must be an ANR (although M need not be). If h is monotone open then A must be ULC^1, and if h is light open then A must be almost ULC^1 in a certain sense. Necessary and sufficient conditions that A be ULC^n are given in terms of deformation-free maps. Examples of a 3-gcm which is not 1-lc over the integers and an LC^3 3-gcm which is not a manifold are given. This paper contains the results on homotopy-regular convergence reported on previously (Bull. Amer. Math. Soc. Abstract 57-1-49). (Received November 13, 1951.)

253t. Mary-Elizabeth Hamstrom: *Concerning continuous collections of continuous curves.*

Let G be a continuous collection of mutually exclusive continuous curves filling up a compact metric continuum M such that G is an arc with respect to its elements. It is shown that if G has a nondegenerate element, then M is not irreducible. This is an extension of a result of E. E. Moise (*A theorem on monotone interior transformations*, Bull. Amer. Math. Soc. vol. 55 (1949) pp. 810–811). It is also shown that under the same conditions, if M is a subset of the plane it is a domain plus its boundary and if furthermore M is a continuous curve, then it is a simple closed curve plus its interior. (Received December 18, 1951.)

254t. S. T. Hu. *Cohomology relations in principal fiber spaces.*

If X is a compact metrizable principal fiber space over B with projection $p: X\to B$ and a compact zero-dimensional structural group G, then the homomorphism $p^*: H(B) \to H(X)$ of the Čech cohomology rings with real coefficients induced by the projection $p: X\to B$ maps $H(B)$ isomorphically onto the invariant subring $H_*(X)$ of $H(X)$. Hence, up to an isomorphism, $H(B)$ is completely determined by $H(X)$ and the operations of G on $H(X)$. It follows also that $H(B)$ is isomorphic with $H(X)$ if and only if G operates trivially on $H(X)$. If, for a given integer $n\geqq 0$, the dimension of the Čech cohomology group $H^n(X)$ with real coefficients is finite, then G operates on the real vector space $H^n(X)$ by means of a natural linear representation. For each $g\in G$, let $\chi^n(g)$ denote the character of the linear transformation determined by g in this linear representation. Then the dimension $p^n(B)$ of the Čech cohomology group $H^n(B)$ is also finite and is given by the formula $p^n(B)=\int_G \chi^n(g)dg$. Since every principal fiber bundle is a principal fiber space in the sense of Bourbaki, the foregoing formula generalizes a similar formula of Beno Eckmann on regular covering polyhedra. (Received November 2, 1951.)

255t. V. L. Klee: *Convex bodies and periodic homeomorphisms in Hilbert space.*

This paper contains work described in two earlier abstracts (Bull. Amer. Math. Soc. Abstract 56-4-339 and *A proof that Hilbert space is homeomorphic with its solid sphere*, Proceedings of the International Congress of Mathematicians, Cambridge, 1950), and also establishes some previously unannounced results concerning periodic homeomorphisms of Hilbert space H. Suppose $n \geqq 2$. Then: (1) *If X is a finite polytope in H, H admits a homeomorphism of period n with fixed-point set X*; (2) *If Y is a relatively open subset of X, H admits a homeomorphism of period n whose set of fixed points is homeomorphic with Y*; (3) *For each $d>0$, H admits a homeomorphism of period n whose fixed-point set is $\{x \mid \|x\| \geqq d\}$*. Also included is an extension to H of the "nearest-point" characterization of convex sets (Jessen, Matematisk Tiddskrift (1940) pp. 66–70) and an extension to more general spaces of Keller's results (Math. Ann. vol. 105 (1931) pp. 748–758) on compact convex subsets of H. (Received November 13, 1951.)

256t. V. L. Klee: *Invariant metrics in groups (solution of a problem of Banach).*

By use of Sierpinski's theorem [Fund. Math. vol. 11 (1928) pp. 203–205] that a topologically complete metric space is an absolute G_δ set (relative to metric spaces), the following is proved: *If G is a group with two-sided invariant metric ρ such that the space (G, ρ) is topologically complete, then G is complete under ρ.* In conjunction with the theorem of Kakutani [Proc. Imp. Acad. Japan vol. 12 (1936) pp. 82–84] and G. Birkhoff [Compositio Math. vol. 3 (1936) pp. 427–430] on the existence of invariant metrics, this result guarantees a complete invariant metric (compatible with the topology) for every first countable abelian Hausdorff group which is topologically complete. As applied to linear metric spaces, this answers affirmatively a question of Banach [*Théorie des opérations linéaires*, p. 232]. Also included is a characterization of those groups with associated topology which admit a left-invariant metric, and of those which admit a two-sided invariant metric, thus sharpening slightly the result of Kakutani and Birkhoff. (Received November 13, 1951.)

257t. E. E. Moise: *Affine structures in 3-manifolds.* III. *Tubular neighborhoods of linear graphs.*

Let L be a polyhedral linear graph in Euclidean 3-space E^3 with vertices v and edges e. Let T be a sum of closed spheres U_v with centers at v and closed solid cylinders C_e with axes e. T is a tubular neighborhood of L if, v' and v'' being the end points of e, $C_e-(U_{v'} \cup U_{v''})$ is connected and intersects no U_v or $C_{e'}$, $e' \neq e$. If $K, K' \subset E^3$ are homeomorphic, then $D(K, K')$ is the greatest lower bound of the numbers ϵ for which there is a homeomorphism f, throwing K onto K', such that for each $p \in K$ the distance from p to $f(p)$ is less than ϵ. *Theorem.* If f is a homeomorphism throwing a neighborhood U of L into E^3, and $\epsilon>0$, then for every sufficiently small tubular neighborhood T of L, lying in U, there is a homeomorphism f', throwing T onto a polyhedron $K \subset E^3$, such that (1) K is a neighborhood of $f(L)$ in E^3, (2) the U_v's used in defining T are thrown by f' onto polyhedra of diameter less than ϵ, and (3) $D(K, f(T))<\epsilon$. Forthcoming in Annals of Mathematics. (Received November 13, 1951.)

258t. E. E. Moise: *Affine structures in 3-manifolds.* IV. *Piecewise linear approximations of homeomorphisms.*

A separable metric space is a 3-manifold with boundary if each of its points has a closed neighborhood homeomorphic to a 3-cell. If K is a (locally finite simplicial) complex, then a homeomorphism f throwing K into Euclidean 3-space E^3 is piecewise linear if there is a simplicial subdivision G of K such that f is linear over every cell of G. *Theorem.* Let K be a finite simplicial complex which (considered as a space) is a 3-manifold with boundary, let f be a homeomorphism throwing K into E^3, and let ϵ

be a positive number. Then there is a piecewise linear homeomorphism f', throwing K into E^3, such that for each point p of K, the distance from $f(p)$ to $f'(p)$ is less than ϵ. This is the solution, for $n=3$, of a problem of J. W. Alexander (*Some problems in topology*, Verhandlungen des Internationalen Mathematiker Kongresses, Zürich, 1932, 1932, vol. 1, pp. 249–257). Forthcoming in Annals of Mathematics. (Received November 13, 1951.)

259. E. E. Moise: *Affine structures in 3-manifolds.* V. *The triangulation theorem and Hauptvermutung.*

A 3-manifold is a separable metric space each of whose points has an open neighborhood homeomorphic to Euclidean 3-space E^3. A space is triangulable if it is homeomorphic to a (locally finite simplicial) complex. Two complexes K_1 and K_2 are isomorphic if there is a one-to-one correspondence between the simplices of K_1 and those of K_2, preserving incidence relations. K_1 and K_2 are combinatorially equivalent if there are simplicial subdivisions K_1' and K_2' of K_1 and K_2 respectively, such that K_1' and K_2' are isomorphic. *Theorem.* Every 3-manifold is triangulable. This is the solution (for $n=3$) of a problem of H. Poincaré. *Theorem.* If the complexes K_1 and K_2 are homeomorphic 3-manifolds, then they are combinatorially equivalent. This is the case $n=3$ of the *Hauptvermutung* of Steinitz. It implies that every combinatorial invariant of a triangulated 3-manifold is a topological invariant. Reidemeister's torsion invariant, which is sufficient to classify the 3-dimensional lens spaces combinatorially, is therefore also sufficient to classify these spaces topologically. (Reidemeister, *Homotopieringe und Linsenräume*, Abh. Math. Sem. Hamburgischen Univ. vol. 11 (1935) pp. 102–109.) Forthcoming in Annals of Mathematics. (Received November 13, 1951.)

260*t*. E. E. Moise: *On the nonseparation of the Antoine set by 2-spheres.*

L. Antoine (*Sur la possibilité d'étendre l'homéomorphisme de deux figures à leur voisinage*, C. R. Acad. Sci. Paris vol. 171 (1920) pp. 661–663) has described a 0-dimensional compact set A in 3-space, such that the complement of A is not simply connected. It was asserted by Antoine, with extremely brief indications of proof, that any 2-sphere which separates two points of A from one another in 3-space must intersect A. In the present note, a detailed argument for Antoine's statement is given, using Vietoris linking theory. This property of the set A is used to give an example of two 2-spheres in 3-space which cannot be separated by any third 2-sphere, polyhedral or not. (Received November 13, 1951.)

261. G. D. Mostow: *On the topology of homogeneous spaces.* II.

Let G be a simply connected solvable Lie group, and S a closed subgroup of G. Let N be the connected subgroup determined by the maximum nilpotent ideal of the Lie algebra of G. Then either S is in a Cartan subgroup of G or SN is closed. It follows that a homogeneous space arising from a solvable Lie group is the direct product of a euclidean space and a fibre bundle which has a toroid as base and a nilmanifold as fibre (i.e., homogeneous space of a nilpotent group; cf. Malcev, Izvestiya Akademii Nauk SSSR. Ser. Mat. (1949), Amer. Math. Soc. Translation No. 39). Necessary and sufficient conditions on the fundamental group of G/S in order that the bundle above admit a continuous cross-section are given. If G/S is compact and its fundamental group is abelian, then G/S is homeomorphic to a toroid. (Received November 13, 1951.)

262. O. M. Nikodým: *Universal locally convex linear topological spaces.*

Locally convex linear topological spaces whose vectors are some real-valued functions $M(U, \alpha)$ of two variables are constructed, into which vast classes of locally convex linear topological spaces L can be isomorphically embedded with preservation of Hyer's pseudonorm. These applications transform the convergence of directed-set-sequences in L into a generalized uniform convergence. This paper constitutes part of the work done by the author for the A.E.C. under a nonclassified project. (Received November 13, 1951.)

263. C. N. Reynolds: *The present status of the four color problem.*

Traditionally, the reductions of the four color problem have been part of synthetic geometry. Their implications, on the other hand, have been developed by arithmetic and algebraic inequalities. This paper generalizes the algebraic formulae, proves the generalizations by algebra, and shows that they lead to this proposition: A map which is irreducible with respect to the four color problem must contain a denumberably infinite set of regions. If, by definition, irreducible maps are restricted to maps with a finite set of regions, this "solves" the "four color problem." (Received November 13, 1951.)

264*t*. S. K. B. Stein: *Homology of the symmetric product.*

Let X be a topological space. The symmetric product of X is obtained from the cartesian product $X \times X$ by identifying the points (x, x') and (x', x). Now assume that X is triangulable. P. A. Smith (Proceedings of the Academy of Science (1933)) obtains the mod 2 homology of the symmetric product from that of X. Moses Richardson (Duke Math. J. (1935)) obtains the homology mod p^n where p is a prime other than 2. His proof for the case $p=2$ seems to contain errors. In the present paper the author computes the homology over the integers. The method is a generalization of that employed by Smith. Three exact sequences and Smith's mod 2 result are used. (Received December 6, 1951.)

265. H. C. Wang: *One-dimensional Lefschetz group of locally compact homogeneous spaces.*

Let X be a locally compact, noncompact space, and $X' = X + I$ be its one-point compactification. Following H. Cartan, by Lefschetz group $L_r(X)$ of X we mean the Čech homology group $H_r(X')$ of the compact space X'. The following is proved: Theorem. Let X be a connected, locally compact, separable metric space admitting a transitive group of isometries (in particular, X is a connected, locally compact group satisfying the first countability axiom). If X is noncompact and locally connected, the 1-dimensional Lefschetz group $L_1(X)$ of X is either 0 or isomorphic with the coefficient group. (Received November 13, 1951.)

266. M. T. Wechsler: *A characterization of certain topological spaces by means of their groups of homeomorphisms.*

A topological space F is n-homogeneous if for every pair of n-tuples $(x_1, \cdots, x_n)$ and $(y_1, \cdots, y_n)$, where the x_i and y_i are points of F and $x_i \neq x_j$ and $y_i \neq y_j$ for $i \neq j$, there exists a homeomorphism g of F onto F such that $g(x_i) = y_i$, $i = 1, \cdots, n$. F is said to be ω-homogeneous if it is n-homogeneous for each n. Let $\mathcal{J}$ be the class of all spaces F with the property that F is ω-homogeneous, nondiscrete, and that there exists at least two open sets of F which are nonempty and disjoint. Note that the class of all manifolds of dimension two or greater is a subclass of $\mathcal{J}$. Let $G(F)$ be the group of all homeomorphisms of F onto itself and give $G(F)$ the point-open topology. The result of this paper is that if F and F' are two spaces belonging to the class $\mathcal{J}$ and if there exists a mapping of $G(F)$ onto $G(F')$ which is both an isomorphism and a homeomorphism, then the spaces F and F' are homeomorphic. (Received November 16, 1951.)

267. G. W. Whitehead: *Fibre spaces and the Eilenberg homology groups.*

Let X be a 0-connected, locally equi-connected space. Then for any integer n, there exists a fibre space T_n over X such that (1) T_n is n-connected; (2) the fibre map of T_n on X induces isomorphisms onto: $\pi_i(T_n) \approx \pi_i(X)$ for $i \geqq n+1$. The space T_n is constructed as follows: let $X^* \supset X$ be a space such that $\pi_i(X^*) = 0$ for $i > n$, while the inclusion map of X into X^* induces isomorphisms onto: $\pi_i(X) \approx \pi_i(X^*)$ for $i \leqq n$. Then T_n is the space of all paths in X^* starting at a fixed point $x_0 \in X$ and ending in X. As an application it is shown that if X is a 0-connected space and $S_n(X)$ is the subcomplex of the total singular complex consisting of all singular simplexes whose n-skeletons are mapped into x_0, then $H_q(S_n(X), S_{n+1}(X)) \approx H_{q-1}(\pi_{n+1}(X), n)$ for $q \leqq 2n$. (Received October 29, 1951.)

L. W. COHEN,
Associate Secretary

APPENDIX

EXCERPTS FROM REPORT OF TREASURER

TO THE BOARD OF TRUSTEES OF THE
AMERICAN MATHEMATICAL SOCIETY:

Gentlemen:

I have the honor to submit herewith the report of the Treasurer for the fiscal year ended November 30, 1950, as is customary in two parts, (1) exhibits and schedules, and (2) a discussion and commentary.

On November 30, 1950, the market value of securities held for invested funds exceeded book value (normally cost) by $45,008, while the market value of securities held for current funds was less than book value by $444.00.

It has been our policy in recent years to keep current funds only in Government securities, as in all probability the Society will be obliged either to liquidate many of these securities or to incur bank loans secured by pledging these assets as collateral. Liquidity and safety have been our sole objectives in the investment of these funds.

On the other hand, in dealing with invested funds, which may be thought of with reasonable accuracy to be the permanent funds of the Society, we have sought a diversified portfolio of high grade securities yielding as liberal a return as is consistent with sound investment policy. Our return this year on these funds has been 5.47%, computed on average book value of investments.

The sale of the Library placed $66,000 in the hands of the Trustees. It was decided to set aside $50,000 as a fund to be known as the "Library Proceeds Fund," and securities were purchased to implement this decision of the Board. The income from these investments should cover the rent on the new headquarters building in Providence.

The balance of $16,000 was made available for current use in reconditioning and furnishing the headquarters building, and in meeting expenses incurred in moving from New York to Providence. The Board voted that the sum should be amortized over five years and repayments added to the Library Proceeds Fund until this Fund should have been built up to the sum of $66,000. Interest will be charged on the unamortized balance.

Following is a summary of the changes in the security holdings made during the year, including the securities purchased with the monies placed in the Library Proceeds Fund.

Acquired

\$5,000	American Telephone and Telegraph Co. Deb $3\frac{3}{8}$s 1973
5,000	Consolidated Edison Co. of New York 1st Ref Ser G $3\frac{1}{4}$s 1981
3,000	Oregon Washington RR and Navigation Co. Ref A 3s 1960
9,000	United Gas Corp. 1st Mtg and Coll Trust $3\frac{5}{8}$ 1971
2,500	United States Treasury $2\frac{1}{4}$s 1962–1959
50	shares American Tobacco Co. Cum 6%
100	shares Ingersoll Rand Co. Common
25	shares Northern States Power Co. of Minnesota Cum \$3.60
100	shares Phillips Petroleum Co. Common
100	shares Westinghouse Electric Common
5	shares Standard Oil Co. of New Jersey

Acquired as result of stock splits

100	shares Continental Oil Co.
50	shares Texas Company
2	shares Standard Oil Co. of New Jersey

Sold

25	rights American Telephone and Telegraph Co.
100	rights Consumers Power Co.
30/40	share Standard Oil of New Jersey

It will be noted that, in accordance with a recommendation made by the Treasurer in the Annual Report for 1950 and subsequently approved by the Board, the *Proceedings* and the *Transactions* have been set up as publication funds, rather than being treated as general membership activities. The deficits incurred in the publication of these journals have thus been made startlingly apparent. A reserve to cover the deficit has been set up in anticipation of an appropriation from surplus by the Board at the annual meeting to cover these deficits—approximately \$23,000.

The increase in dues, both individual and institutional, was of substantial help in meeting the obligations of the Society, having produced about \$12,000.

As a matter of record, it should be noted that the Society accepted Federal Social Security benefits for its staff as of January 1, 1951, in accordance with an Act of Congress offering this opportunity to scientific and educational organizations hitherto excluded from these benefits. The cost to the Society in the resulting taxes was just under $750.00.

Fortunately for the Society, a substantial balance remains in the International Congress account, for the Proceedings of the Congress have not yet been published. The fact that these funds have been held in the treasury pending the receipt of bills for printing and distributing the Proceedings has enabled us to avoid throughout the entire year bank borrowing to obtain current cash.

The Society has about one more year during which it can continue to conduct its extensive program of publication on its present scale without securing new sources of income. If additional income of $15,000 to $25,000 per year cannot be obtained, publication of our journals will then have to be curtailed. Our reserves, built up when the volume of publication was abnormally low, will have been exhausted by what is evidently not a temporarily abnormally high volume, but a new expanded normal level.

In addition, every economy in operation must be sought and embraced. The move to Providence will help. Our unified operations can be conducted at less expense than was the case when our offices were dispersed. It may be necessary for us to allow some of our older publications to go out of print, or to adopt a policy of maintaining files of back numbers for only a limited period of years.

Above all, the Society must be sure that any new activities undertaken are fully supported by clearly foreseeable income before being begun.

The Treasurer believes that the basic financial policies of the Society are soundly conceived and wisely executed, and in particular, holds that the action of the Board in authorizing unbalanced budgets during the postwar years, was necessary to the welfare of American mathematics and the furtherance of the objectives of the Society. But the time has now come when we must return, not later than fiscal 1953, to a strictly balanced budget and engage only in enterprises we can currently support.

Reference has already been made to the increased dues authorized by the membership, effective in 1951. The members, with commendable realism, began the program of finding increased income with themselves. The universities, as institutional members, have also recognized the reality of increased costs and assumed their share of the burden. Thus individual mathematicians and institutions of higher learning have assumed and are carrying at least as much of the responsibility as can rightfully be expected of them. If American mathematical research is not to be hampered and, indeed, curtailed at a time when its importance to Government and industry is greater than ever, Government and industry must accept also their rightful share of its support.

In doing this, such agencies and corporations should recognize the importance of the claims of the American Mathematical Society, at once the voice of the mathematicians themselves, the principal agency of research publication in America, and the one organization in the Country whose principal purpose—indeed, whose sole purpose—is the furtherance of mathematical research. It is believed that contributions to the work of the American Mathematical Society will be more effective in securing the desired result than would similar contributions made in any other way.

Respectfully submitted,
ALBERT E. MEDER, JR.
Treasurer

BALANCE SHEET

Assets

	November 30, 1951	November 30, 1950
CURRENT FUNDS:		
Cash	$ 39,634.51	$ 33,332.51
Account Receivable (United States Government)	3,137.82	5,842.66
Investments	67,329.31	67,328.66
	$110,101.64	$106,503.83
INVESTED FUNDS:		
Cash	$ 1,427.00	$ 629.83
Investments	244,972.71	194,242.32
	$246,399.71	$194,872.15
TOTAL ASSETS	$356,501.35	$301,375.98

Liabilities

CURRENT FUNDS:		
Publications	$ 5,667.37	$ 26,348.27
International Congress	41,992.58	45,531.48
Policy Committee	778.30	(648.55)
Prize Funds and Other Special Funds Accumulated Income	4,346.02	3,524.71
Sinking Fund		54.04
Profit on Sales of Securities	2,049.85	2,049.85
Miscellaneous (credit)	2,033.45	1,644.02
Surplus and Reserves	46,655.93	28,000.01
Account Payable (United States Government)	28.95	
Office Transfer Fund	6,549.19	
	$110,101.64	$106,503.83
INVESTED FUNDS:		
Endowment Fund Principal	$ 71,000.00	$ 71,000.00
Library Proceeds Fund	50,000.00	
Prize Funds and Other Special Funds	33,683.22	33,683.22
Life Membership and Subscription Reserve	4,162.64	2,635.08
Mathematical Reviews	65,000.00	65,000.00
Reserve for Investment Losses	4,385.89	4,385.89
Profit on Sales of Securities	18,167.96	18,167.96
	$246,399.71	$194,872.15
TOTAL LIABILITIES	$356,501.35	$301,375.98

GENERAL RECEIPTS AND DISBURSEMENTS

GENERAL RECEIPTS:

Dues from Ordinary Memberships		$45,677.64	
Dues from Contributing Memberships		776.75	
Dues from Institutional Memberships		14,323.46	
Publication Charges from Non-Member Institutions		213.75	
Initiation Fees		799.62	
Income from			
Henderson Estate	$ 6,850.00		
Current Funds Investments	1,509.60		
Invested Funds	5,321.96	13,681.56	
Miscellaneous			
Donations	357.90		
Meeting Fees	516.00	873.90	
TOTAL GENERAL RECEIPTS			$76,346.68

GENERAL DISBURSEMENTS:

Secretaries and Executive Director	$27,396.78	
Treasurer	7,859.00	
Librarian	1,056.81	
Policy Committee	785.03	
Furniture and Fixtures, N. Y.	2,198.56	
Rent	1,021.20	
Headquarters Utilities	113.94	
Miscellaneous	1,796.75	
TOTAL GENERAL DISBURSEMENTS		$42,228.07
EXCESS OF GENERAL RECEIPTS OVER GENERAL DISBURSEMENTS		$34,118.61

ANALYSIS OF SURPLUS

	Additions	Deductions	
SURPLUS AT DECEMBER 1, 1950			$28,000.01
Excess of General Receipts over General Disbursements (see above)	$34,118.61		
Bulletin	3,262.66	$13,431.63	
Appropriations:			
Accumulated Income—Reilly	2,300.00		
Mathematical Reviews		500.00	
American Journal		2,091.16	
Canadian Journal		1,000.00	
Pacific Journal		1,300.00	
Annals		1,175.00	
Miscellaneous	374.47	1,902.03	
	$40,055.74	$21,399.82	
Net Additions			$18,655.92
Surplus before Reserves			$46,655.93
Reserves for Proceedings and Transactions Deficit			23,170.67
SURPLUS, November 30, 1951			$23,485.26

SUMMARY OF CURRENT FUNDS

	Balance 12/1/50	Additions	Deductions	Balance 11/30/51
CURRENT FUNDS				
Publications				
Mathematical Reviews	$ 6,912.09	$ 53,911.30	$ 56,280.27	$ 4,543.12
Colloquium	11,530.24	13,962.33	12,231.28	13,261.29
Mathematical Surveys	(3,457.77)	5,474.54	7,669.35	(5,652.58)
Proceedings		3,515.34	19,129.50	(15,614.16)
Transactions		9,963.35	17,519.86	(7,556.51)
Symposia on Applied Mathematics	(1,901.77)	1,191.02	168.12	(878.87)
Birkhoff's Collected Papers	(3,145.63)	2,015.30	117.19	(1,247.52)
Memoirs	4,290.44	1,653.00	3,614.99	2,328.45
Bulletin Reprinting	7,631.43	1,523.09		9,154.52
Transactions Reprinting	5,159.28	2,435.15		7,594.43
Russian Vocabulary	(49.72)	284.33	128.64	105.97
Translations	(620.32)	810.62	561.07	(370.77)
TOTAL PUBLICATIONS	$26,348.27	$ 96,739.37	$117,420.27	$ 5,667.37
International Congress	$45,531.48	$ 421.17	$ 3,960.07	$41,992.58
Office Transfer Fund		16,000.00	9,450.81	6,549.19
Policy Committee	(648.55)	1,823.55	396.70	778.30
Prize Funds and Other Special Funds Accumulated Income	3,524.71	3,121.31	2,300.00	4,346.02
Sinking Fund	54.04		54.04	
Profit on Sales of Securities	2,049.85			2,049.85
TOTAL	$76,859.80	$118,105.40	$133,581.89	$61,383.31

SUMMARY OF INVESTED FUNDS

	PRINCIPAL				INCOME			
	Balance 12/1/50	Additions	Deductions	Balance 11/30/51	Balance 12/1/50	Additions	Deductions	Balance 11/30/51
INVESTED FUNDS								
Endowment	$ 71,000.00			$ 71,000.00		$ 3,887.55	$ 3,887.55*	
Library Proceeds		$50,000.00		50,000.00		1,277.02		$1,277.02
Prize Funds								
Böcher	$ 1,188.00			$ 1,188.00	$ 748.89	$ 65.04		$ 813.93
Brown	1,000.00			1,000.00	56.07	54.74		110.81
Cole	2,093.13			2,093.13	1,055.33	114.62		1,169.95
Henderson	1,000.00			1,000.00	56.07	54.74		110.81
Hutchinson	1,000.00			1,000.00	56.07	54.74		110.81
Merrill	650.00			650.00	52.19	35.57		87.76
Moore	2,100.62			2,100.62	117.78	115.05		232.83
Reilly	23,651.47			23,651.47	1,326.24	1,295.05	2,300.00†	321.29
Whittemore	1,000.00			1,000.00	56.07	54.74		110.81
TOTAL	$ 33,683.22			$ 33,683.22	$3,524.71	$ 1,844.29	$ 2,300.00	$3,069.00
Mathematical Reviews	$ 65,000.00			$ 65,000.00		$ 3,559.04	$ 3,559.04‡	
Life Membership and Subscription Reserve	2,635.08	1,902.03	374.47	4,162.64		201.81	201.81*	
Reserve for Investment Losses	4,385.89			4,385.89		238.09	238.09*	
Profit and/or Loss on Sales of Securities	18,167.96			18,167.96		994.51	994.51*	
TOTAL	$194,872.15	$51,902.03	$374.47	$246,399.71	$3,524.71	$12,002.31	$11,181.00	$4,346.02

* Transferred to General Receipts.
† Transferred to Surplus.
‡ Transferred to Mathematical Reviews.