466*t*. L. A. Zadeh: *Initial conditions in linear varying-parameter systems.*

Consider a linear varying-parameter system N whose behavior is described by an nth order linear differential equation $L(p; t)v(t) = u(t)$. Let $u(t)$ be zero for $t<0$ and let the initial values of $v(t)$ and its derivatives be $v^{(\nu)}(0) = \alpha_\nu$ $(\nu = 0, 1, \cdots, n-1)$. Let $H(s; t)$ be the system function of N. When the system is initially at rest (that is, all α_ν are zero), the response of N to $u(t)$ may be written as $v(t) = \mathcal{L}^{-1}\{H(s; t)\,U(s)\}$ (see abstract 56-6-465). When, on the other hand, some of the α_ν are not zero, the expression for the response to a given input $u(t)$ becomes $v(t) = \mathcal{L}^{-1}\{H(s; t)\,[U(s) + \Delta(s)]\}$, where $\Delta(s)$ is a polynomial in s and p_0 given by $\Delta(s) = \{[L(s; 0) - Lp_0; 0)]/(s - p_0)\}v$ (p_0 represents a differential operator such that $p_0^\nu v = v^{(\nu)}(0) = \alpha_\nu$). $\Delta(s)$ is essentially the Laplace transform of a linear combination of delta-functions of various order (up to $n-1$) such that the initial values of the derivatives of the response of N to this combination are equal to α_ν. (Received September 14, 1950.)

Topology

467*t*. A. L. Blakers and W. S. Massey: *Generalized Whitehead products.*

J. H. C. Whitehead has defined (Ann. of Math. vol. 42 (1941) pp. 409–428) a product which associates with elements $\alpha \in \pi_p(X)$ and $\beta \in \pi_q(X)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(X)$. The authors show how to define three new products, as follows: (a) A product which associates with elements $\alpha \in \pi_p(A)$ and $\beta \in \pi_q(X, A)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(X, A)$. (b) A product which associates with elements $\alpha \in \pi_p(A/B)$ and $\beta \in \pi_q(A \cap B)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(A/B)$. Here the sets A and B are a covering of the space $X = A \cup B$, and $\pi_p(A/B)$ is the p-dimensional homotopy group of this covering which has been introduced by the authors (Bull. Amer. Math. Soc. Abstract 56-3-208). (c) Let $(X; A, B)$ be a triad (see A. L. Blakers and W. S. Massey, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) p. 323), then there is a product which associates with elements of $\pi_p(A/B)$ and $\pi_q(X, A \cap B)$ an element of $\pi_{p+q-1}(X; A, B)$. The bilinearity of these three new products is established under suitable restrictions, and relationships between the various products are proved. The behavior of the products under homomorphisms induced by a continuous map or a homotopy boundary operator is also studied. (Received August 30, 1950.)

468*t*. A. L. Blakers and W. S. Massey: *The triad homotopy groups in the critical dimension.*

Let $X^* = X \cup \xi_1^n \cup \xi_2^n \cup \cdots \cup \xi_k^n$ be a space obtained by adjoining the n-dimensional $(n>2)$ cells ξ_i^n to the connected, simply connected topological space X. Let $\xi^n = \xi_1^n \cup \xi_2^n \cup \cdots \cup \xi_k^n$, and $\dot{\xi}^n = X \cap \xi^n$. Assume that the space $\dot{\xi}^n$ is arcwise connected, and that the relative homotopy groups $\pi_p(X, \dot{\xi}^n)$ are trivial for $1 \leqq p \leqq m$, where $m \geqq 1$. Then it is known that the triad homotopy groups $\pi_q(X^*; \xi^n, X)$ are trivial for $2 \leqq q \leqq m+n-1$. The authors now show that under the assumption of suitable "smoothness" conditions on the pair $(X, \dot{\xi}^n)$ (for example, both X and $\dot{\xi}^n$ are compact A.N.R.'s), there is a natural isomorphism of the tensor product $\pi_n(\xi^n, \dot{\xi}^n) \otimes \pi_{m+1}(X/\dot{\xi}^n)$ onto the triad homotopy group $\pi_{m+n}(X^*; \xi^n, X)$. This isomorphism is defined by means of a generalized Whitehead product. The Freudenthal "Einhängung" theorems in the critical dimensions can easily be derived from this theorem;

also, some results of J. H. C. Whitehead follow as a corollary (see Ann. of Math. vol. 42 (1941) pp. 409–428, especially Lemma 4). The proof makes use of the functional cup product of Steenrod, and the theory of obstructions to deformations of continuous mappings. (Received August 30, 1950.)

469t. Tibor Radó: *On the identifications in singular homology theory.*

According to the classical formulation of singular homology theory, a singular p-cell of a topological space X is an aggregate $[(x_0, \cdots, x_p, T)]$ of a geometrical p-simplex $(x_0, \cdots, x_p)$ and of a continuous mapping $T: (x_0, \cdots, x_p) \to X$, subject to the following identifications. (1) If $(y_0, \cdots, y_p)$ is an even permutation of $(x_0, \cdots, x_p)$, then $[(x_0, \cdots, x_p), T] = [(y_0, \cdots, y_p), T]$. (2) If m is an affine map such that $m(y_i) = x_i$, $i = 0, \cdots, p$, then $[(x_0, \cdots, x_p), T] = [(y_0, \cdots, y_p), Tm]$. The main result in this paper is the theorem that the singular homology groups of X are unchanged (up to isomorphisms) if both of these identifications are eliminated, provided that the geometrical simplexes $(x_0, \cdots, x_p)$, $p = 0, 1, 2, \cdots$, are taken from a fixed Hilbert space. This theorem completes previous work by Eilenberg, who obtained an analogous result concerning the identification (1) alone. (Received September 5, 1950.)