462t. Werner Leutert: *On the convergence of approximate solutions of the heat equation to the exact solution.*

It is shown that an approximate solution of the heat equation can be obtained from a three line difference equation by using only half of the particular solutions of the form $e^{i\beta x}e^{\alpha i}$. The approximate solution will converge to the exact solution for all positive values of the mesh ratio $r=\Delta t/(\Delta x)^2$ and it will be stable in the sense that small changes in the initial condition vanish as the time t is increased. von Neumann's test shows instability for all values of $r>0$. (Received July 31, 1950.)

463t. Bertram Yood: *On fixed points for semi-groups of linear operators.*

Let G be a semi-group of bounded linear operators on a normed linear space X, and G^* be the family of adjoints of elements of G. Sets of conditions are given on G which imply the existence of a nonzero fixed element for G^* (in X^*). In particular if X is the space of bounded functions on a set S, the results show, as a special case, the existence of a finitely-additive measure defined for all subsets of S invariant under a solvable group of 1-1 transformations of S onto S. This fact is due to von Neumann (Fund. Math. vol. 13 (1929)). (Received September 14, 1950.)

Applied Mathematics

464t. C. N. Mooers: *Automata with learning.* Preliminary report.

The automata moves in an artificial environment having positions or states $q_i (i=1, \cdots, N_q)$. It has a repertory of moves that it can make, each called m_{ij} $(j=1, \cdots, N_i)$. From state q_i by move m_{ij} it goes to a new uniquely determined state q_k, that is, $(q_i, m_{ij})=q_k$. Each state q_i is characterized by an aspect a_i having the value $+1$ or -1. The a_i is a "drive" in the psychological sense, and when a_i is positive the automata is active. In state q_i the automata initially randomly chooses an m_{ij} where all the m's have an equal probability. In the case $(q_i, m_{ij})=q_{i+1}$ whose a_{i+1} is negative (drive extinguished), then the probability is increased for choice m_{ij} when in state q_i. In (q_i, m_{ij}) there is a transfer relation such that when some $m_{i+1,k}$ of q_{i+1} has a probability greater than $2/N_{i+1}$, then the probability of taking m_{ij} in q_i is also increased. The automata as postulated can learn its way through a maze, learning from the goal backwards; it can remember the solution to two or more mazes; it forgets nonused information; and its behavior is not predictable. (Received September 5, 1950.)

465t. L. A. Zadeh: *On stability of linear varying-parameter systems.*

Starting with the definition of stability in the case of linear varying-parameter systems: a system is stable if and only if every bounded input produces a bounded output, it is shown that the necessary and sufficient condition for stability is that the impulsive response of the system $W(t, \tau)$ should belong to $L(0, \infty)$ for all t ($W(t, \tau)$ is the response at t to a unit impulse applied at $t-\tau$). The system function of a linear varying-parameter system is related to $W(t, \tau)$ through $H(s; t)=\int_0^\infty W(t, \tau)e^{-s\tau}d\tau$. From this it follows that the system function of a stable system is analytic in the right half and on the imaginary axis of the s-plane for all t. This result can be applied with advantage to the investigation of stability of linear varying-parameter systems. In particular, it yields useful criteria of stability for differential equations having periodic coefficients. (Received September 14, 1950.)

466t. L. A. Zadeh: *Initial conditions in linear varying-parameter systems.*

Consider a linear varying-parameter system N whose behavior is described by an nth order linear differential equation $L(p; t)v(t) = u(t)$. Let $u(t)$ be zero for $t<0$ and let the initial values of $v(t)$ and its derivatives be $v^{(\nu)}(0) = \alpha_\nu$ ($\nu = 0, 1, \cdots, n-1$). Let $H(s; t)$ be the system function of N. When the system is initially at rest (that is, all α_ν are zero), the response of N to $u(t)$ may be written as $v(t) = \mathcal{L}^{-1}\{H(s; t)\, U(s)\}$ (see abstract 56-6-465). When, on the other hand, some of the α_ν are not zero, the expression for the response to a given input $u(t)$ becomes $v(t) = \mathcal{L}^{-1}\{H(s; t)[U(s) + \Delta(s)]\}$, where $\Delta(s)$ is a polynomial in s and p_0 given by $\Delta(s) = \{[L(s; 0) - Lp_0; 0)]/(s - p_0)\}v$ (p_0 represents a differential operator such that $p_0^\nu v = v^{(\nu)}(0) = \alpha_\nu$). $\Delta(s)$ is essentially the Laplace transform of a linear combination of delta-functions of various order (up to $n-1$) such that the initial values of the derivatives of the response of N to this combination are equal to α_ν. (Received September 14, 1950.)

TOPOLOGY

467t. A. L. Blakers and W. S. Massey: *Generalized Whitehead products.*

J. H. C. Whitehead has defined (Ann. of Math. vol. 42 (1941) pp. 409–428) a product which associates with elements $\alpha \in \pi_p(X)$ and $\beta \in \pi_q(X)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(X)$. The authors show how to define three new products, as follows: (a) A product which associates with elements $\alpha \in \pi_p(A)$ and $\beta \in \pi_q(X, A)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(X, A)$. (b) A product which associates with elements $\alpha \in \pi_p(A/B)$ and $\beta \in \pi_q(A \cap B)$, an element $[\alpha, \beta] \in \pi_{p+q-1}(A/B)$. Here the sets A and B are a covering of the space $X = A \cup B$, and $\pi_p(A/B)$ is the p-dimensional homotopy group of this covering which has been introduced by the authors (Bull. Amer. Math. Soc. Abstract 56-3-208). (c) Let $(X; A, B)$ be a triad (see A. L. Blakers and W. S. Massey, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) p. 323), then there is a product which associates with elements of $\pi_p(A/B)$ and $\pi_q(X, A \cap B)$ an element of $\pi_{p+q-1}(X; A, B)$. The bilinearity of these three new products is established under suitable restrictions, and relationships between the various products are proved. The behavior of the products under homomorphisms induced by a continuous map or a homotopy boundary operator is also studied. (Received August 30, 1950.)

468t. A. L. Blakers and W. S. Massey: *The triad homotopy groups in the critical dimension.*

Let $X^* = X \cup \xi_1^n \cup \xi_2^n \cup \cdots \cup \xi_k^n$ be a space obtained by adjoining the n-dimensional ($n>2$) cells ξ_i^n to the connected, simply connected topological space X. Let $\xi^n = \xi_1^n \cup \xi_2^n \cup \cdots \cup \xi_k^n$, and $\dot{\xi}^n = X \cap \xi^n$. Assume that the space $\dot{\xi}^n$ is arcwise connected, and that the relative homotopy groups $\pi_p(X, \dot{\xi}^n)$ are trivial for $1 \leqq p \leqq m$, where $m \geqq 1$. Then it is known that the triad homotopy groups $\pi_q(X^*; \xi^n, X)$ are trivial for $2 \leqq q \leqq m+n-1$. The authors now show that under the assumption of suitable "smoothness" conditions on the pair $(X, \dot{\xi}^n)$ (for example, both X and $\dot{\xi}^n$ are compact A.N.R.'s), there is a natural isomorphism of the tensor product $\pi_n(\xi^n, \dot{\xi}^n) \otimes \pi_{m+1}(X/\dot{\xi}^n)$ onto the triad homotopy group $\pi_{m+n}(X^*; \xi^n, X)$. This isomorphism is defined by means of a generalized Whitehead product. The Freudenthal "Einhängung" theorems in the critical dimensions can easily be derived from this theorem;