ABSTRACTS OF PAPERS

The abstracts below are abstracts of papers presented by title at the October Meeting in New York and the November Meeting in Evanston. Abstracts of papers presented in person at these meetings will be included in the reports of the meetings which will be published in the January issue of this BULLETIN.

Abstracts are numbered serially throughout this volume.

ALGEBRA AND THEORY OF NUMBERS

446*t*. R. H. Bruck and Erwin Kleinfeld: *New identities for alternative rings.* Preliminary report.

Let R be an alternative ring with (skew-symmetric) commutator $(x, y)=xy-yx$ and associator $(x, y, z)=xy\cdot z-x\cdot yz$. Known identities for the associator are mainly based upon (*) $(wx, y, z)-(w, xy, z)+(w, x, yz)=w(x, y, z)+(w, x, y)z$. Instead, define f by $(wx, y, z)=(x, y, z)w+x(w, y, z)+f(w, x, y, z)$, and use (*) to prove f skew-symmetric. Skew-symmetry of f, and one of several identities such as (i) $f(w, x, y, z)=((w, x), y, z)+((y, z), w, x)$, are equivalent to (*). Other functions are introduced in the same spirit; these and f are powerful tools in the study of alternative rings. Among the significant byproducts are: (ii) $((x, y, z)^2, y, z)=0$; (iii) $((w, x)^2, y, z) =(w, x)f(w, x, y, z)+f(w, x, y, z)(w, x)$. (Received July 21, 1950.)

447*t*. D. O. Ellis: *An algebraic characterization of lattices among semilattices.*

A semilattice is an associative, commutative, idempotent groupoid. The set-theoretic product of the sets of units for each of two elements of a semilattice is a sub-semilattice. Cosets of this sub-semilattice by its own elements are considered. The original semilattice is said to have property M if at least one such coset consists of a single element for each two elements of the original semilattice. The principle result is: *Let L be a semilattice under $a+b$. It is possible to define a second operation, ab, in L so that L forms a lattice under $a+b$ and ab if and only if L has Property* M. *Moreover, the introduction of this second operation may be accomplished in exactly one way; namely, by construction from Property* M. (Received June 2, 1950.)

448*t*. D. O. Ellis and J. W. Gaddum: *On solutions of systems of linear equations in a Boolean algebra.*

Let B be a Boolean algebra with meet, join, and complement denoted by ab, $a+b$, and a', respectively. Consider the system of equations (1) $\sum_{j=1}^{n} a_{ij}x_j=k_i$; $i=1, 2, \cdots, m$, where the a_{ij} and k_i are constants. The cardinals m and n are finite, $\leqq\aleph_0$, or arbitrary according as B is arbitrary, σ-complete, or complete. Define $d_{ij}=\prod_{\lambda=1,\lambda\neq i}^{m}(a'_{\lambda j}+k_\lambda)$; $j=1, 2, \cdots, n$. The principle results are: 1. If $k_i=0$; $i=1, 2, \cdots, m$ in (1), the complete solution is the n-parameter form $x_j=a_jd_{ij}a'_{ij}$; $j=1, 2, \cdots, n$. 2. A necessary and sufficient condition that (1) have a solution is that $k_i\sum_{j=1}^{n}a_{ij}d_{ij}=k_i$; $i=1, 2, \cdots, m$. 3. The solutions of (1) are those values of the n-parameter form $x_j=a_jd_{ij}(a'_{ij}+k_i)$; $j=1, 2, \cdots, n$, for which the parameter values satisfy $k_i\sum_{j=1}^{n}a_{ij}d_{ij}a_j=k_i$; $i=1, 2, \cdots, m$. Some of the results have been obtained for the finite case by Löwenheim (Math. Ann. vol. 79 (1919) pp. 223–236). (Received June 2, 1950.)

449t. Oscar Goldman: *On the theory of algebraic surfaces.*

Algebraic foundations for a theory of algebraic surfaces without singular points are developed. The considerations are essentially limited to that part of the subject which has direct bearing on the so-called Riemann-Roch Theorem. For the purpose of providing an algebraic proof of this theorem, the theory of double differentials is developed in some detail, enabling one to avoid the necessity of considering the adjoint surfaces. A proof of the birational invariance of the entire theory is included. (Received July 6, 1950.)

450t. B. W. Neumann: *Embedding nonassociative rings in division rings.*

Rings are here understood nonassociative, that is, with associativity of multiplication omitted from the usual ring postulates. Necessary and sufficient condition for a ring to be embeddable in a division ring (no matter whether one-sided or two-sided division be required) is that every nonzero element generate additively a cycle of the same order; this order, if finite, is a prime. The same condition is equivalent with embeddability in an algebra (of possibly infinite rank) over a field. Necessary and sufficient condition for a ring to be embeddable in a ring with *unique* division (on one side or on both sides) is that it possess no zero divisors. A one-sided or two-sided neutral element can be adjoined, and an obviously necessary condition is also sufficient to ensure that such adjunction does not disturb the uniqueness of division. Analogous results hold when multiplication is commutative ($xy=yx$ for all x, y), anticommutative ($xy=-yx$), or idempotent ($xx=x$). The method consists in the adjunction, as freely as possible, of single left quotients, and so forth, together with the usual transfinite building-up process. The author believes the results may be known already, but has been unable to find any references. (Received July 17, 1950.)

Analysis

451t. J. G. Herriot: *The polarization of a lens.*

Consider a conductor placed in an electrostatic field uniform at infinity and having a given direction there. The intensity of the disturbance produced by the conductor may be measured by a quantity called the polarization in the given direction. (See M. Schiffer and G. Szegö, Trans. Amer. Math. Soc. vol. 67 (1949) pp. 130–205.) The mean polarization P_m is the average of the polarizations in any three mutually orthogonal directions. Let V denote volume and C electrostatic capacity. For the bowl (limiting case of lens) the author showed earlier (see Bull. Amer. Math. Soc. Abstract 54-11-470) that $P_m \geq (8\pi/3)C^3$. It is now shown that $P_m \geqq 4\pi C^3$ for the bowl. The inequality $P_m + V \geqq 4\pi C^3$ previously proven for a lens with dielectric angle $\pi/2$ and for two tangent spheres is now established for the symmetric lens. (Received July 20, 1950.)

Topology

452t. A. D. Wallace: *An isomorphism theorem.*

For the Alexander-Kolmogoroff cohomology groups (arbitrary coefficient group)