VIRGIL SNYDER

1869–1950

Professor Virgil Snyder of Cornell University, President of the American Mathematical Society in 1927 and 1928, died on January 4, 1950, at the age of eighty-one. It is appropriate, therefore, to record at this time some account of his services to the Society and of his contributions to mathematics over his active career of forty-three years. Since a biography of Professor Snyder appears in Volume One of the Semicentennial Publications of the Society,[1] the following account is devoted more particularly to his activity in research.

Professor Snyder began his work at a time when geometers were exploring the superstructures of their subject, particularly in space and hyperspace. By adding the radius of a sphere to its coefficients, Lie had defined a sphere by six homogeneous coordinates subject to a non-singular quadratic relation. This situation also occurs with the Plücker line-coordinates so that the parallel between line geometry in three-space and Lie's "Kugelgeometrie" was apparent. Snyder's doctoral dissertation (Göttingen, 1895) was concerned with linear complexes of spheres. Of twenty-one papers he published in the next ten years, twelve were concerned with the metric side of this parallel and dealt with annular, tubular, and developable surfaces, their asymptotic lines, and lines of curvature, or with the development of collateral algebra. In [3] he gives a metric classification of the Dupin cyclide, the envelope of a quadratic system of spheres. The other nine have to do with algebraic ruled surfaces or scrolls, largely from the line geometry, or projective, point of view. After a preliminary check on a classification by Schwartz of quintic scrolls he undertook in three papers [18, 19, 20] in vol. 25 of the American Journal the classification of sextic scrolls according to the multiplicity of directrix curves. This classification yielded 68, 32, 11, 5, 2 types of genus (genus of a plane section) 0, 1, 2, 3, 4, respectively. Two years later in vol. 27, two papers [22, 24] amplified and corrected this classification with reference to earlier works of A. Wiman. This initial preoccupation with line geometry colored much of Snyder's thinking in later years, particularly in connection with a variety of birational and Cremona transformations which he introduced.

[1] Pp. 218–223. To this biography there is appended a bibliography with eighty-five numbered entries. For brevity we use the brackets [ ] to indicate the number of a specific item in this bibliography and add thereto [30A], *On birational transformations of curves of high genus*, AJM, v. 30, 1908, pp. 10–18.

In five papers published in 1907–1908 Snyder turned his attention to the birational geometry of plane curves. In [29] he constructs a curve C_{2n} which has a complete but composite $g^2_{2(n-1)}$, analogous to the hyperelliptic curve, and shows that it is the projection of a space curve on a quadric surface. In [30A] he proves that curves of sufficiently high genus can be transformed into curves of the same order only by collineations, and in [32] the range of possible transforms is explored. In [31] the groups of birational transformations of curves of genus six are obtained. Finally in [33] there is given a proof of the existence of curves of order n with any prescribed number of double points up to the maximum number $(n-1)(n-2)/2$.

The cubic primal in S_4 with nine nodes, and the one with a double line, had been studied synthetically by Segre and Castelnuovo. In [34] and [35], respectively, Snyder treats these cases analytically, thus obtaining more precise results particularly with respect to the surface in S_3 which yields the apparent contour of the primal with respect to a point. It is indeed somewhat characteristic of his work in general that, while he would use synthetic methods for a first approximation, he would not seem to be entirely happy until he had obtained an analytic formulation as a basis for a more comprehensive treatment of individual cases.

In 1910 Snyder opened a long series of articles on surfaces invariant under infinite discontinuous groups of birational transformations. In [36, 39, 40] these groups are generated by involutorial elements such as the projection of a quartic surface into itself from a node, or as the interchange of the two contacts of a properly chosen system of bitangents. The surfaces in general are those which can be obtained as focal surfaces of line congruences, a number of which he had discussed in earlier papers from other points of view.

In the period 1914–1923, Snyder, in collaboration with his colleague F. R. Sharpe, also an accomplished geometer, continued the above series along with other topics in eight memoirs published in the Transactions of the Society. In [53] and [55] the birational groups of quartic surfaces with exceptional curves, either rational curves, or sextics of genus three, or sextics of genus two, are obtained, the birational transformations in the last case being excerpts from Cremona transformations. In these papers a notable increase in elegance and sophistication accrues from the use of a Severi basis for the curves on the surfaces. In [59] the mapping of one three-space on another by means of a web of quadrics is discussed and birational transformations of the jacobian of the web and its related symmetroid are given, again in terms of a Severi basis. These memoirs furnish

interesting examples of the effectiveness of the basis theory for a study of the geometry on a surface. In [58] there is given an exhaustive discussion of (2, 2) point correspondences between two planes which yields two general types with respectively five and six subtypes. Another interesting paper [61] in this group relates to the construction of (n, n') correspondences between two algebraic curves. In the two memoirs [60], the very comprehensive class of Cremona involutions in space which arise from the pairs of a (2, 1) correspondence with another space is completely treated up to the cases where the web of surfaces which defines the correspondence has the order six. In [63] the inverse problem of determining the (2, 1) correspondence when the involution is given is considered in some special cases.

Following a determination [64] of the five types of monoidal Cremona involutions in space corresponding to the five types of ternary Cremona involutions, Snyder, in an address [65] to the Society, presented his reflections on the state (then quite unsatisfactory) of the theory of such involutions in space as contrasted with that of the ternary involutions. A number of his later papers, not reported here, deal with particular involutions. Some of these papers were done in collaboration with graduate students and he inspired a number of others which were published independently.

Two problems seemed to intrigue Snyder in his later years, these relating respectively to the rationality of the pairs of a space Cremona involution and to the rationality of the cubic primal in S_4. His address [70, cf. also 62, 74] as retiring president is devoted to a survey of the literature which bears on these problems. Here [cf. also 79] he frequently indicates the place of his own contributions in the complex situation which he portrays.

One of Snyder's major contributions was made as chairman of the committee of the National Research Council which prepared their Bulletin No. 63, entitled *Selected topics in algebraic geometry*. Of this digest of journal articles up to the date of publication he personally wrote about one quarter of the text and he took on with energy and enthusiasm the entire responsibility for editing and publishing the volume. The supplementary Bulletin, No. 96, covering the next six years, was almost entirely his own work.

Mr. and Mrs. Snyder traveled a great deal both on vacations and on leaves of absence for study and research. On these occasions he attended foreign mathematical meetings and made frequent reports to the Society concerning them. His acquaintance with foreign scholars and with national attitudes made him an especially useful member of the committee which represented the Society at the

Toronto Congress where international tensions resulting from the first world war were still acute [cf. History, pp. 19–20].

Professor Snyder retired from active teaching in 1938. He was fortunate in that he lived to see the Society which he had served so faithfully and so well become one of the leading organizations of its kind.

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