

The serious mathematics begins with the third chapter in which Coxeter introduces the symmetry groups of the Platonic solids. After a full discussion of this important topic, he turns to degenerate polyhedra such as tessellations and honeycombs and their groups. These lead to results of crystallographic importance. Under the heading "The Kaleidoscope" he then describes the discrete groups generated by reflections. The exposition is greatly illuminated by his own "graphical notation" which makes complicated relations self-evident. The treatment of three-dimensional solids closes with a chapter on regular solids which are not quite polyhedra in the strict sense. These are obtained from the Platonic solids by "stellating" (adding pointed solid pieces) or "faceting" (removing solid pieces). This process raises the number of regular polyhedra from five to nine.

The remaining two-thirds of the book is devoted to polytopes of higher dimensions. The general program is similar to that carried out for ordinary polyhedra. It is shown that in four-space there are six regular polytopes and that in n -space ($n > 4$) there are only three regular polytopes. Explicit constructions are given for these and metrical properties are derived. There are even photographs of models of three-dimensional projections of some of these hypersolids. The methods used are increasingly analytic, but the underlying geometry is never lost among the formulae.

I have only one regret about this theory, and Coxeter should not be blamed for this. I refer to the formidable terminology, doubtless invented by mathematicians with a far better education in classical languages than myself. Dry-sounding words like "enantiomorphous" and "great stellated triacontahedron" tend to obscure the geometrical treasures of the subject. This is a place where a judicious use of American slang would greatly improve the situation.

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Mathematical theory of human relations. An approach to a mathematical biology of social phenomena. By N. Rashevsky. Bloomington, Principia, 1947. 14+202 pp. \$4.00.

The name Rashevsky is virtually synonymous with the term "mathematical biophysics," a term which he coined to designate a field created and developed by himself and under his own direction. In this field he is the author of two books, one recently reissued in revised and greatly expanded form, and the editor of a quarterly journal, the *Bulletin of Mathematical Biophysics*.

Professor Rashevsky entered this field by way of a theoretical study of the behavior of liquid drops within which constituents drawn from

the surrounding medium undergo chemical transformation. The analogy to a metabolizing biological cell is obvious. From the "vegetative" cell, he passed quickly to the "sensitive" cell, which has lost the capacity to grow and divide, but undergoes characteristic reversible changes when subjected to external influences of certain types.

The next step was to consider an organized aggregate of cells of either type. Concentrating attention upon the vegetative constituents, he developed a theory of organic form. In considering the sensitive constituents one has a model of the central nervous system, whose structure imposes certain general patterns upon its own activity.

The mathematical theory of human relations applies a similar methodology to the consideration of interacting organisms. Much of the content has appeared over the past 10 years in a series of papers published in *Psychometrika*. The point of departure is the observation that the actions of an individual are dependent jointly upon his own desires and the influence of others. In qualitative terms the observation is as trite as the "law of supply and demand," but with the assignment of numerical parameters and specific functional dependences it yields a tool that can be worked with.

Of course, there are horrible complexities already present at the start. There are many types of activity; there are attitudes toward each ranging all the way from psychopathic aversion to equally psychopathic drive; and there are all degrees of influence, varying with both of the parties and with the activity. These complexities the author sidesteps in characteristic fashion by renouncing any claim to realism, indulging merely in the mathematical exercise of manipulating a few systems of the simplest conceivable type. He considers a single type of activity, usually economic, and generally considers two or three groups, each characterized by only a single coefficient of influence averaged over the group.

Herein is exhibited Rashevsky's cardinal methodological principle. Any approximation is better than none; any guess is better than none, if it is quantitative. Simplify equations until they become manageable, study these, and add complexities as you are able to take care of them. By progressing in this fashion one may hope ultimately to arrive at estimates that are reliable. In the present instance the dichotomies and abstractions accepted as temporary makeshifts are very little different from, and scarcely worse than, those commonly accepted in either the forum or the academy.

In the main course of the development, individuals are classed into two groups, the "actives," having a high coefficient of influence, and the "passives," having a low one. The actives may, themselves, be split on some issue, in which case the passives are found to drift,

ultimately, all into the same party. This is a secular trend: presidential elections are not an example!

In the economic field, the actives are the organizing and directing group, whose skills can greatly enhance the productivity of the passives. For this reason the passives will accept their direction, receiving for their own labors a certain portion of the output. Or one may consider a governing group in addition to the two industrial groups, exacting payment (taxes) from both.

The development becomes most dramatic when consideration is given to the fact that once the social lines are drawn, membership in a social group may be hereditary while the characteristics are not. Over the generations there will be born into each class individuals who are members of this by birth but have characteristics that would place them in the other. Such a course of events may lead, as one illustration, to the eventual passage of control into the hands of the active subclass of the "passive" group, or, as another example, to the shift of control from one group of actives to another. The significant feature of the discussion is not the qualitative conclusion, which is, perhaps, obvious enough, but the possibility of estimating the duration of an epoch.

As the subtitle indicates, this is "an approach" to the problem, rather than a systematic development. The endeavor throughout is to suggest problems for study, to indicate the relevant variables and how they might be measured, rather than to elaborate any single line. A few graphs are presented (some should have been made as scatter plots or bar graphs) to show that the predicted variation goes in the right direction. But the success of the book must be measured ultimately by the extent to which others will pick up and carry on the suggestions in it. For this reason, since its incompleteness is repeatedly pointed out by the author himself, the book is scarcely a subject for legitimate criticism. One may feel that the approach is fruitless, and with this feeling there can be no argument until there are fruits to show.

The type of mathematics can be fairly well guessed in advance. The integral and integro-differential equations are generally pared down by simplifying assumptions to ordinary finite or differential equations. Statisticians may be startled to find essentially no statistical concepts whatever, but this is only an aspect of the early embryonic phase in which the subject is left.

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