BOOK REVIEWS

Finite differences and difference equations in the real domain. By Tomlinson Fort. London, Oxford University Press, 1948. 7+251 pp. $8.00.

The subject-matter of this book, as the title indicates, falls into two general categories; roughly half of the book is devoted to each of them.

In the first category, *finite differences*, are the following topics: difference operators and their elementary properties; the problem (in its most elementary form) of the *sum* of a function; the Bernoulli polynomials and numbers, and the Euler-Maclaurin and Euler summation formulas, and generalizations of all these concepts (the method of generalization is a brief and very inclusive one due to the author); numerical differentiation; interpolation formulas; numerical integration.

The second category, *difference equations in the real domain*, is made up almost exclusively of topics in the theory of the linear *recurrent relation*, that is, of the linear difference equation where the domain of the independent variable is a set of *integers*. The section of the book devoted to these topics consists in large part of published and unpublished research work of the author. For the homogeneous nth-order linear recurrent relation the concept of *fundamental system* of solutions is studied, and for the nonhomogeneous equation the method of variation of constants is described; special techniques for the case of constant coefficients are treated. Study is made of the question of the number of linearly independent solutions of that difference system which consists of a linear recurrent relation together with n linear boundary conditions, and the concept of a *Green's function* for an incompatible system is introduced.

For the *second order* linear recurrent relation, with coefficients dependent on a parameter, a Sturm-Liouville theory is developed. The manner in which the rapidity of oscillation of the solutions varies with the parameter is studied; the existence of characteristic values and characteristic functions is proved; the orthogonality of characteristic functions (orthogonality defined in terms of finite sums instead of integrals) is shown; and the expansion of an arbitrary function in terms of characteristic functions is obtained. Application is made to the problem of the vibration of a mass-less string loaded with n particles, and the vibration of a material string is studied as a limiting case. For second-order recurrent relations with periodic coefficients the question of the existence of periodic solutions is considered in itself and also in connection with the Sturm-Liouville theory. In addition the problem of the existence of unbounded solutions is treated.

One chapter of the book is devoted to the finite analogue of the calculus of variations; a necessary condition analogous to the Euler equation is derived, and sufficient conditions are obtained also.

The final chapter provides a very brief summary (totalling only 14 pages) of some topics in the theory of linear difference equations with *continuous* independent variable. Among these topics are the gamma, digamma, trigamma functions, and so on, the Nörlund sum, and the adjoint equation.

This book is a welcome addition to the literature. In the first place, its presentation of the elements of the calculus of finite differences, being compact and clean-cut, and augmented by numerous exercises, makes it a serviceable and attractive text for a course in finite differences on about the first-year graduate level. In the second place, its treatment of the theory of linear difference equations emphasizes various interesting aspects of which there has been hitherto no unified account.

WALTER STRODT

The differential geometry of ruled surfaces. By Ram Behari. (Lucknow University Studies, no. 18.) Lucknow University, 1946. 94 pp.

This book is the basis of a series of Extension Lectures on *The differential geometry of ruled surfaces in euclidean space of three dimensions*, delivered by the author at the Lucknow University during 1942.

It is a concise and comprehensive survey of the differential geometry of ruled surfaces including ample references to the literature of the subject. The notation adopted is that of ordinary cartesian coordinates as used by Forsyth in his *Lectures on the differential geometry of curves and surfaces*, and by L. P. Eisenhart in his *Differential geometry*, Boston, 1909.

The book consists of six chapters. The first three form an introduction to the subject and the last three contain the investigations of the author.

The first three chapters are concerned with the standard theory of ruled surfaces. Chapter IV is devoted in the first part to the determination of ruled surfaces whose curved asymptotic lines can be found by quadratures. This problem has also been studied by Picard, Buhl, Goursat, Srinivasiengar, Hayashi. A systematic study of the osculating quadrics of a ruled surface is developed. The following new