properties of the solutions and their expansions in Fourier series and the corresponding expansions of solutions that are not periodic; and the expansions in series of Bessel functions and of products of Bessel functions and the asymptotic expansions for large argument values. Powerful methods are given for computing the characteristic numbers and the coefficients in the expansions.

The applications fall naturally into two classes. Parametric resonance in dynamical systems where Mathieu's equation is to be considered the simplest special form of Hill's equation—also treated in Part I; and the solution of two-dimensional problems by separation of variables in elliptic coordinates, described for the wave equation also in Part I. The representative applications in Part II of the first class include treatments of amplitude distortion, frequency modulation, and sub-harmonics; of the second class the vibrations of elliptical membranes, plates, and lakes; eddy currents and thermal diffusion in elliptic cylinders; the propagation of electromagnetic waves in elliptical wave guides; and the diffraction of waves by elliptical cylinders. These, and the other applications given, should enable the reader to apply the theory to new cases as they arise.

The only large part of the theory not given is that connected with the treatment of the equation in terms of the singular points of its algebraic form.

The book is beautifully printed and a pleasure to read; it fulfills a long felt want and will take its place alongside Watson's *Bessel functions* as a necessary part of the equipment of an applied mathematician.

L. H. THOMAS

*Algèbre et analyse linéaires*. By A. Lichnerowicz. Paris, Masson, 1947. 316 pp. 800 fr.

This book is an introduction to linear mathematics for physicists based on lectures given by the author at Strasbourg. It is designed to fill in part the gap left in French scientific literature by the absence of works in French similar to the German treatises of Courant and Hilbert and of Frank and Mises.

The book has two parts, one on linear algebra and one on linear analysis, and each part is divided into four chapters. The first three chapters of Part I and the last three chapters of Part II present the standard elementary material on linear operators and bases in vector spaces with applications to the theory of expansions in orthogonal systems of functions and to the theory of integral equations. The middle two chapters present the algebra and analysis leading up to and including the formulation and proof of the generalized Stokes theorem for multiple integrals.

Keeping in mind his audience of physicists, the author treats his material with somewhat less generality and completeness than he might otherwise consider desirable. Occasionally he seems to go a little too far in this direction. For example one is astonished to find on page 266 a discussion of the spectrum of an operator in Hilbert space with no mention of anything but the point spectrum. Although he accepts the abstract approach to the extent of writing down the axioms for a vector space, he works as a rule with concrete vector spaces and in general uses coordinates and concrete representations more than an uncompromising abstractionist would consider necessary or desirable. On the other hand he usually, if not always, observes mathematical standards of rigor and at times assumes a degree of mathematical sophistication on the part of the reader uncommon among physicists, at least among those educated in the United States.

A more detailed outline of the contents of the book follows. Chapter I of Part I presents the fundamental facts about vector spaces, determinants and the solution of systems of linear equations. Chapter II contains a short treatment of inner products, the Schwartz inequality, the Bessel inequality, Parseval's equation and related matters for finite-dimensional vector spaces. Chapter III deals with linear transformations, the algebra of matrices, bilinear forms, characteristic values and the diagonalization of matrices. Chapter IV treats tensor products of vector spaces, the related algebra of tensors and the Grassman algebra of exterior forms. Chapter I of Part II begins with a treatment of exterior differential forms, their differentials and their integrals over $m$ spreads in $n$-space. The familiar notions of gradient, divergence and curl in vector analysis are exhibited as examples of the differentials of differential forms and the chapter concludes with a proof of the generalized Stokes theorem. The principal topics presented in Chapter II include orthogonality for functions, convergence in the mean, completeness of systems of functions, the Riesz-Fischer theorem, the Weierstrass approximation theorem and its use in proving the completeness of systems of orthogonal polynomials and the trigonometric functions, properties of orthogonal polynomials, and the convergence of Fourier series and integrals. Chapter III deals with the elementary facts about linear operators and infinite matrices associated with vector spaces of functions. Chapter IV contains an exposition of the theory of the Fredholm integral equation.

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