

## THE FEBRUARY MEETING IN NEW YORK

The four hundred thirty-second meeting of the American Mathematical Society was held at The City College, New York, on Saturday, February 28, 1948. The attendance was about two hundred fifty, including the following two hundred fifteen members of the Society:

Milton Abramowitz, C. R. Adams, E. B. Allen, C. B. Allendoerfer, R. L. Anderson, H. A. Arnold, L. A. Aroian, R. N. Ascher, W. A. Asprey, F. E. Baker, S. F. Barber, Joshua Barlaz, F. S. Beckman, F. P. Beer, E. G. Begle, Richard Bellman, Stefan Bergman, P. G. Bergmann, Lipman Bers, W. A. Blankinship, J. H. Blau, R. P. Boas, H. W. Bode, G. L. Bolton, Enrico Bompiani, Paul Brock, A. B. Brown, R. C. Buck, W. R. Callahan, H. H. Campaigne, P. G. Carlson, K. Chandrasekharan, K. K. Chen, K. T. Chen, Herman Chernoff, W. L. Chow, Randolph Church, A. B. Clarke, E. A. Coddington, I. S. Cohen, L. W. Cohen, R. M. Cohn, W. H. H. Cowles, D. A. Darling, F. H. Davidson, M. D. Davis, C. H. Dowker, Y. N. Dowker, Nelson Dunford, Aryeh Dvoretzky, W. F. Eberlein, Samuel Eilenberg, Bernard Epstein, Paul Erdős, R. M. Exner, A. L. Fass, Herbert Federer, J. M. Feld, C. D. Firestone, Irwin Fischer, W. B. Fite, D. A. Flanders, Edward Fleisher, L. R. Ford, R. M. Foster, R. H. Fox, Bernard Friedman, C. S. Gardner, L. L. Gavurin, Abe Gelbart, J. H. Giese, H. E. Goheen, V. D. Gokhale, A. W. Goodman, M. J. Gottlieb, F. G. Gravalos, E. J. Gumbel, M. M. Gutterman, P. R. Halmos, G. J. Haltiner, R. W. Hamming, Gerald Harrison, Philip Hartman, K. E. Hazard, M. H. Heins, G. C. Helme, L. H. Herbach, T. H. Hildebrandt, L. S. Hill, Einar Hille, Joseph Hilsenrath, I. I. Hirschman, A. J. Hoffman, Banesh Hoffmann, E. M. Hull, Witold Hurewicz, Solomon Hurwitz, R. E. Huston, L. C. Hutchinson, E. T. Jabotinsky, R. A. Johnson, R. E. Johnson, F. E. Johnston, A. W. Jones, Mark Kac, Samuel Kaplan, M. E. Kellar, J. R. Kline, E. R. Kolchin, B. O. Koopman, M. S. Kramer, S. H. Lachenbruck, Solomon Lefschetz, R. A. Leibler, Howard Levene, Howard Levi, Samuel Lial, M. A. Lipschutz, E. R. Lorch, Lee Lorch, R. B. Lowe, Myles McConnon, N. H. McCoy, S. W. McCuskey, Janet McDonald, L. A. MacColl, G. R. MacLane, Dis Maly, Murray Mannon, A. J. Maria, M. H. Maria, F. I. Mautner, D. G. Mead, A. N. Milgram, S. Minakshisundaram, W. L. Mitchell, Don Mittleman, C. R. Morris, I. R. Moses, F. J. Murray, D. S. Nathan, C. A. Nelson, Paolo Nesbeda, John von Neumann, Morris Newman, A. V. Newton, P. B. Norman, Paul Olum, A. F. O'Neill, J. C. Oxtoby, Anna Pell-Wheeler, B. J. Pettis, C. R. Phelps, E. L. Post, M. H. Protter, H. W. Reddick, Mina Rees, Irving Reiner, Eric Reissner, J. F. Ritt, M. S. Robertson, S. L. Robinson, J. B. Rosser, Herman Rubin, Charles Saltzer, H. E. Salzer, Arthur Sard, I. J. Schoenberg, Lowell Schoenfeld, Pincus Schub, George Shapiro, H. N. Shapiro, R. H. Shaw, Max Shiffman, S. S. Shü, D. N. Silver, James Singer, P. A. Smith, J. J. Sopka, E. P. Starke, N. E. Steenrod, S. K. Stein, Fritz Steinhardt, Wolfgang Sternberg, F. M. Stewart, J. J. Stoker, E. G. Straus, M. M. Sullivan, Fred Supnick, Otto Szász, Olga Taussky-Todd, J. M. Thomas, D. L. Thomsen, Hing Tong, C. A. Truesdell, A. W. Tucker, Bryant Tuckerman, J. W. Tukey, J. R. Van Andel, W. G. Warnock, Alexander Weinstein, Louis Weisner, E. T. Welmers, J. H. White, G. W. Whitehead, K. B. Whitehead, D. V. Widder, Albert Wilansky, John Williamson, Jacob Wolfowitz, L. C. Young, J. R. Ziegler, J. A. Zilber, H. J. Zimmerberg, O. J. Zobel.

On Saturday morning there were two sections for contributed pa-

pers: one in Analysis, in which Professor E. R. Lorch presided, and one in Algebra, Topology and Applied Mathematics, in which Professor P. A. Smith presided.

At 2:00 P.M. Professor Eric Reissner of Massachusetts Institute of Technology gave an address on *Boundary value problems in aerodynamics of oscillating wings*. Professor L. R. Ford presided.

The Council met at 1:00 P.M. in the Faculty Room of Main Building.

The Secretary announced the election of the following two hundred and nineteen persons to ordinary membership in the Society:

Mr. Asger Hartvig Aaboe, Washington University;  
 Professor Joseph Jesse Abernethy, Texas State University for Negroes, Houston, Tex.;  
 Mr. Richmond G. Albert, Sampson College, Sampson, N. Y.;  
 Mr. Arlo Dean Anderson, University of Oregon;  
 Miss Florence Rosamond Anderson, University of Southern California;  
 Mr. Rodney E. Anderson, Northern Illinois State Teachers College, DeKalb, Ill.;  
 Mr. Silas Stuart Anderson, Ohio University, Athens, Ohio;  
 Mr. Donald Norris Armstrong, Massachusetts Institute of Technology;  
 Natascha Artin (Mrs. Emil), Washington Square College, and Institute for Mathematics and Mechanics, New York University;  
 Dr. Rafael Artzy, The Galilee Secondary School, Tiberias, Palestine;  
 Mr. William George Bade, University of California at Los Angeles;  
 Professor Harry Millard Beatty, The Ohio State University;  
 Mr. Leonard Becker, New York University;  
 Professor Guy Gaillard Becknell, University of Tampa, Tampa, Fla.;  
 Mr. Earl Louis Bell, North Central College, Naperville, Ill.;  
 Mr. Robert Louis Belzer, University of Santa Clara;  
 Professor Robert John Bickel, Drexel Institute of Technology;  
 Mr. Dewey Blair, University of Cincinnati;  
 Dr. Albert Laurence Blakers, Princeton University;  
 Mr. Bartolomé C. Blanco, General Electric Company, Schenectady, N. Y.;  
 Mr. Daniel Block, Yeshiva University;  
 Mr. Francis Ronald Britton, McMaster University, Hamilton, Ontario;  
 Mr. John Ben Butler, 3rd, Franklin and Marshall College;  
 Mr. Lewis Clark Butler, Alfred University;  
 Miss Willie Bee Campbell, Southern University, Baton Rouge, La.;  
 Mr. Philip Noah Cassen, Sampson College, Sampson, N. Y.;  
 Mr. Arthur I. Chalfant, Pratt and Whitney Aircraft Company, East Hartford, Conn.;  
 Mr. Pei Ping Chen, Providence, R. I.;  
 Mr. Jack Chernick, Brookhaven National Laboratory, Upton, N. Y.;  
 Mr. Morris I. Chernofsky, The College of the City of New York;  
 Mr. Allen Bruce Clarke, Brown University;  
 Mr. Robert Francis Cotellessa, Stevens Institute of Technology;  
 Mr. Micha Cotlar, Facultad de Ciencias Exactas, Buenos Aires, Argentina;  
 Mr. Earl Hicks Crisler, West Virginia University;  
 Professor John Cecil Currie, Louisiana State University;  
 Mr. Robert Fred Deniston, Washington University;  
 Mr. Verne Eugene Dietrich, Purdue University;

Professor Helen Walter Dodson, Goucher College;  
Mr. Joseph Vincent Downey, Regis College, Denver, Colo.;  
Mr. Chester Lee Dunsmore, University of California at Los Angeles;  
Dr. Agustin Duranona y Vedia, Instituto de Matematicas de la Universidad de la Plata, Buenos Aires, Argentina;  
Dr. Aryeh Dvoretzky, Einstein Institute of Mathematics, Hebrew University;  
Mr. Sherman Eldon Dyer, University of Texas;  
Mr. Harvey Nelson Ebersole, Lafayette College, Easton, Pa.;  
Miss Myrtle Edwards, Mars Hill College, Mars Hill, N. C.;  
Mr. (Francis) Edward Ehlers, Brown University;  
Capt. Harry Van Horn Ellis, Jr., United States Military Academy;  
Mr. Daniel Jacob Ewy, Bethel College, North Newton, Kan.;  
Professor Basil M. Fedorovsky, St. Joseph's College, Philadelphia, Pa.;  
Mr. Arlin Martin Feyerherm, State University of Iowa;  
Miss Sarah Grace Fleming, University of Alabama;  
Mr. José Salvador Florio, Mendoza, Argentina;  
Mr. Andrew Coulthard Free, State University of Iowa;  
Mr. John Clinton Freeman, Jr., Brown University;  
Professor John E. Freund, Alfred University;  
Professor Arthur Earl Fulton, Georgia School of Technology;  
Mr. Bernard Aaron Galler, University of California at Los Angeles;  
Professor Robert Lee Garrett, Athens College, Athens, Ala.;  
Professor Conklin Blain Gentry, Oakwood College, Huntsville, Ala.;  
Mr. Merton Taylor Goodrich, Keene Teachers College, Keene, N. H.;  
Mr. George Elihu Gourrich, University of California at Los Angeles;  
Mr. John Wagner Graham, Ripon College, Ripon, Wis.;  
Professor Laura Zazle Greene, Washburn Municipal University, Topeka, Kan.;  
Mr. Ronald LeRay Greene, Clarkson College, Potsdam, N. Y.;  
Mr. Robert Todd Gregory, Iowa State College of Agriculture and Mechanic Arts;  
Mr. Emil Grosswald, University of Puerto Rico;  
Professor Blanche Bennett Grover, University of Houston;  
Miss Elizabeth Hahnemann, University of Minnesota;  
Mr. Edmond Emerson Hammond, Jr., Brown University;  
Mr. Arthur Gene Hansen, Purdue University;  
Professor Nathan Warren Harter, Greenville, Pa.;  
Dr. Thomas Watkins Hatcher, Virginia Polytechnic Institute;  
Professor David Hawkins, University of Colorado;  
Mr. Robert Mayo Hayes, University of California at Los Angeles;  
Mrs. Emma Marie Henderson, University of Southern California;  
Professor Charles Haelig Higgins, Monmouth Junior College, Long Branch, N. J.;  
Miss Winifred Adelia Hill, Alabama Polytechnic Institute;  
Miss Ruth Bisgrove Hofstra, Syracuse University;  
Mr. Ernest Ikenberry, Louisiana State University;  
Mr. Wallace Henry Ito, University of Minnesota;  
Professor Paul Frederick Iverson, Potomac State School at West Virginia University;  
Mr. Russell Yaroslav Iwanchuk, St. Basil's College, Stamford, Conn.;  
Mr. Roscislaw Michael Iwanowski, Sweet Briar College;  
Mr. Woodrow Allan Jaffee, Roosevelt College, Chicago, Ill.;  
Mr. John Xavier Jamrich, Marquette University;  
Professor Vojtech Jarnik, Carol University, Praha, Czechoslovakia;

Miss Marion Bruce Jeffries, Blackstone College, Blackstone, Va. ;  
Miss Bessie L. Jones, University of Alabama ;  
Mr. Harold T. Jones, Pacific Union College, Angwin, Calif. ;  
Professor Margaret Eloise Jones, Ohio State University ;  
Miss Reba Merle Jordan, Oklahoma Baptist University, Shawnee, Okla. ;  
Mr. Asher Dale Kantz, Southwestern College, Winfield, Kan. ;  
Mr. Samuel Noah Karp, Brown University ;  
Mr. Gilbert Kaskey, University of Delaware ;  
Mr. Robert Herman Kasriel, University of Virginia ;  
Professor Leo Katz, Michigan State College ;  
Mr. Murray S. Klamkin, Carnegie Institute of Technology ;  
Dean Olaf C. Kjosness, Western State College, Gunnison, Colo. ;  
Mr. Barron George Knechtel, Aurora College, Aurora, Ill. ;  
Miss Louise Murrell Knifley, Marshall College, Huntington, W. Va. ;  
Mr. C. Frederick Koehler, St. Joseph's College, Philadelphia, Pa. ;  
Mr. Sidney Kravitz, Newark College of Engineering ;  
Rev. Benedict P. Kremer, John Carroll University, Cleveland, Ohio ;  
Mr. William Edward Kruse, St. Peter's College, Jersey City, N. J. ;  
Mr. Lamar Layton, University of Illinois ;  
Mr. William Thomas Lenser, University of Nebraska ;  
Miss Marian Amelia Leshner, Fort Scott Junior College, Fort Scott, Kan. ;  
Mr. Charles Hadley Lewis, Bakersfield College, Bakersfield, Calif. ;  
Mr. H. Melvin Lieberstein, Washington University ;  
Mr. Robert Bernard Lowe, Polytechnic Institute of Brooklyn ;  
Mr. Walter Joseph Lyche, Wartburg College, Waverly, Iowa ;  
Professor Myles McConnon, Norwich University, Northfield, Vt. ;  
Miss Betty McKnight, Centenary College of Louisiana ;  
Professor Brown Lee Mackin, East Central State College, Ada, Okla. ;  
Dr. Emilio Antonio Machado, Instituto de Matematicas, La Plata, Buenos Aires, Argentina ;  
Mr. Joseph Simon Mamelak, Carnegie Institute of Technology ;  
Mr. Joseph Anthony Marano, Manhattan College, New York, N. Y. ;  
Miss Margaret Evelyn Martinson, Washburn Municipal University, Topeka, Kan. ;  
Professor José L. Massera, F. de Ingenieria, Montevideo, Uruguay ;  
Miss Rose Mary Miller, Vermont Junior College, Montpelier, Vt. ;  
Professor Christopher Raymond Mitchell, Rhode Island College of Education, Providence, R. I. ;  
Mr. Alex John Mock, Porterville Junior College, Porterville, Calif. ;  
Professor Richard Luther Moenter, Midland College, Fremont, Neb. ;  
Mr. Carlos Andrews de Moraes, Rensselaer Polytechnic Institute ;  
Mr. Robert H. Morris, Color Control Department, Eastman Kodak Company, Rochester, N. Y. ;  
Professor Morris Edward Mosely, Agricultural, Mechanical and Normal College, Pine Bluff, Ark. ;  
Mr. Simon Mowshowitz, University of Bridgeport ;  
Miss Elizabeth M. Muller, University of Bridgeport ;  
Mr. Edgar Raymond Mullins, University of Illinois ;  
Mr. John Paul Murray, Fairfield University, Fairfield, Conn. ;  
Mr. Dennis Mercer Nead, University of Cincinnati ;  
Miss Florence Nusim, Oswego State Teachers College, Oswego, N. Y. ;  
Mr. Edward Franklin Ormsby, Union College, Schenectady, N. Y. ;

Mr. Donald Bruce Owen, University of Washington;  
Mr. Alvin Jewel Owens, State University of Iowa;  
Mr. Carl Ralph Partington, Purdue University;  
Mr. Edward Paulson, University of Washington;  
Mr. Lawrence Edward Payne, Iowa State College of Agriculture and Mechanic Arts;  
Miss Mae Perlestein, New York, N. Y.;  
Mr. Norman Conrad Perry, University of Southern California;  
Professor Hector Alberico Persico, Universidad Nacional La Plata, Buenos Aires, Argentina;  
Mr. Samuel Everett Peters, Long Beach City College, Long Beach, Calif.;  
Mr. Lee Willard Petersen, La Salle-Peru High School, La Salle, Ill.;  
Mr. Raymond Paul Peterson, University of California at Los Angeles;  
Mr. Richard J. Phelps, The Teleregister Laboratories, New York, N. Y.;  
Rev. Albeni Poitras, St. Joseph's University, New Brunswick, Canada;  
Professor Herman Henry Price, Pasadena College;  
Mr. Carl Pride, University of Oregon;  
Professor William Walker Proctor, Morgan State College, Baltimore, Md.;  
Mr. Charles Newton Putt, Kansas State College;  
Mr. (Thomas) Benjamin Ripton, Sampson College, Sampson, N. Y.;  
Mr. Murray B. Ritterman, Long Island University;  
Miss Martha Ethyl Rivers, University of Tennessee;  
Mr. Nathan Henry Rochmes, University of Illinois;  
Miss Marguerite Katherine Roscoe, Montana State College;  
Professor Ralph M. Ross, Rose Polytechnic Institute, Terre Haute, Ind.;  
Dr. William C. Roudebush, New Mexico Military Institute;  
Rev. Charles Harry Rust, St. Louis University;  
Mr. Donald Raleigh Ryan, Gonzaga University;  
Mr. Louis Sacks, Carnegie Institute of Technology;  
Mr. Charles T. Salkind, Brooklyn College;  
Mr. John Wesley Sawyer, University of Missouri;  
Mr. George Van Schlietett, Office of Naval Research;  
Professor Lilah Godfrey Schlotthauer, Walla Walla College, College Place, Wash.;  
Mr. Nicholas Christ Scholomiti, De Paul University;  
Carol S. Scott (Mrs. W. C.), St. Petersburg Junior College;  
Mr. Fariebee Parker Self, Centenary College of Louisiana;  
Mr. Edward I. Shapiro, Brooklyn College;  
Mr. Julius Canoy Shepherd, University of Maryland;  
Mr. Bernard Sherak, Newark Colleges, Rutgers University;  
Miss Miriam Elmore Shi, Alabama Polytechnic Institute;  
Professor Edwin Theodore Sheffield, University of Alberta, Edmonton, Alberta, Canada;  
Professor William Archibald Sherratt, Presbyterian College, Clinton, S. C.;  
Mr. Marvin R. Sitts, Flint Junior College, Flint, Mich.;  
Professor Floyd B. Sloat, Kansas State College of Agriculture and Applied Science;  
Miss Georgia Caldwell Smith, Spelman College, Atlanta, Ga.;  
Mr. Roland Frederick Smith, LeMoyne College, Memphis, Tenn.;  
Mr. Guilford Lawson Spencer, II, Massachusetts Institute of Technology;  
Professor Lois Leavitt Splinter, Nebraska Wesleyan University, Lincoln, Neb.;  
Mr. Philip Charles Stanger, Oklahoma Agricultural and Mechanical College;  
Mr. Marvin Leonard Stein, University of California at Los Angeles;

Mr. Harold William Stephens, University of Florida;  
 Mr. Joel Eric Strandberg, Riverside College, Riverside, Calif.;  
 Mr. George Ralph Strohl, Jr., United States Naval Academy;  
 Professor Edward W. Suppiger, Princeton University;  
 Mr. William Charles Taylor, Jr., University of Tennessee Junior College;  
 Professor William Benson Temple, Agricultural and Mechanical College of Tex.;  
 Professor Ruth Fike Terry, Florida Southern College;  
 Miss Florence Gertrude Tetreault, University of Detroit;  
 Mr. Layton Oscar Thompson, University of Detroit;  
 Mr. Marvin L. Tomber, University of Pennsylvania;  
 Mr. Charles J. Tremblay, Bard College;  
 Miss Evelyn Ladene Trennt, Springfield Junior College, Springfield, Ill.;  
 Mr. Deonisie Trifan, Brown University;  
 Miss Frances Belle Tunstall, Stratford College, Danville, Va.;  
 Mr. Ethan M. Turley, University of Illinois;  
 Miss Lona Lee Turner, University of Chicago;  
 Mr. Veras Dean Turner, University of Illinois;  
 Mr. Harold Robert Uhl, Champlain College, Plattsburg, N. Y.;  
 Professor Louisa Amelia Van Dyke, Flora Macdonald College, Red Springs, N. C.;  
 Mr. Robert Zeno Vause, Jr., Clemson Agricultural College, Clemson, S. C.;  
 Mr. Sumner Ira Vrooman, Rennselaer Polytechnic Institute;  
 Professor Theodore Weaver, Michigan State Normal College, Ypsilanti, Mich.;  
 Mr. Herschel Weil, Brown University;  
 Miss Frances Weisbecker, Milwaukee-Downer College;  
 Mr. Ralph Martin Whitmore, Southwestern University;  
 Professor Mary Elizabeth Wilcox, Southwestern University;  
 Professor Ross Brooke Wildermuth, Capital University, Columbus, Ohio;  
 Mr. John Fox Williams, University of Tennessee;  
 Professor Joseph Bassett Williams, Champlain College, Plattsburg, N. Y.;  
 Mr. Harold Witz, Chicago Technical College;  
 Mr. John George Wozencraft, University of Illinois;  
 Mr. Lawrence Edgar Yancey, Morehouse College, Atlanta University;  
 Professor John Edward Yarnelle, Hanover College, Hanover, Ind.;  
 Mr. Chester Grant Young, University of Santa Clara;  
 Professor Neil Ferguson Young, Emory and Henry College, Emory, Va.;  
 Mr. Ai-ting Yu, Lehigh University;  
 Professor Harold LeRoy Zeiders, Midland College, Fremont, Neb.;  
 Mr. Joshua Vung-yuan Zia, University of Maryland.

It was reported that the following had been elected to membership on nomination of institutional members as indicated:

California Institute of Technology: Messrs. Paolo Gustavo Comba, John Edward Denby-Wilkes, and Daniel Talbot Finkbeiner, Dr. Samuel Karlin, Mr. Jack Enloe McLaughlin.  
 University of Chicago: Messrs. Matthew Page Gaffney, Jr., Clair Eugene Miller, Hugh Brent Prendergast, and Bethuhne Vanderburg.  
 University of Illinois: Mr. Richard Francis Burns.  
 State University of Iowa: Mr. Royal Keith Zeigler.  
 Kenyon College: Mr. Vasile Gh. Gorciu.  
 Purdue University: Messrs. Dean Norman Arden and John S. Lomont.

Stanford University: Mr. Michael I. Aissen, Miss Helen Ann Jackson.  
Wesleyan University: Mr. Frank Dyer Bosworth.

The Secretary reported that a reciprocity agreement had been established between the Society and the Matematisk Forening i København.

Duquesne University (Pittsburgh, Pennsylvania) was elected to institutional contributing membership in the Society.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: London Mathematical Society: Mr. John Todd, Kings College, University of London, and National Bureau of Standards, Washington, D. C.; Matematisk Forening i København: Professor Aksel Frederik Andersen, Danmarks Tekniske Højskole, Copenhagen; Lektor Kai Rander Buch, Danmarks Tekniske Højskole, Copenhagen; Lektor Svend Bundgaard, Matematisk Institut, Copenhagen; Mr. Thoege Busk, University of Copenhagen; Professor Frederik Fabricius-Bjerre, Danmarks Tekniske Højskole, Copenhagen; Dr. Werner Fenchel, Danmarks Tekniske Højskole, Copenhagen; Professor David Føg, Den kgl. Veterinär- og Landbohøjskole; Mr. Anders Hjorth Hald, University of Copenhagen; Mr. Hans Henrik Hansen, Danish Meteorological Institute, Copenhagen; Dr. Svend Edvard Lauritzen, Danmarks Tekniske Højskole, Copenhagen; Professor Jakob Nielsen, The Technical University of Denmark, Copenhagen; Professor Richard Petersen, Danmarks Tekniske Højskole, Copenhagen; Dr. Georg William Rasch, State Serum Institute, Copenhagen; Société Mathématique de France: Professor Albert Chatelet, Faculté des Sciences de Paris; Mr. Lucien Droussent, Ingenieur civil au Ministère de l'Armement, Clermont-Ferrand; Professor André Lichnerowicz, University of Strasbourg; Professor Frederic Amede Emile Roger, University of Bordeaux.

The following appointments by President Einar Hille were reported: Professor B. P. Gill as a member of the Library-Housing Committee, to replace Dr. Warren Weaver; Professor T. F. Cope and Dean A. E. Meder as auditors of the Society's accounts for 1948; Professor L. M. Graves as representative of the Society on the Editorial Board of the American Year Book for the period 1948-50; Professors R. G. Hesel (Chairman), H. M. Beatty, H. M. Gehman, L. H. Miller and C. V. Newsom as a Committee on Arrangements for the 1948 Annual Meeting; Professor Guy Stevenson as representative of the Society at the Sesquicentennial and inauguration of the President of the University of Louisville on February 9-10, 1948; Profes-

sor J. M. Clarkson as representative of the Society at the inauguration of John Decatur Messick as President of East Carolina Teachers College on March 6, 1948; Dean T. M. Simpson as representative of the Society at the inauguration of Joseph Hillis Miller as President of the University of Florida on March 5, 1948; Professor J. A. Clarkson as representative of the Society at the Fifty-second Annual Meeting of The American Academy of Political and Social Science on April 2-3, 1948; Professor E. W. Chittenden as representative of the Society at the Ninetieth Anniversary of the founding of Iowa State College of Agriculture and Mechanic Arts on March 22, 1948.

The Secretary reported that Professors L. M. Graves and O. E. Neugebauer will be the voting members on the Council during 1948 for the American Journal and Mathematical Reviews, respectively.

The resignation of Associate Secretary R. H. Bruck, to be effective after the 1948 Summer Meeting, was reported.

The Secretary reported that Dean A. E. Meder has been appointed as the representative of the Association for Symbolic Logic on the Policy Committee for Mathematics, to succeed Professor Alonzo Church.

For the Committee to Select Hour Speakers for Western Sectional Meetings, it was reported that Professors P. R. Halmos and Saunders MacLane have been invited to deliver hour addresses at the November, 1948, meeting in Chicago.

The election by the Trustees of Professor P. A. Smith as a member of the Board of Trustees to fill the unexpired term of Professor G. W. Mullins (to December 31, 1948) was reported.

The Council approved December 28-30, 1948, as the dates for the Annual Meeting and authorized the President to include December 27, if such action proved necessary.

The Council accepted the following slate of officers and committees for the International Congress of Mathematicians, presented by the Committee on Nomination of Officers and Committees of the International Congress:

*Secretariat:* Professor J. R. Kline, Secretary; Dr. R. P. Boas, Associate Secretary.

*Editorial Committee:* Professors Salomon Bochner (Chairman), Einar Hille, P. A. Smith, Oscar Zariski.

*Financial Committee:* Professors M. H. Stone (Chairman), J. L. Coolidge, Dean M. H. Ingraham, Professors John von Neumann, W. L. G. Williams.

*Organizing Committee:* Professors Garrett Birkhoff (Chairman), W. T. Martin (Vice Chairman), G. C. Evans, J. R. Kline, Solomon

Lefschetz, Saunders MacLane, Dean R. G. D. Richardson, Professors J. L. Synge, Oswald Veblen, J. L. Walsh, D. V. Widder, Norbert Wiener, R. L. Wilder.

President Hille was authorized to appoint a representative of the Society on the Cooperating Committee for the International Congress of Theoretical and Applied Mechanics, to be held in London in 1948.

The Council accepted for publication in the Colloquium Series a volume entitled *Differential algebra* by Professor J. F. Ritt. The Council also voted its approval of Professor G. A. Hedlund's request that Professor W. H. Gottschalk be made a co-author of his Colloquium volume.

The Council voted to recommend to the Board of Trustees that the number of pages in the 1948 volumes of the Transactions be increased to 1600.

A report of the Committee on the Role of the Society in Mathematical Publication, dealing with the possibility of inaugurating a project for the publication of translations of important Russian mathematical articles, was presented. The Council voted as follows: (1) To authorize President Hille to appoint a committee to serve as a committee of selection for articles (in Russian and other languages to be chosen at the discretion of the committee) of sufficient importance and demand to warrant a translation into English and publication thereof and to handle such other details as are necessary to inaugurate the project. (2) To authorize President Hille to appoint a committee to prepare a Russian-English vocabulary of important mathematical terms, supplemented by a pamphlet to include the Russian alphabet, rules of syntax, and short bibliography of works available in parallel Russian and English versions. (3) To recommend to the Board of Trustees that the President and Secretary be authorized to request the Office of Naval Research for \$25,000 for the purpose of carrying out the projects mentioned above.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "t" were presented by title. Paper number 188 was read by Dr. Chandrasekharan and paper number 193 by Dr. Hirschman. Professor Wright was introduced by Professor Antoni Zygmund.

#### ALGEBRA AND THEORY OF NUMBERS

##### 180. I. S. Cohen: *Preservation of descending chain condition.*

Let  $R$  and  $S$  be commutative integral domains with identity,  $R \subseteq S$ . Assume that  $R$  satisfies the ascending chain condition and that every prime ideal is maximal; this is equivalent to the assumption of the descending chain condition modulo every non-

zero ideal. It is proved that if  $S$  is integrally dependent on  $R$  and if the quotient field of  $S$  is finite over that of  $R$ , then  $S$  will also satisfy the restricted descending chain condition. Special cases of this theorem have been proved by Krull, Grell, F. K. Schmidt, and Akizuki. A result recently announced without proof by Grell is shown to be equivalent to the one given here. (Received February 20, 1948.)

181*t.* H. E. Goheen: *An elementary application of the Hamilton-Cayley theorem.*

Using the fact that a square matrix satisfies its own characteristic equation, the author develops the theory of the linear system with constant coefficients,  $de/dt = Ae$ , in which  $A$  is an  $n \times n$  matrix of constants and  $e$  a column vector of variables. The results are believed new in case  $A$  is a singular matrix, but the point of the paper is its use of the Hamilton-Cayley theorem as a tool. Similar methods may be used to discuss the nonhomogeneous system,  $de/dt = Ae + p(t)$ , in which  $p(t)$  is a column vector of functions of  $t$ . (Received January 21, 1948.)

182*t.* L. K. Hua and Irving Reiner: *On the generators of the symplectic modular group.*

Let  $\Gamma$  denote the group of  $2n$ -rowed matrices  $\mathfrak{M}$  with rational integral elements for which  $\mathfrak{M}\mathfrak{S}\mathfrak{M}' = \mathfrak{S}$ , where  $\mathfrak{M}'$  denotes the transpose of  $\mathfrak{M}$  and  $\mathfrak{S}$  is the matrix of the coefficients of the bilinear form  $\sum_{i=1}^n (x_i y_{i+n} - y_i x_{i+n})$ . The structure of the symplectic modular group  $\Gamma_0$  (L. K. Hua, Trans. Amer. Math. Soc. vol. 57 (1945) pp. 441-481) defined as the factor group of  $\Gamma$  over its centrum is investigated. It is shown that there are exactly four (multiplicatively) independent generators of  $\Gamma_0$  if  $n > 1$ , and these generators are found explicitly. (For  $n = 1$ , it is well known that there are two independent generators.) The same method is used to find independent generators of the generalized Picard group  $\Gamma'_0$  obtained by permitting the elements of  $\mathfrak{M}$  to be Gaussian integers, and also to find independent generators of the group of automorphisms of the quadratic form  $\sum_{i=1}^n 2x_i x_{2n-i+1}$ . (Received January 23, 1948.)

183. Eri Jabotinsky: *On Newtonian sequences of polynomials.* Preliminary report.

A sequence of polynomials  $P_n(z)$  is said to be Newtonian if: (1)  $P_0(z) = 1$  and  $P_n(x+y) = \sum_{k=0}^n P_k(x) \cdot P_{n-k}(y)$  for  $n = 0, 1, 2, \dots$ . As particular cases of Newtonian sequences we mention:  $P_n(z) = z^n/n!$ ;  $P_n(z) = C_{z,n}$ ;  $P_n(z) = C_{z+n-1,n}$  and Nielsen's Stirling polynomials suitably normalized. It can be proved that conditions (1) are equivalent to: (2)  $P_0(z) = 1$  and  $\sum_{k=0}^n P_k(-z-k) \cdot P_{n-k}(z+k)/(z+k) = 0$  for  $n = 1, 2, \dots$ . The proof is obtained by constructing the most general Newtonian sequence and using identities of which the following is typical:  $(x+a+b+c+d)^3 - \sum (x+a+b+c)^3 + \sum (x+a+b)^3 - \sum (x+a)^3 + x^3 = 0$ . The particular case of this theorem for the Stirling polynomials is contained implicitly in Nielsen, *Recherche sur les polynômes de Stirling*, Host and Son, Kobenhavn, 1920. Application: Since  $P_n(z) = z(z+n)^{n-1}/n!$  satisfy (2), they also satisfy (1) so that: (3)  $(x+y)(x+y+n)^{n-1} = \sum_{k=0}^n C_{n,k} x(x+k)^{k-1} \cdot y(y+n-k)^{n-k-1}$ , whence, identifying the coefficients of  $xy$ : (4)  $\sum_{k=1}^{n-1} C_{n,k} k^{k-1} \cdot (n-k)^{n-k-1} = 2(n-1)n^{n-2}$ . Identity (3) can also be deduced from a result of Pólya. (Received January 26, 1948.)

184. R. E. Johnson: *Conjugate modules.*

The present paper is the outgrowth of an attempt to examine more closely the

relationship between the right and left primitive ideal structures of a ring  $R$ . If  $M$  is a right  $R$ - and left  $S$ -module, the conjugate,  $C(M)$ , of  $M$  relative to the ring  $S$  is the set of all  $S$ -homomorphisms of  $M$  into  $S$ . The natural definitions of operations make  $C(M)$  into a left  $R$ - and right  $S$ -module. If  $S$  is taken to be a division ring, the elements of  $C(M)$  correspond to the maximal  $S$ -submodules of  $M$ . Finally, by restricting  $M$  to be a simple right  $R$ -module and  $S$  the centralizer of  $R$  relative to  $M$ , every non-zero submodule of  $C(M)$  is shown to have the same  $R$ -annihilator as  $M$ , and thus, if  $C(M)$  has a simple left  $R$ -submodule, the  $R$ -annihilator of  $M$  is both right and left primitive. In case elements of  $R$  have finite nonzero  $S$ -order over  $M$ ,  $C(M)$  is shown to be subdirectly irreducible so that a simple left  $R$ -submodule does exist. (Received January 22, 1948.)

185t. B. H. Neumann: *On ordered division rings*. I, II.

Using results previously presented (Bull. Amer. Math. Soc. Abstract 53-9-305) the first part shows that every ordered group can be imbedded in the multiplicative group of an ordered division ring of formal power series. The method elaborates a proof sketch of O. F. G. Schilling (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 297-304). It follows that the free group of any number of generators can be so embedded, which answers a question raised by R. Moufang (J. Reine Angew. Math. vol. 176 (1937) pp. 203-223). The group ring of the free group of two generators over any field of coefficients is, however, shown to contain two elements without nontrivial common left-multiples. In the second part it is shown that every ordered division ring can be so extended as to contain in its centre a field order-isomorphic to the field of all real numbers. It is also shown that in an ordered division ring an element which is algebraic over the centre lies in the centre. (Received January 20, 1948.)

186t. B. J. Pettis: *Remarks on the extension of lattice functionals*.

The chief result established is the following: a lattice  $L$  is distributive if and only if any bounded modular functional on a sublattice can be extended (mod a constant) to be bounded and modular on  $L$ . (Received February 2, 1948.)

#### ANALYSIS

187t. Richard Bellman: *On the existence and boundedness of solutions of nonlinear partial differential equations of parabolic type*.

The nonlinear partial differential equation of parabolic type is studied. The space variables range over the interior of a cube, and the time variable over  $(0, \infty)$ . The partial differential equation is transformed into a nonlinear integral equation, and the method of successive approximations is applied. Crucial for the method is the result that the solution of the linear problem is a continuous operation on the initial value (the temperature of the cube at  $t=0$ ) under slight restrictions on the type of solution considered. This result is a consequence of an inequality of Eversull. Further applications of the method are indicated. (Received January 24, 1948.)

188. Salomon Bochner and K. Chandrasekharan. *On the localization property for multiple Fourier series*.

If the function  $f(x_1, \dots, x_k)$  is of period  $2\pi$  in each of the  $k$  variables, belongs to  $L_1$  in  $0 \leq x_r < 2\pi$ ,  $r=1, \dots, k$ , and belongs to  $L_2$  in a neighborhood of the point  $(x_1, \dots, x_k)$ , and if  $\int_0^t \{f_p(y)\}^2 dy = o(t)$  as  $t \rightarrow 0$ , then  $\int_0^R \{S^\delta(R)\}^2 dR = o(R)$  as  $R \rightarrow \infty$ , for  $\delta = p + (k-1)/2$ , where  $f_p(t)$  is the spherical mean of order  $p$  of the function  $f$  at

the point  $(x_1, \dots, x_n)$  and  $S^\delta(R)$  is the Riesz mean of order  $\delta$  of the multiple Fourier series of the function (when summed spherically). (Received January 14, 1948.)

189*t*. John DeCicco: *Functions of several complex variables and multiharmonic functions.*

This paper is a brief introduction to the theory of polygenic functions of several complex variables. The importance of the mean and phase derivatives is demonstrated in the study of the Kasner clocks. The author considers how analytic polygenic functions can be extended into spaces of double the original number of dimensions. Applications are made to the theory of monogenic functions. The author proves that if a multiharmonic function (the real or imaginary part of a monogenic function) is rational, algebraic, or entire, then the associated monogenic function is rational, algebraic, or entire. A characterization of multiharmonic functions is studied. Finally, a discussion is given of several polygenic functions of several complex variables. An expression for the jacobian is obtained in terms of the mean and phase derivatives. Application is made to the pseudo-conformal group which may be defined as the group preserving the pseudo-angle of Kasner. (Received December 29, 1947.)

190. Aryeh Dvoretzky. *On changing the signs of the coefficients of Taylor series and of Dirichlet series.*

It is well known that if (1)  $\sum_0 a_n z^n$  has a finite positive radius of convergence  $R$ , then a sequence  $\{\epsilon_n\}$  ( $\epsilon_n = \pm 1$ ;  $n = 0, 1, 2, \dots$ ) can be found such that (2)  $\sum \epsilon_n a_n z^n$  has  $|z| = R$  as a natural boundary. Pólya (C. R. Acad. Sci. Paris vol. 184 (1927) pp. 1526-1528) has shown that when  $R = \infty$  a sequence  $\{\epsilon_n\}$  can be found so that every direction will be a Julia direction for (2), provided that (1) represents a function of infinite order. The author proved (Ph.D. thesis, Jerusalem, 1941) that the above result holds for every transcendental integral function. When information on the growth of the function is available much more can be said, and essentially narrower domains which exhibit the Picard phenomenon given. Also when  $0 < R < \infty$  similar results may be obtained. Thus if  $\limsup |a_n| R^n = \infty$  a sequence  $\{\epsilon_n\}$  can be found so that (2) assumes every value with one exception at most infinitely many times in every sector  $|z - Re^{i\phi}| < \delta$ ,  $\psi_1 < \arg(z - Re^{i\phi}) < \psi_2$  lying inside  $|z| = R$ . These and more general and precise theorems are obtained through the use of the theory of normal families. The methods and results carry over mutatis mutandis to general Dirichlet series. (Received January 22, 1948.)

191. Paul Erdős: *On the strong law of large numbers.*

Kac, Salem and Zygmund proved that if  $f(x+1) = f(x)$ ,  $\int_0^1 f(x) = 0$ ,  $\int_0^1 f(x)^2 = 1$ ,  $n_{k+1}/n_k > c > 1$  and if, for any  $\epsilon$ ,  $\int_0^1 (f(x) - \phi_n(f))^2 = O(1/(\log n)^\epsilon)$ , where  $S_n(f)$  denotes the  $n$ th partial sum of the Fourier series of  $f(x)$ , then for almost all  $x$ ,  $\lim (1/N) \sum_{k=1}^N f(n_k x) = 0$ . It is shown that the conclusion remains true if  $1/(\log n)^\epsilon$  is replaced by  $1/(\log \log n)^{1+\epsilon}$ . On the other hand some restriction on  $f(x)$  is necessary since an  $f(x)$  is constructed satisfying  $f(x+1) = f(x)$ ,  $\int_0^1 f(x) = 0$ ,  $\int_0^1 f(x)^2 = 1$  such that there exists a sequence  $n_{k+1}/n_k > c > 1$  for which  $\lim (1/N) (\sum f(n_k x)) = \infty$  for almost all  $x$ . (Received January 19, 1948.)

192. G. J. Haltiner: *On the theory of linear differential boundary systems when based upon a new definition of the adjoint.*

The author considers the formal aspects of the theory of an ordinary differential

boundary system and of the associated theory of the expansion of an arbitrary function. Specifically, the boundary problem studied is 1.  $\sum_{j=0}^n p_j(x) d^{n-j}u(x)/dx^{n-j} + \lambda q(x)u(x) = 0$ ,  $p_0(x) \equiv 1$ , 2.  $\sum_{j=1}^n \alpha_{ij}(\lambda) u^{(n-i)}(a, \lambda) + \sum_{j=n-1}^{2n-1} \alpha_{ij}(\lambda) u^{(2n-i)}(b, \lambda) = 0$ ,  $i = 1, 2, \dots, n$ , in which the coefficients  $p_j(x)$  and  $q(x)$  are differentiable functions of  $x$  on some closed interval  $(a, b)$ ;  $\lambda$  is a complex parameter; and the  $\alpha_{ij}(\lambda)$  are polynomials in  $\lambda$ . The treatment of the theory is based upon a new definition of the adjoint system introduced and applied to differential systems of order 2 by R. E. Langer. Some of the features in which the present paper differs from previous discussions of the  $n$ th order linear system are: (a) the new definition affords the means for an explicit and inclusive representation of the entire adjoint boundary problem; (b) specific expressions are derived for the solutions of the adjoint system in terms of the solutions and coefficients of the given problem; and (c) in the derivation of generalized orthogonality and in the determination of the residues of the Green's function for use in the expansion theory, no restriction is placed on the index of the characteristic values. (Received January 26, 1948.)

193. I. I. Hirschman and D. V. Widder: *Inversion of a general class of convolution transforms.*

Let  $E(s) = e^{cs} \prod_1^{\infty} [1 - (s/a_k)] e^{s/a_k}$ , where the roots  $a_k$  are real (not necessarily distinct) and such that  $\sum_1^{\infty} a_k^{-2} < \infty$ . It is shown that in any vertical strip of the complex  $s$ -plane that is free of zeros  $a_k$ ,  $1/E(s)$  is the bilateral Laplace transform of some function  $G(t)$ . With this kernel the integral equation  $f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt$  is solved; in fact  $\phi(x) = E(D)f(x)$ , where  $Df(x) = f'(x)$  and  $e^{cD}f(x) = f(x+c)$ . The above convolution of  $G$  with  $\phi$  behaves very differently in the following two cases: (a) the  $a_k$  are all  $> 0$  or all  $< 0$ , (b) there are infinitely many  $a_k$  of each sign. In case (a) it has convergence properties like a Laplace transform; in case (b), like a Stieltjes transform. Hence proofs of the above inversion are different in the two cases. No conditions are imposed on  $\phi(x)$  except those implicitly imposed by the convergence of the improper integral of the transform. The inversion then holds almost everywhere. In particular if  $E(s) = 1/\Gamma(s)$  the convolution reduces to the unilateral Laplace transform, and the inversion becomes the Post-Widder formula. If  $E(s) = \sin \pi s$  we have the Stieltjes transform, and the inversion reduces to a formula due to Widder. All the iterates of the Laplace transform appear as special cases of our theory since the  $a_k$  may be multiple roots. (Received January 21, 1948.)

194t. J. B. Rosser: *The complete monotonicity of certain functions derived from completely monotone functions.*

By means of a generalization of Rolle's Theorem, several theorems are proved, of which the following is an instance. Let  $F(x)$  be completely monotonic and have derivatives for  $0 \leq x \leq \infty$ . That is,  $(-1)^k F^{(k)}(x) \geq 0$  for  $0 \leq x \leq \infty$ . If  $\lambda_i$  and  $C_i$  are constants,  $C_i \geq 0$  so that  $G(x) = F(x) - \lambda_1 \exp(-C_1 x) - \lambda_2 \exp(-C_2 x) - \dots - \lambda_n \exp(-C_n x)$ , and  $G(x)$  and its first  $2n-1$  derivatives all vanish at the origin, then  $G(x)/x^{2n}$  is completely monotonic for  $0 \leq x \leq \infty$ . (Received January 26, 1948.)

195. Arthur Sard: *Best approximate integration formulas; best approximation formulas.*

Let  $\mathcal{A}_{m,n}$  be the set of approximations  $A = c_0 x(0) + c_1 x(1) + \dots + c_m x(m)$  of the integral  $I = \int_0^m x(t) dt$  which are exact whenever  $x(t)$  is a polynomial of degree  $n$ . Assume that  $x(t)$  is a function with absolutely continuous  $n$ th derivative and such that

$x^{(n+1)}(t)^2$  is integrable on  $[0, m]$ . For each approximation in  $\mathcal{A}_{m,n}$ , the remainder can be expressed as  $R[x] = I - A = \int_0^m x^{(n+1)}(t)k(t)dt$ , where  $k(t)$  is an appropriate function. The best formula in  $\mathcal{A}_{m,n}$  is defined as that one for which  $\int_0^m k(t)^2 dt$  is minimal. The best formula leads to the least appraisal of  $|R[x]|$  in terms of  $x^{(n+1)}(t)$ , if Schwarz's inequality is used. The best formulas are given for the cases  $m=1, 2, \dots, 6$ ;  $n=0, 1, 2, 3$ . More generally, let  $\mathcal{R}$  be a set of functionals  $R[x]$ , each of which is linear (additive and continuous) on the space  $C_q$  of functions  $x(t)$  with continuous  $q$ th derivative on  $a \leq t \leq b$  and each of which vanishes whenever  $x$  is a polynomial of degree  $n \geq q$ . For each functional in  $\mathcal{R}$ ,  $R[x] = \int_a^b x^{(n+1)}(t)k(t)dt$ , where  $k(t)$  is an appropriate function. A best functional in  $\mathcal{R}$  is defined as a functional (if any) which minimizes  $|K| \int_K k(t)^2 dt$ , where  $K$  is the set on  $[a, b]$  on which  $k(t) \neq 0$  and  $|K|$  is the measure of  $K$ . (Received January 14, 1948.)

196. F. M. Stewart: *Integration in noncommutative systems.*

Let  $\mathfrak{D}: \lambda, \mu, \dots$  be a topological space with a binary associative operation,  $\cdot$ , such that  $\lambda \cdot \mu \in \mathfrak{D}$  for all  $\lambda, \mu \in \mathfrak{D}$  and is continuous in  $\lambda$  and  $\mu$  jointly. If  $\mu\{t, \sigma\} \in \mathfrak{D}$  is defined for each  $t$  in an interval  $[a, b]$  and each measurable subset,  $\sigma$ , of  $[a, b]$ , Riemann and Lebesgue integrability of  $\mu$  over  $[a, b]$  are defined. If  $\mathfrak{D}$  is the additive group of real numbers with its usual topology and  $\mu\{t, \sigma\} = f(t) \cdot m(\sigma)$  then the Riemann and Lebesgue integrals of  $\mu$  over  $[a, b]$  coincide with the ordinary Riemann and Lebesgue integrals of  $f(t)$  over  $[a, b]$ . Other specializations of  $\mathfrak{D}$  and  $\mu\{t, \sigma\}$  yield the product integrals of Volterra and Birkhoff, the Riemann-Stieltjes integral, and certain integrals of functions with values in a topological linear space. Generalizations of theorems familiar in some or all of these specific cases are proved when the following conditions are imposed on  $\mathfrak{D}$ : (1) The topology in  $\mathfrak{D}$  is given by a metric,  $\rho(\lambda, \mu)$ . (2)  $\mathfrak{D}$  contains a two-sided identity,  $e$ . (3) Given  $\epsilon > 0, B$ , there is a  $\delta > 0$  such that  $\sum_{i=1}^n \rho(\lambda_i, e) < B$  and  $\sum_{i=1}^n \rho(\lambda_i, \mu_i) < \delta$  imply  $\rho(\prod_{i=1}^n \lambda_i, \prod_{i=1}^n \mu_i) < \epsilon$ . (Received January 21, 1948.)

197t. E. M. Wright: *The asymptotic expansion of integral functions and of the coefficients in their Taylor series.*

Let  $f(x)$  be an integral function and let  $f(x+\lambda) = \sum_0^\infty c_\lambda(n)x^n$ ,  $g(t) = t^\beta e^{\psi(t)}(e/\kappa t)^{\kappa t}$ ,  $\mathcal{R}(\kappa) > 0$ ,  $\psi(t) = \sum_1^j a_i t^{b_i}$ ,  $\mathcal{R}(b_i) < 1$ . If either (i)  $c_0(t) \sim g(t)$  for all large, positive, integral  $t$  or (ii)  $c_0(t) \sim g(t)$  in a sector of the  $t$ -plane enclosing the positive half of the real axis, then  $f(x) \sim (2\pi)^{1/2} \kappa^{-1-\beta} X^{1/2+\beta} e^{P(X)}$  for large  $x$  in part (or all) of the  $x$ -plane. Here  $X$  is a suitable  $\kappa$ th root of  $x$  and  $P(X) - X = o(X)$  is a finite sum of fractional powers of  $X$ . It follows from (ii) that  $c_\lambda(n) \sim c_0(n) e^{\lambda \psi(n, \lambda)}$ , where  $\psi(n, \lambda)$  is a calculable polynomial in  $\lambda$  (and is zero if  $\mathcal{R}(\kappa) > 1$ ). The same result follows very simply from (i) if and only if  $\mathcal{R}(\kappa) \geq 1$ . In the above  $h_1(y) \sim h_2(y)$  denotes that  $h_1(y)/h_2(y) = 1 + O(y^{-K})$ , with  $K > 0$  and  $y$  one of  $x, t, n$ . (Received December 2, 1947.)

198. L. C. Young: *Some applications of the Dirichlet integral to the theory of surfaces.*

A vector function  $x(u, v)$  in the closed unit square  $R$  of  $u, v$  is termed Dirichlet representation ( $N$ ) if  $x(u, v)$  is absolutely continuous on almost every line parallel to the axes of  $u$  and  $v$ , and its Dirichlet integral is less than  $N$ . A Fréchet surface so representable is a Dirichlet surface ( $N$ ), or a Dirichlet surface when  $N$  is not specified. Dirichlet surfaces ( $N$ ) actually coincide with Dirichlet surfaces of Lebesgue area less than  $N$ . Use is also made of intrinsic area. An  $\epsilon$ -modification of  $x(u, v)$  is an  $\bar{x}(u, v)$  co-

incident with  $x(u, v)$  outside an open set on which the total intrinsic area of  $\bar{x}(u, v)$  is less than  $\epsilon$ . An  $\epsilon$ -modification of a surface  $S$  is defined similarly. Actually a Dirichlet representation  $x(u, v)$  of Lebesgue area less than  $A$  possesses for each  $\epsilon > 0$  an  $\epsilon$ -modification  $\bar{x}(u, v)$  of intrinsic area less than  $A$ . One main theorem states: a sequence of Dirichlet representations  $N$ , equi-continuous on the boundary of  $R$ , can be  $\epsilon$ -modified into Dirichlet representations equi-continuous in  $R$ . It is deduced that given a sequence of surfaces  $S_n$  with bounded Lebesgue areas, and with (say) simple boundary curves converging to a simple curve, there exists, for each  $\epsilon > 0$ , a compact sequence of  $\epsilon$ -modifications. (Received February 12, 1948.)

APPLIED MATHEMATICS

199t. Stefan Bergman: *On linear partial differential equations with singular coefficients. I.*

In the present paper solutions of the equation  $L(\psi) = \psi_{Z\bar{W}} + F\psi = 0$ ,  $Z = Z_1 + iZ_2$ ,  $W = Z_1 - iZ_2$ ,  $Z_k = x_k + iy_k$ ,  $k = 1, 2$ , are considered. Let  $F$  be regular in  $B^4$ . Then the singularities of  $\psi$  can lie only on characteristic planes  $Z = c = \text{const.}$ , or  $W = c$ . To every  $Z = c$  (or  $W = c$ ) there exist functions possessing, on  $Z = c$ , singularities of the same order of infinity and the same behavior as in the case of harmonic functions. Suppose  $F$  is regular in  $B^4 = R^4 - \sum_{\mu=1}^n S_\mu^2$  where  $R^4 = \{|Z| < \infty, |W| < \infty\}$ , and  $S_\nu^2$ ,  $\nu = 1, 2, \dots, n$ , are singularity surfaces of  $F$ . To every point  $P_0(Z_0, W_0) \in B^4$  there exists a system of solutions  $\psi_\nu = P(Z - Z_0)^\nu$ ,  $\psi_{2\nu+1} = P[(W - W_0)^\nu]$ ,  $\nu = 0, 1, 2, \dots$ ,  $P(s) = s - \int_{Z_0}^s \int_{W_0}^w F s dz_1 dw_1 + \int_{Z_0}^s \int_{W_0}^w F \int_{Z_0}^{z_1} \int_{W_0}^{w_1} F s dz_2 dw_2 dz_1 dw_1$ , each of which is regular in  $B^4$  and such that every solution  $\psi$  can be developed in a sufficiently small neighborhood of  $P_0$  in the form  $\psi = \sum \alpha_\nu \psi_\nu$ . If  $P_0$  completes a closed circuit along a curve  $C^1$  which cannot be reduced to a point without cutting  $S_\mu^2$ , then the  $\psi_\nu$  undergo a transformation  $\mathcal{A} = \{\alpha_{m,n}\}$ ,  $m = 1, 2, \dots, n = 1, 2, 3, \dots$ . Suppose  $\{\chi_\nu\}$  is another complete system of solutions of  $L$  and  $\mathcal{B}$  is its transformation matrix of the  $\chi_\nu$  corresponding to  $C^1$ . Then  $\mathcal{B} = \mathcal{D}^{-1} \mathcal{A} \mathcal{D}$ , where  $\mathcal{D}$  is a suitably chosen matrix. (Received February 27, 1948.)

200t. Stefan Bergman: *On linear partial differential equations with singular coefficients. II.*

The author considers solutions of the equation  $\psi_{Z\bar{W}} + F\psi = 0$ ,  $Z = Z_1 + Z_2$ ,  $W = Z_1 - iZ_2$ , where  $F$  is an analytic function of two complex variables  $Z_k = X_k + iY_k$ ,  $k = 1, 2$ , which is regular in  $N^4 - S_1^2 - S_2^2$  and single-valued in  $N^4$ . Here  $N^4 = \{|Z_k| < P\}$ ,  $P$  sufficiently small,  $S_k^2 = [Z_k = \sum_{\nu=1}^{\infty} \alpha_\nu^{(k)} Z_{2-k}^\nu]$ . The problem of the determination of functions which are multiplied by a factor  $\lambda_k$  when the variable makes a closed circuit  $C_k^1$  around  $S_k^2$  can be reduced to the determination of functions  $f(Z)$  and  $g(W)$ , satisfying a system of two integral equations, first of which is  $(\lambda_1 - 1)f(\zeta) + (\lambda_1 - 1)g(\tau) - (\lambda_1 - 1) \cdot \int_{Z_2}^{\zeta} v[Z, \tau; Z; W] f'(Z) dZ + (\lambda_1 - 1) \int_{W_2}^{\tau} v(\zeta, W; z, w) g'(W) dW = \int_{\phi=0}^{\phi=2\pi} [v(A, \tau; z, w) f'(A) + i\psi(\zeta, B; z, w) g'(B)] \rho e^{i\phi} d\phi$  and the second has a similar structure. Here  $v(\zeta, \tau; z, w)$  is the "generalized Riemann's function,"  $A = (\rho \cos \phi - Y^{(0)}) + i(\rho \sin \phi + X_2^{(0)})$ ,  $B = (\rho \cos \phi + Y_2^{(0)}) + i(\rho \sin \phi - X_2^{(0)})$ ;  $\rho, X_2^{(0)}, Y_2^{(0)}$  are suitably chosen constants, and  $\lambda_k$  are multiplication factors to be determined from the equations. The author considers two special cases, namely  $F = (WZ)^{-1} C_1 + W^{-1} C_2(Z) + Z^{-1} C_3(W) + C_4(Z, W)$ , and  $F = Z_1^{-2} C_4 + Z_2^{-2} C_5 + Z^{-1} C_6(Z_2) + Z_2^{-1} C_7(Z_1) + C_8(Z, W)$ ,  $C_n$  being regular functions or constants. In the first case the author obtains characteristic solutions of the form  $W^\rho Z^{C_1/\rho} \sum \sum B_{mn} W^n Z^n$ , where  $\rho$  and  $B_{00}$  are arbitrary and in the second case  $Z_1^r Z_2^s \sum \sum B_{mn} Z_1^m Z_2^n$ , where  $r$  and  $s$  are determined from the "indicial equations." In

the second case a denumerable subset of the  $B_{mn}$  are arbitrary. (Received February 27, 1948.)

201. Lipman Bers: *On the continuation of a potential gas flow across the sonic line.*

Let there be given a two-dimensional potential subsonic gas flow occupying some domain  $D$ . Let  $G$  be a continuously curved boundary arc of  $D$  along which the Mach number (ratio of local speed to local speed of sound) attains the value one. It is shown that if the velocity components possess continuous partial derivatives on  $G$ , and if neither the normal derivative of the speed nor the tangential component of the velocity vector vanish along this curve, then the flow may be continued across  $G$  as a potential supersonic flow without shocks or weak discontinuities. The supersonic flow is uniquely determined in some neighborhood of  $G$ . The exact domain of dependence as well as the flow itself can be effectively computed. (The domain of dependence comprises the entire supersonic region in the case of a transonic flow past an obstacle.) The proof of these statements depends upon the solution of a Cauchy problem for a linear partial differential equation of mixed type with Cauchy data on the critical line. A new treatment of this problem is given. It is believed to be simpler and more convenient than the previous ones due to Khristianowitch and Frankl (see Bull. Acad. Sci. USSR. vol. 8 (1944) pp. 195-245). (Received January 19, 1948.)

202t. A. D. Fialkow and Irving Gerst: *The transfer function of an L-network.*

The following theorem is proved: The necessary and sufficient conditions that  $A(p)$  be the transfer function of a passive L-network containing resistance and capacitance only are: (1)  $A(p) = K \prod_{i=1}^N (p + \delta_i) / \prod_{j=1}^M (p + \gamma_j)$  where the  $\gamma_j$  are distinct positive numbers and the  $\delta_i$  are non-negative real numbers and  $M = N$  or  $M = N + 1$ . (2) The number of  $\delta$ 's less than a particular  $\gamma_j$  ( $j = 1, 2, \dots, M$ ) is equal to or one greater than the number of  $\gamma$ 's less than that  $\gamma_j$ . (3) The number  $K$  satisfies the inequalities  $0 < K \leq K_0$  where  $K_0$  is the least of the three quantities  $K_d$ ,  $\prod_{j=1}^M \gamma_j / \prod_{i=1}^N \delta_i$ , 1 if  $M = N$  and of the first two quantities if  $M = N + 1$ . Here  $K_d$  is the least positive root (if it exists) of the equation in  $K$  obtained by equating the discriminant of  $\prod_{j=1}^M (x + \gamma_j) - K \prod_{i=1}^N (x + \delta_i) = 0$  to zero. (Received February 5, 1948.)

203t. A. D. Fialkow and Irving Gerst: *The transfer function of a ladder network.*

The transfer function of a four terminal network is the ratio of the voltage at the output terminals to the voltage at the input terminals. The necessary and sufficient conditions that  $A(p)$  be the transfer function of any ladder network containing resistance and capacitance only are determined. An algorithm for the synthesis of the network corresponding to  $A(p)$  is given. Analogous results obtain for networks which contain resistance and inductance only or inductance and capacitance only. These theorems are applied to various engineering design problems. (Received February 5, 1948.)

204. F. G. Gravalos: *Two new theorems in hydrodynamics and their technical applications.*

Instead of Euler's well known formula  $H_2 - H_1 = \omega(Cu_2r_2 - Cu_1r_1)$  for the energy input (or output) in the flow about a compressor (or turbine) the first theorem gives

$H_2 - H_1 = -\int_{t_1}^{t_2} U \cdot \omega r dt$  where  $U$  is the whirl component of the aerodynamic reaction. The second formula describes the flow *inside* the rotor and applies to *any* two points. Consequently it is shown that  $\omega(Cu_2r_2 - Cu_1r_1) = -\int_{t_1}^{t_2} U \cdot \omega r dt$  and this amounts to a generalization of Euler's formula as obtained in the ordinary way. The second theorem gives a general expression,  $(1/\rho) \text{grad}_{u_1} p = (1/H_1)(1/2)(\partial/\partial u_1 H_2^2)u_2 + (1/H_1)(1/2)(\partial/\partial u_1 H_3^2)u_3$ , for the pressure gradient along the  $u_1$ -direction in terms of intrinsic geometric elements of the coordinate surface of revolution  $u_1 = \text{constant}$ . As a consequence of these two theorems the necessary conditions for the determination of the flow about a compressor (or turbine) of general shape are determined and a method for the calculation of the characteristics of the flow follows together with new relations between the energy changes and the variation of axial velocity. (Received February 24, 1948.)

205. C. A. Truesdell: *The energy theorem for Newtonian continua.*

A formula for the rate of change of the kinetic energy of an arbitrary finite material volume of a Newtonian continuum has been deduced. The general formula is specialized to the case of compressible viscous fluids, generalizing the classical theorem of Bobileff for incompressible viscous fluids. It is shown that in a viscous compressible fluid the vorticity and the rate of change of the density play completely separate and exactly analogous roles in the viscous dissipation of energy. (Received February 2, 1948.)

#### GEOMETRY

206t. Edward Kasner and John DeCicco: *Physical families of curves in space.*

The authors investigate the geometric character of a system of twisted curves  $S_h$  of space connected with an arbitrary positional field of force. Such a system  $S_h$  consists of curves along which a constrained motion is possible such that the osculating plane at each point contains the force vector and the pressure is proportional to the normal component of the force vector. There are four cases of physical interest: (1) The system  $S_0$  of trajectories. (2) The system  $S_{-2}$  of brachistochrones (in a conservative field of force). (3) The system  $S_1$  of general catenaries. (4) The system  $S_\infty$  of velocity curves. After discussing various geometric properties of the general systems  $S_h$ , the transformation theory of such systems  $S_h$  is given. (Received December 29, 1947.)

#### STATISTICS AND PROBABILITY

207. E. J. Gumbel: *Normal extremes.*

The numerical values of the probability functions, averages and moments of the extreme values taken from a normal distribution have been calculated, mainly by Tippett, up to sample sizes  $n = 1000$ . The distribution of these normal extremes converges toward the double exponential distribution  $f(x) = \exp[-\exp(-y)]$  where  $y = \alpha(x - u)$ . Here,  $u$  is the expected largest value for a sample of  $n$ , and  $\alpha$  the corresponding intensity function. The calculated mean (median) differs from the asymptotic mean (median) by 0.6 per cent of the calculated value for  $n = 1000$  ( $n = 10$ ). The asymptotic distribution can be used for relatively small sample sizes by an adequate estimate of the parameters, either from the definitions, or from the calculated distribution of the extremes. Both procedures lead to practically identical results for  $u$ . The probability paper shows that the asymptotic distribution of the normal ex-

tremes may be applied for  $n=365$  up to a 95 per cent level, a fact observed by Baricelli. The probability paper gives a criterion for the regularity of observed normal extremes. The theoretical straight line may be obtained from the means and standard deviations of the observations and of the reduced values  $y$ , the latter depending exclusively upon  $n$ . The modal  $m$ th extremes are obtained from the mode of the extreme through a recurrent procedure. The distribution of the intervals between extremes converges toward an exponential function. (Received February 2, 1948.)

### TOPOLOGY

208*t*. E. E. Floyd: *On the extension of homeomorphisms on regions.*

Let  $P$  be a region in a compact space  $A$  and let  $Q$  be a subset of a compact space  $B$ . Let  $f$  be a continuous map from  $\bar{P}$  to  $\bar{Q}$  such that  $f|P$  is a homeomorphism from  $P$  onto  $Q$ . A study is made of the action of the map  $f|F(P)$ ,  $F(P)$  the boundary of  $P$ , when various restrictions are placed on  $Q$ . In the first place, if  $Q$  is a region of an  $n$ -sphere  $S_n$ , then the map  $f|F(P)$  is shown to be non-alternating. Secondly, attention is given to the case in which  $Q$  is  $ul\ i-c$ ,  $0 \leq i \leq k$ . If  $P$  is also  $ul\ i-c$ ,  $0 \leq i \leq k-1$ , then  $f|F(P)$  is  $n$ -monotone. If  $P$  is  $ul\ i-c$ ,  $0 \leq i \leq n$ , then a necessary and sufficient condition that  $Q$  be  $ul\ i-c$ ,  $0 \leq i \leq n$ , is that  $f|F(P)$  be  $n$ -monotone. (Received December 4, 1947.)

209. R. H. Fox: *On the imbedding of polyhedra in 3-space.*

By a *tubular figure* is meant a regular neighborhood of a finite (but not necessarily connected) linear graph. Consider a system of  $m$  non-intersecting surfaces of genus  $h_1, \dots, h_m$  polyhedrally imbedded in spherical 3-space  $S$ . It was shown by Alexander (Proc. Nat. Acad. Sci. U.S.A. vol. 10 (1924) pp. 6-8) that in the case  $m=1$ ,  $h=0$  each of the two (closed) complementary domains is a tubular figure and in the case  $m=1$ ,  $h=1$  at least one of them is. In the general case, although there need be no tubular figures among the complementary domains, each complementary domain is homeomorphic to the complement of some tubular figure. By means of this partial extension of Alexander's theorem necessary conditions for a given polyhedron of dimension not greater than 3 to be a polyhedral subset of 3-space may be derived. For example, from a theorem about 3-dimensional manifolds due to Reidemeister (Monatshefte für Mathematik und Physik vol. 43 (1936) pp. 20-28) one obtains the following theorem: If the first Betti number of a connected polyhedral subset of  $S$  is greater than 2 then its fundamental group is non-abelian. (Received January 21, 1948.)

210*t*. Deane Montgomery: *Dimension of factor spaces.*

Let  $G$  be a locally compact topological group of finite dimension and let  $H$  be a closed abelian subgroup, then the dimension of  $G/H$  is finite. (Received December 24, 1947.)

211*t*. Deane Montgomery: *Subgroups of locally compact groups.*

Let  $G$  be a locally compact connected group of dimension  $n$ . If  $H$  is an  $n$ -dimensional closed subgroup of  $G$ , then  $H$  is all of  $G$ . (Received December 24, 1947.)

212*t*. Hing Tong (National Research Fellow): *Note on minimal bicom pact spaces.* Preliminary report.

In his *Treatise on set topology*, Part I (Madras, India, 1947) R. Vaidyanathaswamy

raised the question whether there exist minimal bicomact non-Hausdorff spaces (loc. cit. Preface). It is not difficult to see that a minimal bicomact space is necessarily a  $T_1$ . On the other hand, it can be shown readily that the space  $R$  constructed below is minimal bicomact non-Hausdorff: The points of  $R$  are the symbols  $\alpha_1, \alpha_2$  and all ordered pairs  $(m, n)$  of natural numbers. A neighborhood system of  $R$  consists of all sets of single point  $(m, n)$ , all sets of the form  $\{\alpha_1$  and all points of the form  $(m_{2i+1}, 2i+1)$  for  $m_{2i+1} > M_{2i+1}, i=0, 1, 2, \dots\}$ , and all sets of the form  $\{\alpha_2,$  all points of the form  $(m_{2i}, 2i)$  for  $m_{2i} > M_{2i}, i=1, 2, 3, \dots,$  and all points of the form  $(m, n)$  for  $n > N\}$ . Other properties of minimal bicomact spaces are also discussed. (Received January 24, 1948.)

213. G. W. Whitehead: *On spaces aspherical in dimensions less than  $n$ .*

Let  $X$  be a pathwise connected topological space whose  $i$ th homotopy group  $\pi_i(X)$  vanishes for  $i < n$ . If  $n > 2$ , superposition on the essential element of  $\pi_{n+1}(S^n)$  defines a homomorphism  $\eta: \pi_n(X) \rightarrow \pi_{n+1}(X)$ . Let  $H_k(X)$  be the integral singular homology group of  $X$ ,  $S_k(X)$  the group of spherical homology classes. Then if  $n > 2$ ,  $H_{n+1}(X) \approx \pi_{n+1}(X)/\text{Image } \eta$ , and  $H_{n+2}(X)/S_{n+2}(X) \approx \text{Kernel } \eta/2\pi_n(X)$  if  $n > 3$ . If moreover  $\pi_{n+2}(X) = 0$ , then  $H_{n+3}(X)/S_{n+3}(X) \approx H_{n+1}(X)/2H_{n+1}(X) + {}_2(\pi_n(X))$ . By means of these results the groups  $H_{n+k}(\pi, n)$  defined by Eilenberg and MacLane (Ann. of Math. vol. 46 (1945) pp. 480-509) are calculated for  $k \leq 3$  and  $n > 3$ . (Received January 5, 1948.)

J. R. KLINE,  
*Secretary*