THE NOVEMBER MEETING IN ST. LOUIS

The four hundred twenty-ninth meeting of the American Mathematical Society was held at Washington University, St. Louis, Missouri, on Friday and Saturday, November 28–29, 1947. All sessions were held in Brown Hall. The total attendance was about 150, including the following 115 members of the Society:

V. W. Adkisson, J. J. Andrews, W. L. Ayres, Reinhold Baer, R. W. Ball, M. A. Basoco, P. O. Bell, Herman Betz, H. D. Block, L. M. Blumenthal, D. G. Bourgin, H. R. Brahana, P. B. Burcham, R. H. Bruck, A. V. Bushkovitch, R. H. Cameron, J. E. Case, Abraham Charnes, Harold Chatland, H. J. Cohen, Byron Cosby, M. E. Daniells, M. M. Day, Hubert Delange, J. L. Doob, T. L. Downs, Otto Dunkel, John Dyer-Bennet, J. D. Elder, Benjamin Epstein, Paul Erdös, D. H. Erkiletian, G. M. Ewing, J. W. Gaddum, R. E. Gaskell, B. E. Gatewood, Michael Golomb, S. H. Gould, Cornelius Gouwens, L. M. Graves, R. E. Graves, Franklin Haimo, Marshall Hall, O. H. Hamilton, Charles Hatfield, F. F. Helton, G. A. Herr, D. L. Holl, L. A. Hostinsky, F. B. Jones, L. M. Kelly, Marie Kernaghan, L. A. Knowler, J. C. Koken, Max Kramer, M. Z. Krzywoblocki, Joseph Landin, J. S. Leech, Solomon Lefschetz, Walter Leighton, A. J. Lorenz, W. S. Loud, Szolem Mandelbrojt, C. W. Mathews, Kenneth May, S. W. McCuskey, W. C. McDaniel, J. V. McKelvey, R. J. Michel, Josephine Mitchell, E. E. Moise, F. V. E. Morfoot, K. L. Nielsen, Rufus Oldenburger, Isaac Opatowski, Gordon Pall, S. T. Parker, P. M. Pepper, B. J. Pettis, G. B. Price, M. O. Reade, Francis Regan, T. M. Renner, P. R. Rider, R. E. Roberson, W. H. Roever, Arthur Rosenthal, R. G. Sanger, A. C. Schaeffer, H. M. Schaerf, Robert Schatten, K. C. Schraut, M. G. Segner, M. E. Shanks, Marlow Sholander, Annette Sinclair, M. F. Smiley, G. W. Smith, C. E. Springer, R. H. Stark, M. H. Stone, E. B. Stouffer, H. P. Thielman, W. J. Thron, E. A. Trabant, H. L. Turritin, F. E. Ulrich, G. B. Van Schaack, W. A. Vezeau, Bernard Vinograde, G. L. Walker, S. E. Walkley, S. E. Warschawski, M. F. Willerding, L. C. Young.

At 10:00 A.M. Friday Professor Marshall Hall of the Ohio State University delivered an hour address entitled *Foundations of geometry—retrospect and prospect.* Professor Reinhold Baer presided. At 9:00 A.M. Saturday Professor Szolem Mandelbrojt of the College de France and the Rice Institute delivered an hour address entitled *Analytic continuation and infinitely differentiable functions—a general principle*, with Professor L. C. Young presiding. Two sessions for short research papers were held at 11:15 A.M. Friday, and additional such sessions at 2:00 P.M. Friday and at 10:15 A.M. Saturday. Presiding officers for these sessions were, respectively, Professors Gordon Pall, L. M. Blumenthal, M. H. Stone, and G. B. Price.

An informal gathering of midwestern applied mathematicians was arranged for lunch on Friday for the purpose of a preliminary discussion of the future needs and plans for applied mathematics in the Midwest. It was the sense of the group that no specific conclusions should be reached at that time. A committee of three midwestern applied mathematicians was elected by those present, with instructions to consider, in cooperation with the Committee on Applied Mathematics of the American Mathematical Society, what questions, if any, might warrant further discussion by a more representative group of midwestern applied mathematicians, and to initiate such a discussion if deemed desirable.

Those attending the meetings were the guests of Washington University at a very successful tea, held at 4:00 P.M. Friday in the Women's Building. At 6:30 P.M. Friday, a dinner was held in the dining room of McMillan Hall, with an attendance of 112. Professor J. E. Case was toastmaster. Assistant Dean Graham welcomed the Society on behalf of Washington University. Dean Stouffer expressed the appreciation of the Society for the hospitality of Washington University, and warmly commended Professor P. R. Rider and the other members of the Local Committee on Arrangements for their very efficient work.

Abstracts of all papers presented at the meeting are given below. Papers read by title are indicated by the letter "*t*." Paper number 48 was presented by Professor Chatland, number 54 by Mr. Block, number 58 by Professor Ulrich, number 72 by Mr. Kelly, and number 82 by Dr. Moise.

ALGEBRA AND THEORY OF NUMBERS

47*t*. A. A. Albert: *Power-associative rings*. II.

A power-associative algebra A over a field F is trace-admissible when there is a function $t(x, y)$ on A, A to F such that $t(x, y) = t(y, x)$, $t(xy, z) = t(x, yz)$, $t(x, y) = 0$ if xy is nilpotent, $t(e, e) \neq 0$ if e is idempotent. When the radical is defined to be the maximal nilideal a trace-admissible algebra has the usual structure properties. An algebra A is called a standard algebra if the identities $z(xy) + y(zx) + (zy)x = z(yx) + (zx)y + (yz)x$, $w[x(yz)] + [(wy)x]z + [(wz)x]y = (wx)(yz) + (wy)(xz) + (wz)(xy)$ are satisfied. All associative and Jordan algebras are standard. All standard nilagebras are solvable and all solvable standard algebras strongly nilpotent. If the field is nonmodular standard algebras are trace-admissible and the radical of A is the radical of the Jordan algebra $A^{(+)}$. For every algebra A there is an attached algebra $A(\lambda)$ defined by the product $x \cdot y = \lambda xy + (1 - \lambda)yx$ for λ in F. Then $A^{(+)} = A(1/2)$. The author calls A over F quasiassociative if there exists an extension K such that $A_K = B(\lambda)$ for B associative and λ in K. Then either A is associative or $\lambda^2 - \lambda$ is in F. He calls A J-simple if $A(1/2)$ is a simple Jordan algebra, and shows that all flexible J-simple algebras are either the exceptional Jordan algebra or quasiassociative. In particular, every simple standard algebra is either associative or a Jordan algebra. (Received October 13, 1947.)

48. Harold Chatland and H. B. Mann: *On extensions of domains of integrity.*

Let I, I' denote domains of integrity with unit element. The authors call I' an extension of I if $I' \supset I$ and if $p = qr$, p, q, in I, r in I' imply r in I. Two elements a, b of I are called coprime in I if $d|a$, $d|b$ implies $d|1$. The following theorems are proved. Theorem 1. If a, b in I are coprime in every extension of I then there exists elements x, y in I such that $ax+by=1$. Theorem 2. To every I there exists an extension I' such that every ideal in I is the intersection of a principal ideal of I' with I. Theorem 3. If any two elements in I have a greatest common divisor in I then $(a, d) = 1$, $(b, d) = 1$ implies $(ab, d) = 1$. Received October 20, 1947.

49. Paul Erdös: *On additive number theoretic functions.*

Let $f(n)$ be an additive number theoretic function, $|f(p^\alpha)| < c$ for all p and α. Previously the author proved that the density of integers for which $f(n+1) > f(n)$ is $1/2$. Now it is shown that the necessary and sufficient condition that for every k the density of integers with $f(n+k-1) > f(n+k-2) > \cdots > f(n)$ equals $1/k!$ is that $\sum_p (f(p))^2/p$ diverges. Also the necessary and sufficient condition that for every k the inequalities $f(n+k-1) > f(n+k-2) > \cdots > f(n)$ have a solution is that $\limsup f(n) = \infty$. The condition $|f(p^\alpha)| < c$ can be weakened, but $f(n) = \log n$ shows that $f(p^\alpha)$ must be restricted in some way. It is not yet clear what the necessary condition is. (Received October 18, 1947.)

50t. Franklin Haimo: *Preservation of divisibility under homomorphisms.*

To each element of an additive abelian group G, attach the collection of its heights with respect to the primes (Prüfer, Math. Zeit. vol. 20 (1924) pp. 165–187). Under a homomorphism, these heights will tend to increase. H, a subgroup of G, is said to be a *Liapin subgroup* of G if each element of the factor group G/H has the same set of heights as one of its antecedents in G. (Cf. Liapin, Rec. Math. (Mat. Sbornik) N.S. vol. 8 (1940) pp. 205–237). It is proved that if G is a torsion-free abelian group, every subgroup of which is Liapin, then G is the direct sum of copies of the additive group of rationals. Finite ordered subgroups are always Liapin in any containing group; and sequentially compact subgroups of division-closure groups are Liapin. In particular, closed subgroups of compact groups are Liapin; whence it follows that direct sums of copies of the integers can never be compact groups. (Received October 21, 1947.)

51. Gordon Pall: *The minimum of an indefinite, binary, quadratic form.*

The first two Markoff minima of an indefinite, real binary quadratic form are obtained simultaneously by a simple method. (Received October 24, 1947.)

52t. H. J. Ryser: *Rational vector spaces.*

Two finite-dimensional rational inner product spaces are said to be equivalent if their rational inner products determine the same class of positive definite matrices (Bull. Amer. Math. Soc. Abstracts 52-9-267, 53-1-21). Let V_n be an arbitrary rational inner product space of dimension n and let M_n denote the set of all polynomials of degree less than n with rational coefficients. There exists a rational inner product $\int_0^\infty f(x)g(x)d\psi(x)$ for M_n such that M_n is equivalent to V_n. The function $\psi(x)$ is nondecreasing with $\psi(0) = 0$, has an infinite number of points of increase on $0 \leqq x < \infty$. Every rational inner product space that is denumerable and infinite-dimensional has a basis whose elements are orthogonal and of length one. Thus there exists a set of polynomials $Q_1(x)$, $Q_2(x)$, $\cdots$ with rational coefficients such that every polynomial with rational coefficients is expressible as a finite sum $\sum_{i=1}^{n} a_i Q_i(x)$ where the a's are rational, and, moreover, $\int_{-1}^{1} Q_i(x)Q_j(x)dx=\delta_{ij}$, $i, j=1, 2, \cdots$. Results on algebraic number fields are extended. It is shown that every abelian field is a subfield of a cyclotomic I-field, and for every integer $n>2$, there exists an infinitude of abelian I-fields of degree n. (Received October 29, 1947.)

53. M. F. Smiley: *Application of a radical of Brown and McCoy to non-associative rings.*

The theory of the F_1-radical of Brown and McCoy (Amer. J. Math. vol. 69 (1947) pp. 46–58) is extended to arbitrary non-associative rings. After certain fundamental properties of non-associative rings are established, the proofs given in the associative case require no essential change. In particular, *a non-associative ring R has zero F_1-radical if and only if it is isomorphic to a subdirect sum of simple non-associative rings with unit elements and the sum is direct if the descending chain condition holds for the* (*two-sided*) *ideals of R.* For a non-associative algebra of finite order *which has a unit element*, this radical coincides with the radical defined by A. A. Albert (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 891–897). For a hypercomplex alternative ring (Max Zorn, Ann. of Math. (2) vol. 42 (1941) pp. 676–686), this radical reduces to that defined by Zorn and therefore also to that defined by N. Jacobson (Amer. J. Math. vol. 67 (1945) pp. 300–320, and the author's abstract, *The radical of an alternative ring*, Bull. Amer. Math. Soc. Abstract 53-11-373). (Received October 20, 1947.)

ANALYSIS

54. H. D. Block and H. P. Thielman: *Note on commutative polynomials.*

Two functions $f(x)$, $g(x)$ are said to be commutative with each other if $fg=f[g(x)]=gf=g[f(x)]$. An "entire set of commutative polynomials" is a set of polynomials which contains one of every positive degree, and which is such that any two polynomials of the set are commutative with each other. A linear transformation of a function $g(x)$ is defined as $\lambda g\lambda^{-1}$, where λ is a linear function and the operation is substitution. It is shown in this paper that the only entire sets of commutative polynomials are linear transformations of x^n, and of $\cos(n \arccos x)$. (Received October 23, 1947.)

55. D. G. Bourgin: *An approximate isometry.*

If T is a generalized ϵ isometry of the Banach space $C(Q)$ of bounded continuous functions onto $C(Q_1)$ where Q_2 and Q are completely regular spaces, then there is an isometry within $k\epsilon$ of T. This extends a theorem of Hyers and Ulam who require that T be a homeomorphism and that Q_1 and Q_2 be compacta. In particular, the theorem implies that if Q_1 and Q_2 are Hausdorff compact, then the existence of a generalized ϵ isometry on $C(Q_1)$ onto $C(Q_2)$ implies the topological equivalence of Q_1 and Q_2. (Received November 24, 1947.)

56*t*. D. G. Bourgin: *Some operators.*

Operators of Riesz type are developed for constant coefficient partial differential equations for other than the normal hyperbolic or elliptic cases. For instance, the parabolic equation is included. (Received October 24, 1947.)

57. R. E. Graves: *A generalization of the Riemann-Stieltjes integral appropriate to the Wiener space C.* Preliminary report.

Let $f(t)\in L_2$ on $[0, 1]$, let $\{\alpha_k(t)\}$ $(k=1, 2, \cdots)$ be a complete orthonormal set on $[0, 1]$, each of whose elements is of bounded variation, and let $x(t)$ be an element of the Wiener space C. Define $c_k=\int_0^1 f(t)\alpha_k(t)dt$. The generalized Stieltjes integral $(G)\int_0^1 f(t)dx(t)$ is defined to be $\lim_{n\to\infty}\int_0^1\sum_1^n c_k\alpha_k(t)dx(t)=\sum_1^\infty c_k\int_0^1\alpha_k(t)dx(t)$. The author proves that the limit on the right exists for all $x(\cdot)\in C$ except for a set of Wiener measure zero. He proves further that any two generalized Stieltjes integrals defined with different sets of $\alpha_k(t)$'s agree for all $x(\cdot)\in C$ except for a set of Wiener measure zero. The paper concludes with a proof that the generalized Stieltjes integral is a true generalization in the sense that if $f(t)$ is of B.V., so that the ordinary Stieltjes integral exists for all $x(\cdot)\in C$, then the generalized Stieltjes integral agrees with the ordinary Stieltjes integral for all $x(\cdot)\in C$ except for a set of Wiener measure zero. (Received October 20, 1947.)

58. Szolem Mandelbrojt and F. E. Ulrich: *Concerning regions of flatness for holomorphic functions and their derivatives.*

Let $C(\alpha, R)$ denote the circle $|z-\alpha|<R$. Suppose $f(z)$ is holomorphic and different from zero in each circle $C(\alpha, R)$, where α and R vary over a set S of values (α complex, R real and positive). For a given $0<\eta<1$, let $M(\alpha, R; \eta)=\text{Max } |f(z)|$ for $z\in C(\alpha, \eta R)$, $m(\alpha, R; \eta)=\text{Min } |f(z)|$ for $z\in C(\alpha, \eta R)$. Let $L(\alpha, R; \eta)=\text{Min }[\log M(\alpha, R; \eta)/\log m(\alpha, R; \eta), M(\alpha, R; \eta)/m(\alpha, R; \eta)]$. If to each $0<\eta<1$ there corresponds a finite positive number $A(\eta)$ such that $L(\alpha, R; \eta)<(A\eta)$ for all α and R belonging to S, the circles $C(\alpha, R)$ will constitute regions of flatness for $f(z)$. This definition is somewhat of the same character as that proposed by J. M. Whittaker. The authors have proved the following theorem: Suppose the circles $C(\alpha, R)$ are regions of flatness for the function $f(z)$ and $f'(z)\neq 0$ in any of these circles. If there exist three positive constants A, B, and d such that $R|[\log_2 f(\alpha)]'|>A$, $\log|f(\alpha)|>BR$ and $R\geqq d$, then the circles $C(\alpha, R)$ are regions of flatness for $f'(z)$. (Received October 22, 1947).

59*t*. Szolem Mandelbrojt and Norbert Wiener: *Infinitely differentiable functions on the half line.*

The authors prove a theorem which asserts that an odd, infinitely differentiable function on the half line is identically zero if an infinite sequence of odd derivatives are zero at the origin and if the maxima of the nth derivatives grow suitably (the function and its derivatives are supposed bounded on the half line). The statement is along the same lines as previous theorems by S. Mandelbrojt. The proof differs from those given by the latter for his theorems in that the authors employ an auxiliary function, holomorphic in the half plane with an argument corresponding to an order of differentiation, real or complex, with non-negative real part, and a value given by the value of the derivative of that order at the origin of the infinitely differentiable function. (Received October 22, 1947.)

60*t*. Morris Marden: *The number of zeros of a polynomial in a circle.*

Let $f(z)=\sum_0^n a_k z^k$ and $f^*(z)=z^n f^-(1/z)$. Let the sequence $f_j(z)=\sum_0^{n-j}a_k^{n-j}z^k$ be defined by the recursion formula $f_{j+1}=\bar{a}_0^{(j)}f_j(z)-a_{n-j}^{(j)}f_j^*(z)$, $f_0(z)=f(z)$, $j=1, 2, \cdots, n$. As proved in the present note, if $\delta_j=f_j(0)\neq 0$ for $j=1, 2, \cdots, n$, then the number of zeros of $f(z)$ in the unit circle is equal to the number of negative P_j among the products $P_j=\delta_1\delta_2\cdots\delta_j$, $j=1, 2, \cdots, n$. Furthermore, if there is a $k<n$ such that $f_{k+1}(z)\equiv 0$ and $P_j\neq 0$ for $j=1, 2, \cdots, k$, then $f(z)$ has $n-k$ zeros on the unit circle and a number of zeros in the unit circle equal to the number of negative P_j for $j=1, 2, \cdots, k$. By use of the first theorem, the Schur-Cohn determinant criterion (Math. Zeit. vol. 14 (1922) pp. 110–148) for the number of zeros in the unit circle is given an elementary proof not involving the theory of Hermitian forms, as previous proofs. (Received October 24, 1947.)

61. P. M. Pepper: *Algebraic harmonic functions.*

Using Bergman's integral operator method the author gives sufficient conditions on the coefficients of the development $h=\sum_{\mu=0}^{+\infty}\sum_{\nu=-\mu}^{\mu}A_{\mu\nu}R^{\mu}P_{\mu}^{(\nu)}(\cos\theta)e^{i\nu\phi}$, in spherical harmonics, in order that h be an algebraic harmonic function. In terms of the $A_{\mu\nu}$, the expressions $a(\mu, \nu)=A_{\mu,\nu-\mu}2^{\mu}\nu!/\{(2\mu)!i^{\mu-\nu+1}\}$ for $0\leqq\nu\leqq 2\mu$ and $a(\mu, \nu)=0$ otherwise, are defined. For non-negative integers n and σ determinants $\Gamma_{n\sigma}(\mu_1, \cdots, \mu_\omega; \nu_1, \cdots, \nu_\omega)$ of order $\omega=(n+1)(\sigma+1)$ are formed in which the element of the kth row and the lth column is $a(\mu_k-q_l, \nu_k-r_l)$ where $q_l=[l/(\sigma+1)]$ and $r_l=l-(\sigma+1)q_l$, the brackets denoting the greatest integer less than or equal to the quantity within them. A sufficient condition for h to be algebraic is that there exist four non-negative integers m, n, ρ, σ such that the determinant relations $\Gamma_{n\sigma}(\mu_1, \cdots, \mu_\omega; \nu_1, \cdots, \nu_\omega)=0$ hold for each selection of 2ω integers $\mu_1, \cdots, \mu_\omega; \nu_1, \cdots, \nu_\omega$ for which, in each pair (μ_k, ν_k) either $\mu_k>m$ or $\nu_k>\rho$ (or both). A derivative process of generating harmonic functions is given. New types of singular lines of algebraic harmonic functions are given. (Received October 23, 1947).

62. B. J. Pettis: *Interiority and continuity of homomorphisms between metric groups.*

Necessary and sufficient conditions are given that a homomorphism ϕ between two metric groups X and Y be interior and that it be continuous. Defining ϕ to be permutative if transformations $[S_\alpha]$ and $[T_\beta]$ exist in X and Y respectively such that for each $x\in X$ and each α there is some β such that $\phi(S_\alpha(x))=T_\beta(\phi(x))$, where $[S_\alpha]$ and $[T_\beta]$ have certain topological properties, it follows that if ϕ is permutative and has a closed graph and X is complete then ϕ is interior. A dual definition leads to a dual theorem giving sufficient conditions that ϕ be continuous. From these two theorems there immediately follow, with somewhat weaker hypotheses, the well known interiority and continuity theorems of Schauder, Banach, Freudenthal, and Dunford for spaces of type (F) and for metric groups with separability assumptions. (Received October 21, 1947.)

63. G. B. Price: *Researches in the theory of functions of several real variables.* II. *Derivatives of functions.*

The Bertrand-Peano derivative of a function $F_0^{(n)}$ at $X_0^{(n)}$ is defined to be $\lim \Delta[F^{(n)}; X_0^{(n)}X_1^{(n)}\cdots X_n^{(n)}]/\Delta[X^{(n)}; X_0^{(n)}X_1^{(n)}\cdots X_n^{(n)}]$, the limit being taken as $X_1^{(n)}, \cdots, X_n^{(n)}$ tend to $X_0^{(n)}$ subject to suitable restrictions. The denominator of the fraction is an increment of the independent variable $X^{(n)}:(x_1, \cdots, x_n)$, and the numerator is the corresponding increment of the function $F^{(n)}:(f_1, \cdots, f_n)$. The definition is sufficiently broad to include ordinary partial derivatives and Jacobians in one treatment; it leads to significant advances in the theory of differentiation. This paper develops the theory of BP-derivatives under the following section headings:

sets of increments; BP-derivatives; BP-derivatives and factorable sets of increments; increments of BP-differentiable functions; Stolz differentiable functions; functions with derivatives of dimension 1; functions with derivatives of dimension greater than 1 only; continuity of BP-differentiable functions; derivatives of composite functions; directional derivatives; other limits for derivatives; functions with zero derivatives; linear spaces of differentiable functions; and history and the literature. (Received August 29, 1947).

64t. H. M. Schaerf: *On unique invariant measures.*

Let a measure space in the group G be the union of a sequence of measurable sets of finite measure. Under an assumption weaker than Weil's condition M there are proved (1) the *structure theorem*: If the measure is unique invariant then any measurable set A, whose measure is not greater than that of a measurable set B or equal to it, is almost congruent by finite or denumerable partition respectively with some measurable subset of B or with B; (2) a necessary and sufficient condition for the uniqueness of an invariant measure. The second result furnishes a simple proof for Weil's theorem on the uniqueness of an invariant measure under Weil's condition M and a new similar theorem. Extensions to measures invariant under a transitive group of transformations are indicated. (Received August 13, 1947.)

65t. W. J. Thron: *Twin convergence regions for continued fractions* $b_0+K(1/b_n)$. II.

Let two regions B_0 and B_1 in the complex plane be defined as follows: $re^{i\theta}\in B_0$ if and only if $r\geqq f(\theta)$, $re^{i\theta}\in B_1$ if and only if $r\geqq 4/f(\pi-\theta)$, where $f(\theta)>0$ and moreover $f(\theta)$ is such that the complements of the regions B_0 and B_1 are both strictly convex (that is, the curvature of the boundary is never equal to zero). Then B_0 and B_1 are "best twin convergence regions" for continued fractions of the form $b_0+K(1/b_n)$. This is an improvement over an earlier result of this nature proved by the author (Amer. J. Math. vol. 66 (1944) pp. 428–438). The proof depends on the fact that the regions B_0 and B_1 are of the desired type if and only if $f(\theta)=f_0\cdot\exp\left(\int_{\pi/2}^{\theta}\tan\alpha(\phi)\,d\,\phi\right)$, where $\alpha(\theta)$ has period 2π and satisfies $|\alpha(\theta)|<\pi/2$, $\alpha'(\theta)<1$. (Received October 23, 1947.)

66. L. C. Young: *Area and length.*

The current definitions of area and length have objections which have been repeatedly emphasized, particularly by T. Radó. Moreover, they are still restricted to parametric images of very special figures and they lack analogy with one another and with volume or hyper-volume. Intrinsic definitions, free from these drawbacks, are proposed. For the topological image of a plane set, the new area agrees with the Carathéodory-Hausdorff 2-dimensional measure μ. More generally, consider a "multiplicity-function" $M(x)$ where x denotes a point of the space in which the surface S is situated, and where $M(x)$ takes integer values, positive on S. The new area of S is then the sum $\sum_n\mu(E_n)$ where E_n is the set in which $M(x)\geqq n>0$. Its definition depends on the choice of $M(x)$ and in fact two such multiplicities are introduced: a crude one which leads to the "Banach" area, and a more satisfactory one (the "Morrey" multiplicity) which leads to the "intrinsic area." Various theorems connect these, and corresponding definitions of length, with classical definitions: thus the intrinsic area is not less than the classical double integral and agrees with the latter in the case of a Lipschitzian representation. (Received October 18, 1947.)

Applied Mathematics

67*t*. D. G. Bourgin: *A boundary problem for the damped wave equation.*

This work extends the class of problems for which explicit solutions can be obtained. The data are given in implicit form on two nonparallel time-like straight lines and a space-like third side. The method in part involves co-images and Laplace transforms. The paper will appear in the Quarterly of Applied Mathematics. (Received October 24, 1947.)

68. Abraham Charnes: *Wing-body interaction in linear supersonic flow.*

The problem treated is to determine the steady uniform irrotational supersonic flow about a wing symmetrically affixed to a body. With the usual linearizing assumptions, if ϕ_1 is the velocity potential for flow about the wing in the absence of the body, ϕ_2 is sought such that $\phi_1+\phi_2$ is the potential for the flow about the wing plus the body. Bodies considered are circular and polygonal cylinders. In the former case ϕ_2 is obtained as a Fourier series involving inverse Laplace transforms of ratios of modified Bessel functions and their derivatives. With certain restrictions on ϕ_1, which the known ϕ_1 satisfy, convergence of ϕ_2 and appropriate derivatives is rigorously established. (Received October 24, 1947.)

69. M. Z. Krzywoblocki: *On the two-dimensional steady turbulent flow of a compressible fluid far behind a solid symmetrical body.* II. *Vorticity transfer theory.*

In the theory of incompressible turbulent flow it is the opinion that the velocity distributions obtained by the momentum transfer theory or the vorticity transfer theory are the same. Whereas the temperature distributions based on the vorticity transfer theory fit the results obtained from tests better than the distributions based on the momentum transfer theory. The question arises whether these statements are true for a compressible turbulent flow. To answer this question, the author solved the entire system of equations based on both theories. The first part contained the set of equations based on the momentum transfer theory (Bull. Amer. Math. Soc. Abstract 53-11-409). In the present paper the second part was solved. The following conclusions were drawn: (a) the velocity distributions based on both theories are the same only to the approximation of the first order. Beginning from the approximation of the second order there are increasing discrepancies between both velocity patterns, (b) the temperature distribution is different in both cases beginning from the approximation of the first order, (c) the density and pressure distributions are different in both cases beginning from the first approximation different from zero, that is, approximation of the second order. (Received September 15, 1947).

70*t*. H. E. Salzer: *Coefficients for expressing the first thirty powers in terms of the Hermite polynomials.*

Exact expressions are given for x^n, $n=0, 1, 2, \cdots, 30$, in terms of the Hermite polynomials $H_m(x)\equiv(-1)^m e(x^2)(d^m/dx^m)e(-x^2)$, where $e(x)\equiv e^x$. The computation followed a formula in a paper by E. Feldheim and an overall functional check was performed to guarantee the accuracy of the final manuscript. This table is useful for approximating a polynomial of high degree by a polynomial of much lower degree, which is best in the least square sense where the interval is $[-\infty, \infty]$ with weight factor $e(-x^2)$, by virtue of the well known property that if $q_r(x)$ denotes the partial sum of degree r in the expansion of a function $f(x)$ in terms of Hermite polynomials, and $r_r(x)$ is any polynomial of degree $\leqq r$, distinct from $q_r(x)$, then $\int_{-\infty}^{\infty} e(-x^2)[f(x)-q_r(x)]^2dx < \int_{-\infty}^{\infty} e(-x^2)[f(x)-r_r(x)]^2dx$. (Received October 22, 1947.)

Geometry

71. R. W. Ball: *Dualities of finite projective planes.* Preliminary report.

A duality δ of a finite projective plane is a one-to-one correspondence of a point P (line h) to a line P^δ (point h^δ) such that P is on h if and only if h^δ is on P^δ. If N is the number of absolute points (that is, points P such that P is on P^δ), in a finite plane, $N \equiv 1 \bmod p$ for all primes p that divide n (where there are $n+1$ points on every line). Hence every duality of a finite plane possesses at least one absolute point. If $2o(\delta)$ is the order of δ and if the progression $1+x2o(\delta)$ contains at least one prime p with $(n/p) = -1$, then $N = n+1$. And if $N \neq n+1$ and n not a square, then $o(\delta)$ is not 1, 2, or a power of a prime of the type $4k+3$. (Received October 21, 1947.)

72. L. M. Blumenthal and L. M. Kelly: *New metric-theoretic properties of elliptic space.*

In two recent papers elliptic spaces of finite and infinite dimensions were characterized metrically among the class of semimetric spaces (Trans. Amer. Math. Soc. vol. 59 (1946) pp. 381–400) and criteria for the superposability of congruent subsets were obtained (to appear in Trans. Amer. Math. Soc.). The present paper continues the development of metric-theoretic properties of elliptic space and is principally concerned with the remaining fundamental problem of determining a congruence order of n-dimensional elliptic space with respect to sub-classes of semimetric spaces. Several such theorems are proved. The principal result obtained is that the elliptic plane has congruence order 9 with respect to the class of all separable semimetric spaces. The intrinsic difficulty of these problems is due to the presence in every elliptic space (of dimension exceeding 1) of congruent but not superposable subsets. (Received October 12, 1947.)

73. M. O. Reade: *On the boundary of minimal surfaces.* Preliminary report.

Classic results due to Carathéodory, Fatau, Seidel, et al. (see Seidel, Math. Ann. vol. 104 (1931) pp. 182–243) are used to obtain information concerning the boundary of minimal surfaces. A typical result is the following. Let $\mathfrak{X} = \mathfrak{X}(u, v)$, $u^2+v^2<1$, define a regular minimal surface $\mathcal{M}$, and let $\zeta = \zeta(u, v) \equiv \zeta(z)$ be the normal vector to $\mathcal{M}$. If $\zeta_3(u, v) \leqq 0$, $u^2+v^2<1$, then $\lim \zeta(z)$, $z \to e^{i\theta}$, exists for almost all θ; here z approaches $e^{i\theta}$ along a path not tangent to the unit circle $|z| = 1$. (Received October 24, 1947.)

74. Marlow Sholander: *Contributions to the theory of convex curves.*

Results representative of those contained in this paper are stated below. The midpoints of certain chords of a closed convex curve C are used to define a curve M which has the following property: If P is a point inside C and not on M, if the index of P with respect to M is m, and if the number of pairs of points on C which have P as midpoint is n, then $2m+1=n$. The set of all points which divide the members of a family of parallel chords of C in a fixed ratio form a curve which is called a cross curve of C. Cross curves derived from the same family of chords are said to be parallel. For all C which contain at most one linear segment it is shown that (i) two non-parallel cross curves have one and only one point of intersection, (ii) two points inside C lie on one and only one cross curve. (Received October 27, 1947.)

75. E. A. Trabant: *Some special Riemann spaces.*

The following two theorems are proved. Theorem 1: A three-dimensional Riemann space with element of arc-length $ds^2=g_{ij}dx^idx^j$, $i, j=1, 2, 3$, and the components of the fundamental metric having the form $g_{11}=g_{22}=g_{33}=Af(x')$, $g_{23}=g_{32}=Bf(x')$, $g_{12}=g_{21}=g_{13}=g_{31}=0$ and $|A|\neq|B|$ may possess constant Riemannian curvature of value b if and only if $B=0$ and will possess a zero curvature invariant if and only if $f(x')=((Cx'+D)/k)^k$, where $(3A^2-2B^2)k=8(A^2-B^2)$ and C, D are arbitrary constants. Theorem 2: In a n-dimensional Riemann space with element of arc-length $ds^2=g_{ij}dx^idx^j$ where $g_{ij}=\delta^i_jf(x)$, $\delta^i_j=1$ if $i=j$, $\delta^i_j=0$ if $i\neq j$ and $i, j=1, 2, \cdots, n$, a necessary and sufficient condition in order that the space possess constant Riemannian curvature of value b, be an Einstein space, and may be mapped conformally on a flat space is that $f(x)=K^{-1}(x+C)^{-2}$ where $K=-b$, C is an arbitrary constant and $x=x^1, x^2, \cdots$, or x^n. (Received October 22, 1947.)

Statistics and Probability

76. Benjamin Epstein: *The role of the Mellin transform in the study of the distribution of the product and quotient of independent random variables.*

It is well known that the Fourier transform is a powerful analytical tool in studying the distribution of the sums of independent random variables. In this paper it is pointed out that the Mellin transform is a natural analytical tool to use in studying the distribution of products and quotients of independent random variables. Formulae are given for determining the probability density functions of the product $\xi\eta$ and the quotient ξ/η where ξ and η are independent positive random variables with probability density functions $f(x)$ and $g(y)$, in terms of the Mellin transforms $F(s)=\int_0^\infty x^{s-1}f(x)dx$ and $G(s)=\int_0^\infty y^{s-1}g(y)dy$. An extension of the transform technique to random variables which are not everywhere positive is given. A number of examples including Student's t-distribution and Snedecor's F-distribution are worked out by the techniques of this paper. (Received September 15, 1947).

77t. Dorothy J. Morrow: *On the distribution of the sums of the characteristic roots of a determinantal equation.*

Consider p variates $x_1, \cdots, x_p$ distributed with a multivariate normal distribution. Let $X=\|x_{i\alpha}\|$ and $X^*=\|x^*_{i\beta}\|$ $(i=1, \cdots, p; \alpha=1, \cdots, n_1; \beta=1, \cdots, n_2)$ be observation matrices of two samples of $x_1, \cdots, x_p$ with n_1 and n_2 degrees of freedom and covariance matrices $S=\|XX'\|$ and $S^*=\|X^*X^{*\prime}\|$ respectively. Then the determinantal equation $|S^{-1}S^*-\lambda|=0$ has p roots, $\lambda_1, \cdots, \lambda_p$. Define T_0^2/n_2 to be the sum of the roots. Under the condition that n_1/n_2 is large, the cumulative distribution function of T_0^2 is expressed in series form. The first four terms of the series are found exactly by a method which extends to the general term, the series is shown to be alternating and convergent, and an upper bound for the remainder after k terms is given. The statistic, T_0^2, provides a test of the significance of the difference between covariance matrices for two samples and therefore has many practical applications. It is an extension of Hotelling's generalized T^2 test. (Received October 20, 1947.)

78t. D. N. Nanda: *Certain results on the roots of a determinantal equation.*

If $x=\|x_{ij}\|$ and $x^*=\|x_{ij}^*\|$ are two p-variate sample matrices with n_1 and n_2 degrees of freedom and $S=\|xx'\|/n_1$ and $S^*=\|x^*x^{*\prime}\|/n_2$ are the covariance matrices which under the null hypothesis are independent estimates of the same population covariance matrix, then the joint distribution of the roots of the determinantal equation $|A-\theta(A+B)|=0$ where $A=n_1S$ and $B=n_2S^*$ has been obtained by Hsu in 1939. This also gives the joint distribution of the squares of canonical correlations between two sets with p variates in one set and q in the other. The limiting distribution of any root specified by its rank order has been obtained in the form of linear combinations and products of incomplete gamma functions for the equations of 2, 3, 4, and 5 roots. The distribution of the sum of the roots of an equation of 2, 3, or 4 roots has also been obtained under either of the conditions that $n_1=p\pm 1$ or that the numbers of variates in two sets differ by one. The distribution is expressed as a simple function of algebraic functions and incomplete beta functions. (Received October 20, 1947.)

Topology

79t. H. W. Becker: *On the enumeration of bridge circuits.* Preliminary report.

The pure active bridge circuits a_n^* (n-branch, without free *SPB* sub-circuits) are relatively few, and derivable from a_{n-k}^* by recurrence, $k=1, 2, 3$. The pure passive bridge circuits b_n^* are found in a_{n+1}^*, by deleting an active branch in all possible ways. The b_n^* are classified according to the node, mesh, symmetry, or direct path partitions, to the number of feeders (terminal branches), true bridgers (having negative terms in the transfer conductance), breaks, or makes (products in the hindrance sum, or its inverse), and to the multiplicity (of the general term in the conductance denominator), dimensionality (of surface applicable without crossovers), fracto number (of *SP* circuits in the equivalent fracto-series circuit), or yet other criteria. Then the number of bridge circuits without external (but with possibly internal) free *SPB* branches is $b_n^i=\sum cCpP$, summed over all terms of possible c=census of sub-b^* of given symmetry partition, C=given topological binomial coefficient, P=topological product of the *SPB* circuits inserted, and p=their permutations among the branches chosen; the bridge circuits with external *SPB* branches, $b_n^e=\sum b_{n-k}^i\cdot s_{k+1}'$, s_k' a generalized *SP* number. Finally, the total number of n-branch bridge circuits $b_n=b_n^i+b_n^e$, which is computed through $n=12$, where they outnumber the *SP* circuits about 3 to 1. (Received May 10, 1947.)

80t. R. H. Bing: *Some characterizations of arcs and simple closed curves.*

Each compact continuum contains two points neither of which cuts between two open subsets of the continuum. A compact continuum is a simple closed curve if each pair of its points cuts between open subsets of it. (Received October 24, 1947.)

81. F. B. Jones: *A note on homogeneous plane continua.*

That a homogeneous compact plane continuum exists which is *not* a simple closed curve has been shown by R. H. Bing. However, it is shown in this paper that if a homogeneous compact plane continuum is aposyndetic (or semi-locally-connected), it *is* a simple closed curve. It follows that a homogeneous compact plane continuum which contains no weak cut point is a simple closed curve. (Received October 27, 1947.)

82. E. E. Moise and G. S. Young: *On imbedding continuous curves in 2-manifolds.*

Claytor (Ann. of Math. vol. 38 (1937) pp. 631–646) has proved that a compact locally connected continuum can be imbedded in a 2-sphere if it contains no one of four continua, two of which are the primitive skew curves, and two of which are curves which will be called C-curves. The authors have proved a theorem which seems to cast some light on the role of the C-curves in Claytor's theorem. Their result is somewhat more general than the following: If a compact locally connected continuum does not contain a C-curve and has no local cut points, then it can be imbedded in a 2-manifold. (Received October 24, 1947.)

83*t*. Hing Tong (National Research Fellow): *On some problems of Čech.*

Čech has proposed a few problems in his paper *On bicompact spaces* (Ann. of Math. (1937)). Two of them have been solved by Hewitt (Bull. Amer. Math. Soc. Abstract 52-5-206) and Pospíšil (Ann. of Math. (1937)). The purpose of this note is to discuss the remaining ones. Specifically, answers to the questions on p. 836, line 13, p. 837, footnote, p. 843, lines 27–28, p. 843, footnote, p. 844, lines 14–15, p. 844, lines 16–18 are no, can be weakened but not entirely removed, yes, can be weakened but not entirely removed, no, no respectively. (Received October 24, 1947.)

84*t*. G. S. Young: *The factorization of manifolds.* Preliminary report.

It is proved that no connected open subset of a compact combinatorial n-manifold (without boundary) is homeomorphic to the product of two spaces A and B, such that A is compact and can be continuously retracted onto a nowhere dense subset of itself. From this it follows that if A is a finite complex, it is a manifold. These results are partial extensions of work of Borsuk (Fund. Math. vol. 33 (1945) pp. 273–298) and some of the methods used are due to him. (Received October 24, 1947.)

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