The theory of mathematical machines. By F. J. Murray. New York, Columbia University Press, 1947. 8+116 pp. $3.00.

There is a fairly extensive but widely scattered literature on computing instruments of various types, ranging from simple devices such as slide rules to such highly complex ones as the ENIAC. The present interest in all these types of computers makes Professor Murray's book especially apropos. He has written on a level which should make the book of equal interest to the applied mathematician and to the engineer. Due, unfortunately, to the secrecy restrictions of wartime the author was not, at the time he wrote this book, able to mention all the new developments in the field of digital computers, particularly the electronic ones. Subsequent editions will undoubtedly contain additional chapters describing the new instruments and indicating the present trends in the field.

The first part deals with the field of digital machines—subject to the hiatus mentioned above. It gives a careful discussion of how coincidence noting devices, adders and multipliers can be realized by means of mechanical elements such as gears as well as by electromechanical devices such as relays. Thus the reader is given a picture of how the familiar types of desk multipliers and related devices function in principle, without having to follow intricate engineering details. The chapter concludes with a brief discussion of the principles underlying the operation of the familiar punch card machines.

After this brief but reasonably complete discussion of the field of digital computers prior to the war the author turns his attention in Part II to so-called measurement machines. These are devices in which numerical quantities are represented by the magnitudes of physical quantities such as rotations, displacements, voltages, and so on. They are, in contradistinction to digital instruments, usually special purpose in character. Perhaps the most widely known and general machine in this class is the differential analyzer. They are almost invariably remarkably ingenious and represent quite interesting attempts to use known physical or chemical principles to solve mathematical problems. One particularly interesting machine in this category is the one which solves Laplace's equation in a given region subject to boundary conditions by shaping a container with non-conducting walls to be the given region. The container is then filled with an electrolytic solution and the boundary conditions are satisfied approximately by placing conductors on the boundaries. The electric potential function then satisfies the equation subject to the boundary conditions and is measured at any point in the solution by means of a probe.

Several mechanical adders are discussed including the familiar differential using bevel or spur gears, in which both the addend and augend are represented by rotations of shafts connected to gears and the sum is again such a motion. The use of Kirchhoff's laws also permits one to construct ingenious electrical adders, as is shown. The second chapter of Part II is devoted to a detailed discussion of various methods for achieving the product of two quantities by means of measurement devices. Many of these methods are, of course, now well known from their use in fire control computers and directors.

Having described adders and multipliers, the author next proceeds to a discussion of various means, both mechanical and electrical, for integrating and differentiating. This naturally leads to an inquiry into the nature of amplifiers since the power output levels at which the arithmetic or analytic components of a measurement computer function are usually below those needed for their inputs, for example, the familiar types of integrators, such as the wheel-disk types used in differential analyzers, depend for their output on a small, light wheel rotating as a result of frictional contact with a glass disk. The rotations of this wheel must be used to drive very long and massive shafts. It is clear that some amplification device must intervene between the integrator output and the driven shaft. The chapter on amplifiers gives a very readable account of electronic amplifiers as well as of the mechanical torque amplifiers which played so significant a role in making the original differential analyzer possible. (In this connection it might be remarked that a somewhat more elegant mathematical discussion of this device is given in W. D. MacMillan, *Statics and dynamics of a particle*, New York, 1927, p. 86.) The account of electronic amplifiers should be of particular interest to those not well conversant with circuit theory since the author gives a very lucid and complete introduction to the subject. As he suggests, this chapter should be read in conjunction with the Radio Amateur's Handbook. The second part closes with a discussion on how a function can be expressed as a Fourier series by measurement mechanisms. This includes an account of so-called selsyn systems.

Part III is devoted to showing how the components previously described can be assembled into machines for handling mathematical problems. The author gives extremely interesting accounts of linear equations solvers (including a preliminary account of a machine designed by him of which a small model has been built and a larger one is now in the process of being constructed), of machines for solving linear elliptic equations and of differential analyzers. He incorporates the essentials of a paper by Shannon which gives a complete analysis of the problems to which an analyzer is applicable but from a formal point of view. However this formal treatment presupposes an idealized analyzer, that is, one of unlimited precision, containing as many elements as desired. The difficulties with a differential analyzer however lie precisely in those aspects of the mechanism removed by the idealization.

The book closes with an account in Part IV of various instruments such as planimeters, integrometers and harmonic analyzers. This discussion includes a description of the important cinema integraph of Wiener, Hazen and Brown which may be used to evaluate integrals of the form $\int_b^a f(x+y)g(x)dx$. These, of course, play an important role in the Wiener-Kolmogoroff theory of time series—when $g(x)=f(x)$ the integral above is the familiar auto-correlation function.

Professor Murray and the King's Crown Press are to be highly commended for this book which makes a unique contribution to our literature. There now exists in one thoroughly readable volume a careful discussion by a professional mathematician of the many beautiful instruments that have been designed as tools for the applied mathematician. This book can be recommended to all those who desire to become acquainted with this fascinating subject as well as to the specialists in the field.

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