ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Announcement. Beginning with the report of the 1947 Summer Meeting, this Bulletin will publish the abstracts of papers offered for presentation at a meeting of the Society as part of the report of the meeting. This arrangement will save considerable space in the Bulletin due to the fact that it will no longer be necessary to print the title of a paper and the name of the author as part of the abstract and also as part of the report of a meeting. The present plan of publishing abstracts was inaugurated in 1930 in the expectation that abstracts would appear in the Bulletin before the papers were read at meetings of the Society. Unfortunately a large proportion of the abstracts are not received in time to make such advance printing possible.

The Editors are pleased to announce in this connection that the Secretary of the Society plans to have available for distribution at as many meetings of the Society as possible mimeographed copies of the abstracts of papers to be presented. It is believed that such a distribution of the abstracts will be of considerable assistance to easier understanding of the papers presented.

ALGEBRA AND THEORY OF NUMBERS

164. Grace E. Bates: *Decompositions of a loop into characteristic free summands.*

If the additive loop L is the free sum of proper subloops L_1 and L_2, then neither L_1 nor L_2 is normal in L. Nevertheless, it is possible, with relatively few restrictions, to obtain decompositions of loops into characteristic free summands. The principal theorem of this paper is the following: If the loop L is the free sum of proper subloops A and B, then A is characteristic in L if and only if the following two conditions are satisfied: (i) no proper free summand of A is isomorphic to a free summand of B; (ii) no proper free summand of A is a free loop. There is also a strong refinement theorem for decompositions of loops into characteristic free summands. The concepts of free sums of loops and free loops, as well as existence theorems and subloop theorems, are taken from a previous paper by the author although their statements are here repeated for the convenience of the reader. (Received March 19, 1947.)

165. R. A. Beaumont: *Rings over a group.*

If G is an additive abelian group, G admits I_n, the domain of integers modulo n, as an operator domain. This n is chosen to be a positive integer or 0 according as G contains an element of maximum order n or not. Necessary and sufficient conditions that G be a vector space (finite or infinite-dimensional) over I_n are found. For groups G, which are the direct sum of cyclic groups, necessary and sufficient conditions that G be a ring are found in terms of multiplication constants in I_n. This general theorem is refined for certain classes of groups. Conditions that G be a ring are also obtained for certain groups which are not the direct sum of cyclic groups. (Received February 28, 1947.)

166. Arthur Bernhart: *Quantitative analysis of rings in minimal maps.*

Homogeneous equations are developed for the number of ways of coloring each side of an n-ring according to specified color schemes on the bounding ring. The solution is facilitated by arranging the schemes in magic squares, and by exploiting symmetry. This method provides a program for finding both reducible and irreducible configurations in the 4-color problem. (Received March 20, 1947.)

167. B. A. Bernstein: *Field in terms of multiplication and a unary operation.*

The author gives two sets of mutually independent postulates for fields in terms of multiplication and a unary operation. In one of these sets the result of the unary operation on an element a is a^+ $(=1+a)$, the "successor" of a. In the other set the result of the unary operation is a^* $(=1-a)$, the "complement" of a. The postulates are well adapted to bring out simply and directly properties of certain special elements of a field, properties hidden in the usual treatment of a field as a system of two binary operations. (Received March 17, 1947.)

168. William H. Durfee: *Quadratic forms over fields with a valuation.*

Let K be a field with a non-archimedean valuation which is complete with respect to the valuation and such that the residue class field has characteristic not two. The author considers the problem of the equivalence of quadratic forms over K and the representation of a given number of K by a quadratic form. Both problems are shown to be reducible to the case in which the forms have unit coefficients. Conditions are found under which the Hasse function $c(f)$ will be invariant and with its aid the necessary and sufficient conditions of Hasse for the representation of zero and the equiva lence of two forms over a p-adic field are shown to apply to a larger class of fields. (Received March 10, 1947.)

169. Paul Erdös and Ivan Niven: *On the roots of a polynomial and its derivative.*

N. G. De Brujn has proved (*On the zeros of a polynomial and of its derivative*, K. Akademie van wetenschappen, Proceedings vol. 49 (1946) pp. 1037–1044) that the average of the absolute values of the imaginary parts of the roots of a polynomial with real coefficients is not less than the corresponding average for the derivative polynomial. The authors prove this result for polynomials with complex coefficients. More generally, it is proved that the average distance of the roots of a polynomial from a given straight line in the complex plane is not less than the corresponding aver-

age for the roots of the derivative; the averages are equal only if the roots of the polynomial are not distributed on both sides of the line. The same inequality holds for the average distances of the roots of any polynomial and its derivative from a fixed point in the plane; in this case the averages are equal only if the roots of a polynomial lie on a half line emanating from the fixed point. (Received March 19, 1947.)

170. C. J. Everett and George Whaples: *Representation of classes of finite sets.*

An arbitrary class of finite sets T_α possesses a choice function $f(T_\alpha)=t_\alpha \in T_\alpha$, $f(T_\alpha)\neq f(T_\beta)$ for $\alpha\neq\beta$, if and only if the union of every k of the T_α, $k=1, 2, \cdots$, contains at least k distinct elements. This generalizes the well known result of P. Hall for a finite class of finite sets. Necessary and sufficient conditions are given for the existence of a common representation of two partitions of a set when all components are finite. The existence of a common representation of left and right cosets of a group modulo a finite subgroup is obtained as a special case. This generalizes all results of Bull. Amer. Math. Soc. Abstract 53-1-98 from countable to arbitrary classes of finite sets. (Received March 19, 1947.)

171. Edwin Hewitt: *A characterization of rings of continuous real-valued functions.*

Let A be any commutative, associative algebra over the real number system, with unit e, and with the following properties: (1) for every $x\neq 0$ in A, there is an ideal M in A whose quotient ring is the real number field (such ideals are called *real*) such that $M(x)\neq 0$; (2) if $M(x)\geqq 0$ for all real ideals M in A, then $x=y^2$ for some y in A; (3) (x^2+e) always has an inverse, and there exists a real positive number α such that $M(\alpha e-(x^2+e)^{-1})\geqq 0$ for all real ideals M in A; (4) a topology is introduced into A by the following definition: if α is any positive real number, then $U_\alpha(x)$ is the set of all $y\in A$ such that $|M(y-x)|<\alpha$ for all real ideals M in A; the space A is to be complete under this topology; (5) if x has no inverse, then x is contained in some real ideal; (6) if I_0 and I_1 are intersections of real ideals such that the ideal I_0+I_1 is contained in no real ideal, then $I_0+I_1=A$. Under these hypotheses, it is proved that the algebra A is isomorphic to the ring of all real-valued continuous functions defined on some normal T_1-space. (Received March 3, 1947.)

172. R. E. Johnson: *The modules of a ring.* Preliminary report.

If I is a right ideal of the ring R, the additive groups of I and $R-I$ are well known examples of right R-modules. The author studies the converse problem of relating right, left and two-sided R-modules M to the ideal structure of R. If $M=xR$, then M is R-isomorphic to $R-I$, I a right ideal of R: in case M is irreducible, I is maximal. Any R-module is isomorphic to a subdirect sum of subdirectly irreducible R-modules. A study is made of the subdirectly irreducible R-modules. The Baer-Jacobson and Brown-McCoy radicals of a ring can be defined in terms of the annihilators of irreducible R-modules. These radicals are studied from this standpoint. The anti-radical of Baer is a left annihilator of the Baer-Jacobson radical. (Received March 20, 1947.)

173. G. K. Kalisch: *On extensions of topological fields.*

In the first part transcendental extensions are discussed and it is shown by means of a simple criterion of embeddability of topological integral domains in topological

fields that, if F is a topological field, $F(x)$ can be topologized so as to preserve the topology of F. In the second part finite algebraic extensions are considered. If the ground field is complete, only one way of topologizing the extension is possible. If the ground field is not complete but is completable (that is, its completion is a field) then there are finitely many ways in which the extension can be topologized; these are determined, and their completions which are direct sums of finitely many complete fields are found in the case of separable extensions. In connection with part I of the paper it is pointed out that there exist non-locally compact connected fields, which, incidentally, are not complete, but are completable. (Received March 21, 1947.)

174. Irving Kaplansky and R. F. Arens: *Topological representation of algebras.*

The results obtained can be illustrated by the case of a ring A with unit satisfying $a^4=a$. If A is countable, it is the set of all continuous functions from a compact totally disconnected space X to $GF(4)$, restricted on a closed subset of X to the values 0, 1. An example shows that this theorem fails if A is uncountable. Another kind of representation holds universally: here X has an involutory homeomorphism T and A consists of all functions satisfying $f(xT)=f(x)^2$. Results of this kind are proved for any commutative semi-simple algebraic algebra, and for a certain class of Banach algebras. Some results previously announced by the authors (Bull. Amer. Math. Soc. Abstracts 53-1-2 and 53-1-18) are subsumed. (Received March 15, 1947.)

175. D. H. Lehmer: *On the Tarry-Escott problem.*

In this paper the author considers the problem of finding b sets $S_0, S_1, \cdots, S_{b-1}$ of integers having equal sums of like powers. More explicitly, if $\sigma_k(r)$ denotes the sum of the kth powers of the members of S_r, it is required that $\sigma_k(0)=\sigma_k(1)=\cdots=\sigma_k(b-1)$ hold for $k=0, 1, \cdots, n-1$. The case of $b=2$ is well known (cf. Dickson's *History of the theory of numbers*, vol. 2, chap. 24). The above problem is said to be of order b and degree n. A family of solutions is obtained as follows. Let $m_1, m_2, \cdots, m_n$ be any n integers and let S_r be the set of all integers of the form $a_1m_1+\cdots+a_nm_n$ where each a ranges over all non-negative integers less than b subject only to the condition that their sum be congruent to r modulo b. Then the sets S_r $(r=0, 1, \cdots, b-1)$ form a solution of the general Tarry-Escott problem. Moreover, the degree of this solution is precisely n, except for the trivial case in which the sets S are merely permutations of one another. (Received March 21, 1947.)

176. D. H. Lehmer: *On the vanishing of Ramanujan's function.*

Ramanujan's function $\tau(n)$, defined as the coefficient of x^{n-1} in the expansion of the 24th power of the product $(1-x)(1-x^2)(1-x^3)\cdots$, is a well known numerical function. Nevertheless, the question of the possible vanishing of $\tau(n)$ has never been answered. Tables of $\tau(n)$ for $n\leqq 300$ show no case of $\tau(n)=0$. In this paper it is shown that $\tau(n)\neq 0$ for $n<3316799$ and that at most 8 of the first ten million values of $\tau(n)$ are zero. These results follow from a combination of several congruence properties of Ramanujan's function. (Received March 19, 1947.)

177. M. J. Norris: *Indeterminates over a universal algebra.*

A universal algebra S is considered as a set with an associated family of operations. An element of the algebra is called a polynomial in the elements of S_2 over S_1 if the

element is in the least subset of S containing S_1 and S_2 closed with respect to the operations of S. By means of the notion of polynomial, the notions of algebraic elements and indeterminates are introduced. It is first shown that in the algebra of functions associated with S the identity function on S is an indeterminate over the class of functions constant on S. Other theorems of the above nature that would be expected are proved. The principle method used is that of algebraic induction. (Received March 21, 1947.)

178. L. J. Paige: *Neofields.* Preliminary report.

An additive loop L whose nonzero elements form a multiplicative group G such that multiplication is both right and left distributive with respect to addition is called a *neofield* N. Defining a group G to be admissible if and only if there exists a neofield N having G as its multiplication system, the author determines a necessary and sufficient condition that G be admissible and proves that the following groups are admissible: (i) abelian groups of rank $\rho=0$, (ii) abelian groups of rank $\rho\geqq 1$ and not possessing a unique element of order 2, (iii) groups G, all of whose elements have finite odd order. First proving that the center Z of an admissible group G is the multiplicative system of a sub-neofield $N'\subseteq N$, the author considers an extension problem of abelian groups to noncommutative admissible groups. In conclusion, an application of neofields to the coordinatization of projective planes is presented. (Received March 1, 1947.)

179. G. de B. Robinson: *On a theorem of Nakayama concerning the prime factors of the degrees of the irreducible representations of the symmetric group.*

R. Brauer has expressed the exponent of a prime p dividing the degree of an irreducible representation ϕ of a finite group G in the form $a-d+\epsilon$, where the order of G is $g=p^a g'$, $(g', p)=1$; p^d is the order of the *defect group* corresponding to ϕ, and ϵ is an unknown constant not less than 0 (Proc. Nat. Acad. Sci. U.S.A. vol. 32 (1946) pp. 215–219). By refining Nakayama's theorem it is possible to define ϵ explicitly in the case of the symmetric group. Nakayama's conjecture (Jap. J. Math. vol. 17 (1940) pp. 165–184, 411–424) concerning the characterization of the blocks of characters follows. (Received March 14, 1947.)

180. R. D. Schafer: *The exceptional simple Jordan algebras.*

It is known that the only simple Jordan algebras over a nonmodular field $\mathfrak{F}$ which are not Jordan algebras of linear transformations are those of degree 3 and order 27 over their centers, having $\mathfrak{M}_3^8$ as split algebra. We show that (over their centers) all such exceptional algebras are reduced algebras, and that they consist of the J-symmetric elements of $\mathfrak{M}_3\times\mathfrak{C}$, where $\mathfrak{M}_3$ is the total matric algebra of degree 3, $\mathfrak{C}$ is any Cayley-Dickson algebra over $\mathfrak{F}$, and J is the involution $a\rightarrow pa'p^{-1}$ of $\mathfrak{M}_3\times\mathfrak{C}$ where p is a nonsingular diagonal matrix. If $\mathfrak{F}$ is such that there exist inequivalent Cayley-Dickson division algebras over $\mathfrak{F}$, then there exist inequivalent Jordan algebras over $\mathfrak{F}$ with $\mathfrak{M}_3^8$ as split algebra. (Received March 4, 1947.)

181. M. F. Smiley: *Alternative regular rings without nilpotent elements.*

Forsythe and McCoy (*On the commutativity of certain rings*, Bull. Amer. Math.

Soc. vol. 52 (1946) pp. 523–526) showed, using a general theorem of G. Birkhoff (*Subdirect unions in universal algebra*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 764–768), that an associative regular ring R is a subdirect sum of division rings if and only if R has no nilpotent elements. This result is extended to alternative regular rings. It is also shown that every alternative ring which satisfies the condition $a^{n(a)}=a$ ($n(a)$ *an integer greater than one*) of Jacobson (*Structure theory for algebraic algebras of bounded degree*, Ann. of Math. (2) vol. 46 (1945) pp. 645–707) is associative. The proofs involve essentially new ideas only in case the additive order of some element of the ring is finite and divisible by three. (Received February 18, 1947.)

182. M. F. Smiley: *Binary systems which are almost loops.*

The investigation of H. B. Mann (*On certain systems which are almost groups*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 879–881) on the effect of postulating a right unit and left inverse for associative binary systems is extended to certain non-associative cases. The systems considered are found to be slight generalizations of loops with the inverse property (R. H. Bruck, *Some results in the theory of quasigroups*, Trans. Amer. Math. Soc. vol. 55 (1944) pp. 19–52). (Received February 18, 1947.)

183. B. M. Stewart: *Left associated matrices with elements in an algebraic domain.* II.

In the problem described in the author's Bull. Amer. Math. Soc. Abstract 47-5-202 the sufficiency proof was not completed for all cases. This omission is now remedied by showing that if the enlarged matrices A' and B' are left associates, then the matrices A and B are mutually left divisible, whence by a theorem of Steinitz it follows that the matrices A and B are left associates. This completes the proof, begun in the earlier abstract, that a necessary and sufficient condition that the matrices A and B be left associates over an algebraic domain is that the corresponding enlarged matrices A' and B' be left associates over the rational domain. Some study is made of the construction of the unimodular matrix P such that $PA=B$. (Received March 17, 1947.)

184. L. V. Toralballa: *A generalization of the finite integral.*

Given f, a function of $x_1, x_2, \cdots, x_s$, and A, a positive integer, consider the sum of the values of f over the set of all the ordered sets $(x_1, x_2, \cdots, x_n)$, $1 \leqq s \leqq n$, where for all i, $x \geqq 1$, and $\sum_1^n x_i = A$. Three cases are treated: (1) f is a polynomial of the mth degree in the variables; (2) f is a factorial polynomial, that is, a sum of terms of the form $x_1^{(m_1)}x_2^{(m_2)} \cdots x_s^{(m_s)}$ where $x^{(m)} \equiv x(x-1) \cdots (x-m+1)$; and (3) f is the quotient of two linear polynomials. General formulas are given for cases (1) and (2) and an algorithm for case (3). New formulas are also given for the so-called differences of zero. The methods used throughout are those standard in the calculus of finite differences. (Received March 20, 1947.)

185. J. A. Ward: *Analytic functions in linear algebras.*

This paper extends the results of a previous paper (*Theory of analytic functions in linear associative algebras*, Duke Math. J. vol. 7 (1940) pp. 233–248) by proving additional theorems on differentiation and integration and by giving a method for solving certain types of differential equations by use of linear associative algebras. (Received February 3, 1947.)

186. Morgan Ward: *Elliptic divisibility sequences.*

The author obtains all solutions of the difference equation $\omega_{m+n}\omega_{m-n} = \omega_{m+1}\omega_{m-1}\omega_n^2 - \omega_{n+1}\omega_{n-1}\omega_m^2$ over the field of rationals, and studies the arithmetical properties of integral solutions. These solutions include the well known Lucas function $u_n = (a^n - b^n)/(a-b)$ which satisfies a linear difference equation of order two; the more general numerical functions here investigated have very similar arithmetical properties. The conjecture of Lucas that there exists a connection between solutions of a linear difference equation of order three or four and elliptic functions is shown to be false so far as the difference equation studied is concerned despite Lucas' explicit assertion to the contrary (Amer. J. Math. vol. 1 (1878) p. 203.) (Received March 8, 1947.)

187. N. A. Wiegmann: *Some theorems on normal matrices with analogs of the generalized principal axis transformation.*

The following are among a number of theorems obtained on normal matrices: A necessary and sufficient condition that a matrix be normal is that its polar matrices be expressible as polynomials in the matrix; if A, B, and AB are normal, then BA is a normal matrix; a necessary and sufficient condition that the product AB of two normal matrices be normal is that each commute with the hermitian polar matrix of the other; the product AB of any two square matrices is normal if and only if there exist a unitary U and a nonsingular P such that $UAP = D$ and $P^{-1}BU^{CT} = T$ where D and T are certain diagonal and triangle matrices, respectively. Several analogs of the generalized principal axis transformation for a set of matrices A_α are shown to hold. (Received March 21, 1947.)

188. R. L. Wilson: *A finite method for the determination of the Galois group of an equation with an application to the problem of reducibility.*

This paper gives a sieve process for the determination of the Galois group, which depends only upon the ability to obtain the rational roots of a system of induced equations. If the given equation is of degree n, the Galois group G must be contained in the symmetric group of degree n. Hence, it is only necessary to ascertain which, if any, of the subgroups of the symmetric group contain G. If Γ is any subgroup of the symmetric group, the author has shown how to construct a function, ϕ, of the roots of the given equation which is invariant under precisely the permutations of Γ. The induced equation is then formed, having for its roots the function ϕ together with its conjugate functions. The theorem is then established that $G \subseteq \Gamma$ if and only if the induced equation has at least one rational root. By a different choice of the function ϕ, this same method is applied to the problem of the reducibility of a polynomial. (Received March 15, 1947.)

ANALYSIS

189. J. E. Bearman: *Rotations in the product of two Wiener spaces.* Preliminary report.

Let $X(t)$ and $Y(t)$ be elements of C, the space of all continuous functions on $0 \leqq t \leqq 1$ which vanish at $t=0$. Then if $F[X, Y]$ is a Wiener summable functional over $C \otimes C$, it is shown that the "rotation" $X(t) = x(t) \cos\theta - y(t) \sin\theta$, $Y(t) = x(t) \sin\theta + y(t) \cos\theta$