Laplace transform, $L(s; f)_h$, is meant a linear functional of f with the fundamental property that $L(s; hf)_h = sL(s; f)_h - f(t_0)$. Clearly $\int_{t_0}^{\infty} \exp -s(t-t_0)f(t)dt, \sum_{t_0}^{\infty} f(i)s^{-i-1+t_0}$, $\int_{t_0}^{\infty} t^{-s-1}t_0^s f(t)dt, \sum_{t_0}^{\infty} f(i)(1+s)^{-i+t_0-1}$ are generalized L.T.'s for d/dt, E, td/dt, and Δ respectively. It may be verified that the appropriate expression is in every case given formally by $\lim_{t\to\infty} F(t, s, 0; f) F(t, s, 1; 0)^{-1}$. Tables of transform pairs may be set up for practical use precisely like those of the usual Laplace transform. Regions of convergence must be determined for each different operator; however, in practice it is often possible to get the correct answer by proper "interpretation" even when convergence fails. (Received January 15, 1947.)

149. Fred Supnick: *Cooperative phenomena. II. Structure of the two-dimensional Ising model.*

Let a distribution of A's and B's be made over the vertices of a linear graph G in which any two vertices are joined by at most one edge. Associate with each edge a $(+1)$ or a (-1) accordingly as its end points are the same or different. Denote the sum of the numbers on the edges by E (the energy). The (physical) partition function is obtained by putting E into the Boltzman exponential and summing over all possible states In this paper the author examines from a combinatorial point of view the structure for the case where G is a portion of a (plane or cylindrical) rectangular grating. A method is obtained for constructing all those distributions which have the same energy. An examination of the structure of the three-dimensional model is also made. It is pointed out that the problem of reducing "end effects" is equivalent to certain problems in the topology of sphere clusters. (See Bull. Amer. Math. Soc. Abstracts 52-9-323 and 52-11-386 by the author.) (Received January 17, 1947.)

GEOMETRY

150. John DeCicco: *An extension of Euler's theorem on homogeneous functions.*

The author determines the partial differential equation of order r to be obeyed by a function $\phi(x, y)$ which is the sum of r homogeneous functions with degrees n, $n-1$, $n-2$, $\cdots$, $n-r+1$. It is observed that such a function $\phi(x, y)$ may be said to be of degree n and is a generalization of a polynomial. This is related to the problem of determining all the algebraic curves C_n of degree n such that the $(n-r)$ polars C_r of degree r all pass through a fixed point O. This point O is a singularity of C_n of order $(n-r+1)$. For example, if the polar conics all pass through a given point O, then O is a singularity of C_n of order $(n-1)$. This whole theory is extended quite readily to any number of dimensions. (Received January 31, 1947.)

151. John DeCicco: *New proofs of the theorems of Kasner concerning the infinitesimal contact transformations of mechanics.*

The author submits new proofs of the theorems of Kasner concerning the infinitesimal contact transformations of general dynamics. (See *The infinitesimal contact transformations of mechanics*, Bull. Amer. Math. Soc. vol. 16 (1910) pp. 408–412)). The theorems deal with the nature of two dynamical systems of the same number of degrees of freedom for which the commutator or alternant of the associated infinitesimal contact transformations is a point transformation. The main result is that this situation can arise if and only if the expressions for the kinetic energy are the same or differ merely by a factor. The other proposition is that two infinitesimal contact transformations with the same transversality law will have a point transformation

for commutator if and only if they are associated with two dynamical systems of the type described above. These theorems are proved for homogeneous contact transformations. The methods are those of tensor calculus. (Received January 8, 1947.)

152. Marshall Hall: *Cyclic projective planes.*

A projective plane π is said to be cyclic with respect to a collineation ϕ, if the cyclic group generated by ϕ is transitive on the points of π. It can be shown that this is equivalent to assuming that the cyclic group generated by ϕ is transitive on the lines of π. Every finite Desarguesian plane is a cyclic plane, but there are also infinite cyclic non-Desarguesian planes. Methods are given for constructing the infinite cyclic planes, and their properties are studied. Methods for constructing the representaion of a finite Desarguesian plane as a cyclic plane have been given by Singer and by the author. Restrictions are found here on finite cyclic planes. For example there is no cyclic plane with $n+1$ points on a line and $n \equiv 1 \pmod 3$ unless $n = x^2 + xy + y^2$. This excludes $n \equiv 10 \pmod{12}$. Also every cyclic plane possesses a polarity and results of Baer lead to strong restrictions on even values of n. (Received January 17, 1947.)

153. Edward Kasner and John DeCicco: *Harmonic transformations and velocity systems.*

An arbitrary point transformation of the plane carries the ∞^2 straight lines into the special cubic family $y'' = A + By' + Cy'^2 + Dy'^3$, studied by R. Liouville. The velocity type $y'' = (1 + y'^2)[\psi(x, y) - y'\phi(x, y)]$ appears frequently in geometry and dynamics. (See Kasner, *Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications, vol. 3, 1912, and Kasner and DeCicco, *The geometry of velocity systems*, Bull. Amer. Math. Soc. vol. 49 (1943) pp. 236–245.) In the present paper, the authors inquire when a harmonic transformation (in which the components obey the Laplace equation but not necessarily the Cauchy-Riemann equations) will convert the ∞^2 straight lines into a velocity family. This problem yields on the one hand a special type of harmonic transformation and, on the other, a special kind of velocity system. The resulting harmonic transformations can be analyzed into the products of conformalities by affinities (this obviously includes the conformalities as a special case). The corresponding velocity families can be identified with the Γ_0 families, namely, the conformal image of the ∞^2 straight lines, that is, the isogonals of an arbitrary isothermal family. (Received December 27, 1946.)

154. Edward Kasner and John DeCicco: *New theorems in the polar theory of a general algebraic curve.*

The authors present some new theorems in the polar theory of a general algberaic curve C_n of degree n. If the rth polar curve C_{n-r}, of degree $n-r$, with respect to the algebraic curve C_n of an ordinary point O on C_n, is constructed, then it is known that C_n and C_{n-r} are initially tangent. It is found that the curvatures at O are in general distinct. The ratio ρ of the curvatures is studied and, in the case of higher order contact p, the ratio ρ of the departure of the polar curve C_{n-r} to that of the algebraic curve C_n from their common tangent line. This ratio ρ is given by a simple rational formula involving only the positive integers (n, r, p). Also the case is considered where the point O is a singularity of the algebraic curve C_n. Both C_n and C_{n-r} have a singularity of the same qualitative nature at O. In this case the ratio ρ depends also on the coefficients of the polynomial defining C_n. The new theorems are essentially theorems of projective differential geometry. (Received January 15, 1947.)

155. Buchin Su: *Descriptive collineations in spaces of K-spreads.*

The author studies descriptive collineations in a space of K-spreads as a generalization of projective collineations in the general geometry of paths. After the equations expressing the conditions for an infinitesimal descriptive collineation which the space of Douglas may admit are derived, the notion of the Lie derivative is utilized in reducing the number of the integrability conditions of these equations. The group property of descriptive collineations is established. (Received December 19, 1946.)

STATISTICS AND PROBABILITY

156. P. L. Hsu and H. E. Robbins: *Complete convergence and the law of large numbers.*

A sequence $\{X_n\}$ of random variables is said to converge to 0 *completely* if for every $\epsilon > 0$, $\lim_{n\to\infty}[P(|X_n|>\epsilon)+P(|X_{n+1}|>\epsilon)+\cdots]=0$. Let $\{Y_n\}$ be a sequence of independent random variables with the same distribution function $F(y)=P(Y_n \leq y)$ and such that $\int_{-\infty}^{\infty}ydF(y)=0$, $\int_{-\infty}^{\infty}y^2dF(y)<\infty$. It is proved that the sequence $\{X_n\}$ $=\{n^{-1}(Y_1+\cdots+Y_n)\}$ converges to 0 completely. A partial converse of this theorem is given, and the relation between complete convergence and convergence with probability 1 is clarified. (Received January 18, 1947.)

TOPOLOGY

157. C. H. Dowker: *An imbedding theorem for paracompact metric spaces.*

A space is called paracompact if every covering by open sets has a neighborhood-finite refinement. A Banach space in which every finite-dimensional linear subspace is Euclidean is called a general Euclidean space or a non-separable Hilbert space or a unitary Banach space. It is shown that a metrizable space is paracompact if and only if it is homeomorphic to a subset of a unitary Banach space. (Received January 31, 1947.)

158. Nathan Jacobson: *On the theory of primitive rings.*

In a previous paper a ring $\mathfrak{A}$ has been called *primitive* if it contains a maximal right ideal $\mathfrak{J}$ whose quotient $(\mathfrak{J}:\mathfrak{A})=0$. Here $(\mathfrak{J}:\mathfrak{A})$ is the totality of elements $b\in\mathfrak{A}$ such that $\mathfrak{A}b\subseteq\mathfrak{J}$. An extrinsic characterization of these rings is that $\mathfrak{A}$ is isomorphic to an irreducible ring of endomorphisms. In this paper the author studies the one-sided ideals in primitive rings and defines certain topologies in $\mathfrak{A}$ by using these ideals. Particular attention is given to the primitive rings that contain minimal ideals. For these, the structure theories given previously for simple rings by the author and by Dieudonné are generalized. (Received December 12, 1946.)

159. Everett Pitcher: *Čech homology invariants of continuous maps.*

The effect of a continuous map on the Čech homology (or cohomology) groups under assumptions appropriate to the theory is stated in terms of exact homomorphism sequences. This is done with the aid of new homology invariants of maps, namely homology groups of the Čech type based on chains of the nerve of the domain which vanish and homology groups based on chains of the nerve of the range which are images. The former are analogous to invariants already introduced by the writer (Bull. Amer. Math. Soc. Abstract 46-5-216) for simplicial maps. The whole development is the geometric extension of the algebraic treatment of homomorphisms of