certain functions not polynomials and for homogeneous polynomials of degree n in two real variables. (Received December 24, 1946.)

138. Bertram Yood: *On ideals in operator rings over Banach spaces.* Preliminary report.

Let R be the ring of all continuous linear operators on a Banach space X. For any ideal A in R let $M\subset X$ be the set of all x in X for which $T(x)=0$ for every T in A, and let N be the linear manifold in X generated by the ranges of T in A. It is shown that if A is a left (right) ideal in R then the closure of A in both the weak and strong topology of operators is the set of all T in R which vanish on all of M (set of all T in R whose range is contained in $\overline{N}$). Thus for ideals the topologies are equivalent and the closed ideals characterized. These facts are used to show that if S is a subset of R, the left (right) annihilator of the right (left) annihilator of S in R is the smallest weakly closed left (right) ideal containing S. (Received January 6, 1947.)

139. Bertram Yood: *Regular and singular elements in the ring of operators on a Banach space.* Preliminary report.

The sets G and S of regular and singular elements respectively of the ring R of all bounded linear operators defined on an infinite-dimensional Banach space X are studied in the uniform, strong and weak topologies of R. In the uniform topology, S is decomposed into disjoint sets with topological properties in such a way that the properties of an operator T being an isomorphism or not, having X as its range or not, and the existence or absence of certain projections are correlated with the ring properties of being a left or right generalized null-divisor or not, and the possession or lack of a left or right inverse. If G can be dense in R then an open question of Banach (*Théorie des opérations linéaires*, p. 246) on linear dimensions has a negative answer. Banach's question is further discussed in terms of the concepts of this paper. In both the strong and weak topologies of R, it is shown that G and S are each dense and not open. (Received January 31, 1947.)

140. H. J. Zimmerberg: *Definite integral systems.*

In this paper the notions of definitely self-conjugate adjoint integral systems of Wilkins (Duke Math. J. vol. 11 (1944) pp. 155–166) and those definite systems of the author (Bull. Amer. Math. Soc. Abstract 52-3-76) are extended to integral systems written in matrix form $y(x)=\lambda\int_a^b K(x, t)y(t)dt$, where no restriction is made on the form of the kernel $K(x, t)$. These definite integral systems include the definite systems previously treated as subclasses. With the exception of expansion theorems, most of the properties of the definite integral systems of Wilkins and the author are preserved. (Received January 8, 1947.)

Applied Mathematics

141. J. B. Díaz and Alexander Weinstein: *On an extremal property of the torsional rigidity.*

The torsional rigidity S of a beam with simply or multiply connected cross section can be given by either of the following formulas, which have been hardly explicitly mentioned in the literature: (*) $S=P-D(\phi)$, (**) $S=P-D(\psi)$, where P is the polar moment of inertia, ϕ and ψ are the warping function and its conjugate, and D denotes the Dirichlet integral. From these formulas follows $S<P$, and $S=P$ only for circular or annular sections. Upper and lower bounds for S are obtained from (*) for any section. For simply connected sections estimates are also obtained from (**). (Received December 9, 1946.)

142. Howard Eves: *Some consequences of a simple theorem on torque.*

The simple theorem on torque is: *Let F_i $(i=1, \cdots, n)$ be a set of n coplanar forces. Then the locus of a point P in the plane of the forces such that the sum of the torques of the forces about P is a constant k is a straight line. As we vary k the line moves parallel to itself.* From this theorem a number of physical and geometrical theorems and constructions are derived. Consideration is given to the envelope of the diametral lines of a given system of lines for a given system of multiples, and of the envelope of the diametral planes of a given system of planes for a given system of multiples. Another application leads to a simple construction locating the point about which a system of coplanar forces exerts minimum inertia. This last is a generalization of the symmedian point of a triangle and leads, in turn, to some new theorems in the modern geometry of the symmedian point. (Received January 27, 1947.)

143. Howard Eves: *Systems of particles with a common centroid.*

In general if the members of a system of n weighted particles be independently displaced in space to new positions, the centroid of the new arrangement will not coincide with that of the original arrangement. This paper develops a number of theorems and constructions concerning some rearrangements of n weighted particles for which the centroid remains invariant. New and elementary proofs are supplied for some known theorems due to Brocard, M'Cay, and Weill. Sollerstinsky's elegant construction for the equicenter of two triangles, and Neuberg's analogous construction for the equicenter of two tetrahedra, are very simply established. (Received January 27, 1947.)

144. H. J. Greenberg: *The determination of upper and lower bounds for the solution of the Dirichlet problem.*

Let w be the solution of the Dirichlet problem, that is, harmonic in a domain D and assuming assigned boundary values. A method is given for determining upper and lower bounds for the value of w at any given point of D. The bounds are integral expressions involving arbitrary functions of two classes, U and V; U consists of all functions satisfying the boundary conditions, V is the class of functions harmonic in D. The method is based on the application of two complementary variational principles for w: the first (the Dirichlet principle) applies to U, the complementary principle applies to V. This yields inequalities for $\int_D(w_x^2+w_y^2)dA$. The introduction of an auxiliary boundary value problem leads to bounds for w. By employing sequences of functions in U and V, successively improved bounds are obtained. The method applies to the plane or 3-space and can be made to yield bounds directly for derivatives of the solution. Also treated is the homogeneous boundary value problem associated with Poisson's equation. As an example, bounds are obtained for the central deflection of a square elastic membrane under uniform loading. (Received January 22, 1947.)

145. M. Z. Krzywoblocki: *On certain cases of simple solutions of flow equations in compressible fluid with particular conditions.*

In the most general case of flow in a compressible fluid the density, viscosity and heat conductivity are functions of pressure and temperature. The physical conditions superimpose certain boundary values on velocity, density, pressure and temperature. Besides that one has to take into account a thermodynamic condition expressing usually the relationship between heat added, intrinsic energy and external work. This picture shows that at the present time the difficulties in the solution of equations are probably insurmountable. But it is possible to select cases satisfying only a part of the boundary conditions, which, although uninterpretable from a physical standpoint, are solvable from a mathematical point of view. In the present papers three such cases are solved under the conditions that the coefficients of viscosity and thermal conductivity are constant, that only one boundary condition is superimposed, namely, the one concerning the velocity at infinity and that all other restrictive conditions are neglected. The cases are: source, circular vortex, and spiral vortex. (Received January 30, 1947.)

146. H. E. Salzer: *Table of coefficients for interpolating in functions of two variables.*

When the double Gregory-Newton interpolation formula for functions of two variables is rewritten in terms of the tabular entries $f_{s,t}$, one obtains (1) $f(x+ph_1, y+qh_2) = \sum_{s+t=0}^{n} C_{s,t}(n, p, q) f_{s,t}$, where $C_{s,t}(n, p, q) = {}_{n-p-q}C_{n-s-t}\ {}_pC_s\ {}_qC_t$. Another proof of the corresponding general theorem for any number of variables is given. (See Bull. Amer. Math. Soc. vol. 51 (1945) pp. 279–280.) Formula (1) approximates $f(x, y)$ by a binary n-ic and is particularly applicable whenever Taylor's theorem for two variables holds in a suitable neighborhood of $f(x_0, y_0)$, or when interpolating separately for the real or imaginary part of an analytic function of degree n in $z=x+iy$. Also (1) has the advantages of involving the fewest numbers of points $f_{s,t}$ (that is, 6, 10, or 15 for $n=2$, 3, or 4), and of being applicable near the beginning or end of a table. The present table gives the exact values of $C_{s,t}(n, p, q)$ for $n=2$, 3, or 4, that is, in quadratic, cubic, or quartic interpolation, for all values of s, t whose sum ranges from 0 to n, and for p and q each ranging from 0.1 to 0.9 at intervals of 0.1. The saving in labor by avoiding partial differences or the double Lagrangian formula (latter requiring 12, 20, or 30 multiplications for $n=2$, 3, or 4) is indicated. (Received December 27, 1946.)

147. P. A. Samuelson: *A generalized Newton iteration.*

An unknown simple root, A, of the equation $f(x)=0$ can be approximated by a general iteration of the form $x_{t+1}=H(x_t)$, defined by the implicit relation $y(x_{t+1}) = G[x_{t+1}-x_t; f(x_t), f'(x_t), \cdots, f^{(n)}(x_t)]=0$ where y is a solution of some differential equation system $y^{(n+1)}=g[y, y', \cdots, y^{(n)}]$ and $y^{(i)}(x_t)=f^{(i)}(x_t)$ for $i \leqq n$. It can be shown that $H(A)=A$ and $H^{(i)}(A)=0$, $i \leqq n$, so that the iteration not only converges to the correct root for all sufficiently near initial approximations, but does so at an extremely rapid rate. Special cases include the Newton iteration (when $y''=g\equiv 0$), numerous suggested high-order osculatory extrapolations, truncated Taylor series, inverse interpolation by means of derivatives, and other methods proposed in the literature—often without proof of convergence. (Received January 15, 1947.)

148. P. A. Samuelson: *Generalization of the Laplace transform for any operator.*

Assume given any admissible operator h, such that the first-order linear equation $(h-s)y(t)=f(t)$ has a unique solution $F(t, s, b; f)_h$ for $t \geqq t_0$, for arbitrary initial condition $y(t_0)=b$, and for $f(t)$ a sufficiently limited function. Examples of admissible operators are d/dt, E, td/dt, Δ, $5+\Delta$, and so on, but not d^2/dt^2. By the generalized

Laplace transform, $L(s;f)_h$, is meant a linear functional of f with the fundamental property that $L(s;hf)_h = sL(s;f)_h - f(t_0)$. Clearly $\int_{t_0}^{\infty} \exp -s(t-t_0)f(t)dt$, $\sum_{t_0}^{\infty} f(i)s^{-i-1+t_0}$, $\int_{t_0}^{\infty} t^{-s-1}t_0^s f(t)dt$, $\sum_{t_0}^{\infty} f(i)(1+s)^{-i+t_0-1}$ are generalized L.T.'s for d/dt, E, td/dt, and Δ respectively. It may be verified that the appropriate expression is in every case given formally by $\lim_{t\to\infty} F(t, s, 0; f)F(t, s, 1; 0)^{-1}$. Tables of transform pairs may be set up for practical use precisely like those of the usual Laplace transform. Regions of convergence must be determined for each different operator; however, in practice it is often possible to get the correct answer by proper "interpretation" even when convergence fails. (Received January 15, 1947.)

149. Fred Supnick: *Cooperative phenomena.* II. *Structure of the two-dimensional Ising model.*

Let a distribution of A's and B's be made over the vertices of a linear graph G in which any two vertices are joined by at most one edge. Associate with each edge a $(+1)$ or a (-1) accordingly as its end points are the same or different. Denote the sum of the numbers on the edges by E (the energy). The (physical) partition function is obtained by putting E into the Boltzman exponential and summing over all possible states In this paper the author examines from a combinatorial point of view the structure for the case where G is a portion of a (plane or cylindrical) rectangular grating. A method is obtained for constructing all those distributions which have the same energy. An examination of the structure of the three-dimensional model is also made. It is pointed out that the problem of reducing "end effects" is equivalent to certain problems in the topology of sphere clusters. (See Bull. Amer. Math. Soc. Abstracts 52-9-323 and 52-11-386 by the author.) (Received January 17, 1947.)

Geometry

150. John DeCicco: *An extension of Euler's theorem on homogeneous functions.*

The author determines the partial differential equation of order r to be obeyed by a function $\phi(x, y)$ which is the sum of r homogeneous functions with degrees $n, n-1, n-2, \cdots, n-r+1$. It is observed that such a function $\phi(x, y)$ may be said to be of degree n and is a generalization of a polynomial. This is related to the problem of determining all the algebraic curves C_n of degree n such that the $(n-r)$ polars C_r of degree r all pass through a fixed point O. This point O is a singularity of C_n of order $(n-r+1)$. For example, if the polar conics all pass through a given point O, then O is a singularity of C_n of order $(n-1)$. This whole theory is extended quite readily to any number of dimensions. (Received January 31, 1947.)

151. John DeCicco: *New proofs of the theorems of Kasner concerning the infinitesimal contact transformations of mechanics.*

The author submits new proofs of the theorems of Kasner concerning the infinitesimal contact transformations of general dynamics. (See *The infinitesimal contact transformations of mechanics*, Bull. Amer. Math. Soc. vol. 16 (1910) pp. 408–412)). The theorems deal with the nature of two dynamical systems of the same number of degrees of freedom for which the commutator or alternant of the associated infinitesimal contact transformations is a point transformation. The main result is that this situation can arise if and only if the expressions for the kinetic energy are the same or differ merely by a factor. The other proposition is that two infinitesimal contact transformations with the same transversality law will have a point transformation