within any time Δt be respectively $k_{i+1}\Delta t+o(\Delta t)$ and $g_i\Delta t+o(\Delta t)$. Let the probability of any other transition during Δt be $o(\Delta t)$. Let k_i's and g_i's be constant. Let $P_i(t)$ be the probability of the existence of the state i at the time t if the state 0 existed at the time $t=0$. The paper gives $\int_0^\infty t^m P_i(t)dt$ or $\int_0^\infty t^m P_i'(t)dt$ as algebraic expressions of the k_i's and g_i's. The result is obtained by using the complete homogeneous symmetric functions of the poles of the Laplace transform of $P_i(t)$ or $P_i'(t)$. (Received November 20, 1946.)

91. Gerhard Tintner: *The statistical estimation of the dimensionality of a given set of observations.*

There is a set of N observations of p variables $X_{it}=M_{it}+y_{it}$, where M_{it} is the systematic part and y_{it} the random element ($i=1, 2, \cdots, p$; $t=1, 2, \cdots, N$). The y_{it} are normally and independently distributed with means zero. There are in the population R linear independent relationships of the form: $k_{s0}+\sum_i k_{si}M_{it}=0$ ($s=1, 2 \cdots R$). A method based upon results of R. A. Fisher (Annals of Eugenics, 1938) and P. L. Hsu (ibid. 1941) assumes that we have a large sample estimate of the covariance matrix of the y_{it}: $\|V_{ij}\|$. The determinantal equation is $|a_{ij}-\lambda V_{ij}|=0$, where the a_{ij} are the sample covariances of the X_{it}. λ_1 is the smallest root of the equation, λ_2 the next smallest, and so on. $\Lambda_r=(N-1)\sum_1^r\lambda_i$. These sums of squares are distributed like χ^2 with $r(N-p-1+r)$ degrees of freedom and may be used to estimate the number R of linear relations between the M_{it}. By inserting the R smallest roots into the determinantal equation matrices are found for the computation of the coefficients k_{si} (G. Tintner, Ann. of Math. Statist., 1945). (Received October 15, 1946.)

92. Jacob Wolfowitz: *On the efficiency of unbiased sequential estimates.*

Let $f(x, \theta)$ be the distribution function (density function or probability function) of a chance variable X, which depends upon the parameter $\theta=\theta_1, \cdots, \theta_l$. Let n successive independent observations be made on X, where n is itself a chance variable, and the decision to terminate the drawing of observations depends upon the observations already obtained. Let $\theta_1^*(x_1, \cdots, x_n), \cdots, \theta_l^*(x_1, \cdots, x_n)$ be joint unbiased estimates of $\theta_1, \cdots, \theta_l$. Let $\|\lambda_{ij}\|$ be the nonsingular matrix of their covariances, and $\|\lambda^{ij}\|$ its inverse. Under certain regularity conditions it is proved that the concentration ellipsoid $\sum\lambda^{ij}(k_i-\theta_i)(k_j-\theta_j)=l+2$ always contains within itself the ellipsoid $\sum\mu_{ij}(k_i-\theta_i)(k_j-\theta_j)=l+2$, where $\mu_{ij}=EnE((\partial \log f/\partial\theta_i)(\partial \log f/\partial\theta_j))$. (Received October 21, 1946.)

Topology

93. R. F. Arens: *Pseudo-normed algebras.*

A pseudo-norm defined on a linear algebra A over, for example, the reals, is a real-valued function having the formal properties of a norm in a normed ring except that the pseudo-norm of some nonzero elements may vanish. Consequently, a pseudo-normed algebra is required by definition to have a complete system of pseudo-norms. The continuity of inversion is considered; and the singular elements related to the closed divisorless proper ideals. The space of the latter, and conditions for its compactness, are considered. As a by-product a characterization of rings of all continuous complex-valued functions on a topological space is obtained. Special attention is given to the question of completeness of quotient rings. (Received November 19, 1946.)

94. R. H. Bing: *Solution of a problem proposed by R. L. Wilder.*

It is shown that there exists a locally connected set which is not a simple closed curve but which is the sum of two sets M_1 and M_2 such that each is irreducibly connected from the point A to the point B and such that the common part of M_1 and M_2 is A and B. This answers a question posed by R. L. Wilder (*Concerning simple continuous curves and related point sets*, Amer. J. Math. vol. 53 (1931) pp. 30–55.) (Received October 17, 1946.)

95. R. H. Bing: *Some characterizations of a simple closed curve.*

We say that R cuts A from B in M if R contains neither A nor B, M contains a continuum intersecting both A and B, and each such continuum contains a point of R. If R cuts two points from each other in M, it cuts M. It is shown that a locally peripherally compact nondegenerate continuum is a simple closed curve if it is cut by no one but by each pair of its points. Furthermore, a locally peripherally compact nondegenerate continuum is a simple closed curve provided it is neither cut by any point nor separated by any of its subcontinua. (Received October 1, 1946.)

96. Claude Chevalley and Samuel Eilenberg: *Cohomology theory of Lie groups and Lie algebras.*

Given a Lie algebra L and a representation P of L with representation space V, cohomology groups $H^q(L, P)$ are defined. If L is the Lie algebra of a compact connected Lie group G, V is the field of real numbers and P is the trivial representation, these cohomology groups give the topological cohomology groups of the manifold of G. For semi-simple Lie algebras nothing is lost by limiting the definition of trivial representations. The second cohomology group is related to extensions to Lie algebras; this study yields Levi's theorem. Relative cohomology groups are constructed whenever a subalgebra L' of L is given; with the aid of this definition the main result is extended to homogenous spaces. (Received November 20, 1946.)

97. Samuel Eilenberg: *Simplicial products of simplicial complexes.*

Given two geometric simplicial complexes K_1 and K_2 the simplicial product $K_1 \triangle K_2$ is a simplicial complex defined as follows. The vertices of $K_1 \triangle K_2$ are pairs (v, w) where v is any vertex of K_1 and w any vertex of K_2. The distinct vertices $(v_0, w_0), \cdots, (v_n, w_n)$ of $K_1 \triangle K_2$ determine an n-simplex of $K_1 \triangle K_2$ provided $v_0, \cdots, v_n$ are in a simplex of K_1 and $w_0, \cdots, w_n$ are in a simplex of K_2. The simplicial product has the same invariants as the cartesian product $K_1 \times K_2$ (which is not a simplicial complex) and therefore can be used to simplify various formalisms in homology theory. (Received November 20, 1946.)

98. C. J. Everett: *Representation of a sequence of finite sets.*

A sequence of finite subsets T_i of a set S has a set of distinct representatives $t_i \in T_i$, if and only if, for every $n = 1, 2, \cdots$, the union of every n of the T_i contains at least n distinct elements of S. This modification of Theorem 1 of P. Hall (*On representation of subsets*, J. London Math. Soc. vol. 10 (1935) pp. 26–30) is obtained by use of the Cantor theorem on closed compact sets in a suitably metrized space of sequences $\{t_i\}$, $t_i \in T_i$. It is then shown that Hall's subsequent theorems extend to the infinite case. The existence of a common representation of left and right cosets of a group modulo a finite subgroup of countable index is obtained as a corollary. (Received November 1, 1946.)

99. Edwin Hewitt: *Rings of unbounded continuous functions.*

The set $C(X, R)$ of all continuous functions from a completely regular space X to the real number system R forms a linear space which is also a commutative ring with unit, under the usual definitions of addition and multiplication. For $f \in C(X, R)$, let $Z(f)$ be the set of points where f vanishes. Let $\mathfrak{Z}(X)$, for any completely regular X, be the family of all sets $Z(f)$, for $f \in C(X, R)$. A subfamily $\mathcal{A}$ of $\mathfrak{Z}(X)$ not containing 0, containing finite intersections of its elements, and containing arbitrary supersets (in $\mathfrak{Z}(X)$) of its elements is said to be maximal if no proper superfamily (in $\mathfrak{Z}(X)$) enjoys these properties. A completely regular space X is said to be a Q-space if every maximal family in $\mathfrak{Z}(X)$ has total intersection nonvoid or contains a countable subfamily with void intersection. It is proved that for every completely regular space X, there exists a Q-space αX such that $C(\alpha X, R) = C(X, R)$, $X^- = \alpha X$. Furthermore, if X and Y are Q-spaces such that $C(X, R) = C(Y, R)$, then X is homeomorphic to Y. The ring $C(X, R)$ determines the space X if X is a Q-space satisfying the first axiom of countability. In particular, $C(X, R)$ determines X if X is any separable metric space. (Received November 13, 1946.)

100. F. B. Jones: *Concerning non-aposyndetic continua.*

If a continuum M contains a point P such that $M - P$ is not strongly (continuumwise) connected, P is said to be a (weak) cut point of M. It is shown that every compact metric continuum, which is non-aposyndetic at each of its points, contains a (weak) cut point. Indecomposable continua and related types are characterized by their non-aposyndetic properties. A scheme of classification of continua (which are not locally connected) is introduced and partially investigated. (Received October 25, 1946.)

101. E. E. Moise: *An indecomposable plane continuum which is homeomorphic to each of its nondegenerate subcontinua.*

Using the existence of the continuum described, the author solves a problem of M. Mazurkiewicz, proposed in Fund. Math. vol. 2 (1921) p. 285. (Received November 18, 1946.)

102. W. T. Puckett: *On a problem in connected finite closure algebras.*

In answer to a question left open by J. C. C. McKinsey and Alfred Tarski (Ann. of Math. vol. 45 (1944) p. 160) it is shown that the closure algebra over Euclidean space is a universal algebra for the class of all connected finite closure algebras. The problem is reduced to demonstrating the existence of certain interior transformations by the following results: (1) A connected finite closure algebra is isomorphic with the closure algebra over a connected finite topological space. (2) In order that the closure algebra over a topological space S_1 be isomorphic with a subalgebra of the algebra over a space S, it is sufficient that there exist an interior transformation $f(S) = S_1$. (Received October 23, 1946.)

103. M. H. Stone: *Remarks on metrizability.*

It is proved that a continuous family (that is, an upper semi-continuous collection) of mutually disjoint nonvoid compact subsets of a metric space S is, under its weak topology, *metrizable* (being in addition *separable* whenever S is). From this result and from theorems contained in a paper of the author (Trans. Amer. Math. Soc. vol. 41

(1937) pp. 375–481), it follows immediately that every separable regular space is metrizable (as is, of course, well known). (Received October 12, 1946.)

104. Sister Petronia Van Straten: *Toroidal and non-toroidal graphs.*

While the 10 point-30 arc Desargues configuration is irreducibly non-toroidal (for the definition of the Desargues and Pappus configurations, see Bull. Amer. Math. Soc. Abstract 52-9-345), the 9 point-27 arc Pappus configuration is saturated toroidal. By this we mean that while the configuration can be topologically embedded into the torus, it loses this property if we add any arc joining two points which are not joined in the Pappus configuration. This result is a consequence of the following lemma. Any subset of the torus which is homeomorphic to a Pappus configuration decomposes the torus into triangular regions. (Received November 12, 1946.)

105. G. S. Young: *On compact fiberings of the plane.*

Montgomery and Samelson have conjectured (Duke Math. J. vol. 13 (1946) pp. 49–57) that there exists no compact fibering of Euclidean n-space. This note shows that the conjecture is true for two dimensions by proving the more general theorem: If an interior transformation of the plane, $f(P)=A$, is such that the inverses of each two points of A are homeomorphic, and such that each component of the inverse of a point of A is compact, then f is light. If the entire inverse of a point of A is compact, then f is a homeomorphism. (Received November 20, 1946.)

106. G. S. Young: *On converging sequences of 2-manifolds.*

A new characterization of a 2-manifold (with or without boundary) is given: Let M be a locally compact, locally connected metric space with no local cut points, and such that there is some positive number ϵ such that every simple closed curve of diameter less than ϵ separates M. Then M is a 2-manifold. This result is used to prove that if $\{M_n\}$ is a sequence of compact 2-manifolds converging 1-regularly to a compact set M and either for each n the boundary of M_n is empty or the boundaries of the sets M_n converge 0-regularly, then M is homeomorphic to almost all M_n. The manifold characterization is based on Bing's characterization of a sphere (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 644–653). The result on convergence generalizes, and is based on, results of Begle (Duke Math. J. vol. 11 (1944) pp. 441–450) and of Whyburn (Fund. Math. vol. 25 (1935) pp. 408–426). (Received November 20, 1946.)