385. S. A. Schaaf: *A cylinder cooling problem.*

The temperature distribution is obtained for a heat conducting region consisting of an infinitely long cylinder $0 \leqq r < a$ initially at temperature T_0, immersed in an infinite medium $r > a$ composed of a different substance initially at zero temperature, with a contact resistance condition at the interface $r = a$. The Laplace transform is used and, in inverting, it is necessary to show that $D(z) = I_0'(\alpha z)K_0(\beta z) - \lambda I_0(\alpha z)K_0'(\beta z) - \mu z I_0'(\alpha z)K_0'(\beta z)$, where $I_0(z)$ and $K_0(z)$ are Bessel functions and α, β, λ and μ are positive real numbers, does not vanish for $|\arg z| \leqq \pi/2$. This is done by considering the integral of $D'(z)/D(z)$ around a contour consisting of two semicircular arcs $|z| = R_1$, R_2 and the segments of the imaginary axis joining them. (Received September 26, 1946.)

386. Fred Supnick: *Cooperative phenomena. I. Structure of the linear Ising model.*

The partition function $f(T)$ (the physical term) plays an important part in the theory of crystal statistics (cf. C. H. Wannier, Review of Modern Physics (1945) pp. 50–60). Let the set of spins $u_1, \cdots, u_n$ each capable of two orientations be characterized by $u_i = +1$ or $u_i = -1$, and arranged in cyclic order. It is assumed that only adjacent elements interact. To evaluate $f(T)$ the interaction energy E must be found. E involves the calculation of $\Sigma \equiv \sum_{i=1}^{n} u_i u_{i+1}$ where $u_{n+1} \equiv u_1$. All spin distributions are considered in evaluating $f(T)$. The author calls Σ the interaction constant of the spin distribution. Now, let Σ_i be any integer with $|\Sigma_i| \leqq n$. In this paper the set of all possible spin distributions with interaction constants equal to Σ_i is determined. A method is given for constructing each spin distribution with $\Sigma = \Sigma_i$. Results involving the number of spin distributions with the same interaction constant are obtained. Both cyclic and non-cyclic cases are considered. (Received September 28, 1946.)

GEOMETRY

387. Germán Ancochea: *Zariski's proof of the theorem of Bertini-Enriques in the case of an arbitrary ground field.*

Zariski (Trans. Amer. Math. Soc. vol. 50 (1941)) gave a new proof of the theorem of Bertini-Enriques on reducible linear systems of V_{r-1}'s on an algebraic V_r, by considering this theorem in a larger sense than the customary since irrational pencils are also included. The proof, given for the case of ground fields of characteristic zero, is based on several lemmas concerning the behavior, with respect to irreducibility, of an algebraic variety under ground field extensions. Most of these lemmas have been extended by Chevalley to the case of arbitrary ground fields (Trans. Amer. Math. Soc. vol. 55 (1944)). In the present paper the theorem of Bertini-Enriques, in the sense of Zariski, is extended to ground fields of characteristic $p \neq 0$. The auxiliary lemmas are reconsidered from a different standpoint than that of Chevalley, by using the Chow-van der Waerden concept of an associated form of an algebraic variety. It also has been found necessary to incorporate in Zariski's definition of an irreducible pencil on V_r the extra requirement that the field of functions on V be separably generated over the ground field. With these changes the theorem of Bertini-Enriques is proved essentially as in Zariski's paper, provided that the ground field be an infinite field for the case of linear systems. (Received August 2, 1946.)

388. John DeCicco: *Constrained motion upon a surface under a generalized field of force.*

Some theorems are given concerning the trajectories of a particle moving in a given smooth surface Σ when acted upon only by a generalized field of force. Kasner has developed the differential geometry of the dynamical trajectories of general positional fields of force on a plane, in space, and on a surface, in his Princeton Colloquium Lectures. Recently Kasner and DeCicco have introduced the concept of generalized (anistropic) fields of force which depend not only upon the position of the point but also upon the direction through the point. After discussing the differential equation of third order defining generalized dynamical trajectories on the surface Σ, the ∞^1 lines of force and the ∞^2 rest trajectories are compared. A formula for the ratio ρ of the departure of the rest trajectory to that of the line of force at the initial point is obtained. In the positional case, Kasner proved that $\rho=1/3$. The new formula involves the angular rate and is similar to the one obtained for the plane in the Transactions paper of 1943. (Received September 27, 1946.)

389. John DeCicco: *Union-preserving transformations of higher surface elements*

Transformations in space from differential surface-elements of order n: $(x, y, z, p_{10}, p_{01}, \cdots, p_{j,m-j}, \cdots, p_{0n})$ where $p_{j,m-j}=\partial^m z/\partial x^j \partial y^{m-j}$ for $j=0, 1, 2, \cdots, m$ and $m=1, 2, \cdots, n$, into planar-elements (X, Y, Z, P, Q) are studied. It is shown that the union-preserving transformations are defined by a single directrix equation of the form $\Omega(X, Y, Z, x, y, z, p_{i,m-j})=0$, where $j=0, 1, 2, \cdots, m$ and $m=1, 2, \cdots, n-1$, or by a pair or triplet of such directrix equations. It is noticed that directrix equations contain partial derivatives of z with respect to x and y, of orders up to and including $n-1$, but not n. For $n=1$, this result coincides with the theorem of Lie concerning contact transformations of planar-elements. Finally it is proved that the only available union-preserving transformations in the whole domain of surface-elements are, first, Lie's group of contact transformations and, second, the author's set of union-preserving transformations from surface-elements of order n into planar-elements, together with the extensions of these two types. Union-preserving transformations of curve-elements of higher order have been studied by Kasner and the author. (Received August 15, 1946.)

390. Edison Greer and P. O. Bell: *A study of analytic surfaces by means of a projective theory of envelopes.*

A new formulation of a projective theory of envelopes for the study of algebraic surfaces which are covariantly related to an analytic surface S at a point P_x has been recently developed (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 22–50). The present paper applies this theory of envelopes to investigate relationships of covariantly determined quadrics having contact of the second order with S at P_x with systems of curves on the surface. A characteristic curve is determined on each quadric Q at each point P_x of a curve C_λ of S. The cone which contains this characteristic curve and has as its vertex the point P_x is introduced as the *characteristic cone* $\mathfrak{C}_\lambda$ *of* Q *with respect to* C_λ *at* P_x. The geometric interrelations of the cone $\mathfrak{C}_\lambda$, the quadric Q, and the curve C_λ with respect to the surface S are investigated. By subjecting two of these entities to certain geometric restrictions, properties of the third are determined. The principal results thus obtained are geometric characterizations of significant families of quadrics, systems of hypergeodesics, the canonical pencil, projective curvatures of

a curve C_λ, the general transformation of Čech, the curves of Darboux and Segre, and a class of generalized pangeodesics. (Received September 9, 1946.)

391. Walter Prenowitz: *Characterization of the lattice of convex sets of a descriptive geometry.*

L_1, the lattice of convex sets of a descriptive (ordered linear) geometry, and L_2, that of the linear spaces of a projective geometry, are characterized simultaneously. The geometries are of arbitrary dimension greater than 1. A new lattice concept *linear dependence of a point on a sequence of points* is introduced and used to define *closed* element as one which contains, with any sequence of points, all linearly dependent points. A generalization of modularity involving "closed element" is a common property of L_1 and L_2. Garrett Birkhoff's characterization of L_2 in the finite-dimensional case is deducible from the results. The simultaneous treatment of L_1 and L_2 is based on axiomatizations of descriptive and projective geometry in terms of point and "order" which differ only in a single postulate. (Received September 28, 1946.)

392. Alice T. Schafer: *The neighborhood of an undulation point on a space curve.*

This paper employs the methods of projective differential geometry to study the neighborhood of an undulation point on an analytic space curve. By a suitable choice of the projective coordinate system, canonical power-series expansions representing the curve in the neighborhood of the undulation point are deduced. These expansions are then used to study properties of the curve in the neighborhood of the point, with particular emphasis placed on osculants of the curve, projections of the curve onto the faces of the tetrahedron of reference, and sections of the tangent developable of the curve made by faces of the tetrahedron of reference. (Received September 28, 1946.)

393. Oscar Zariski: *The concept of a simple point of an abstract algebraic variety.*

Let V and W be irreducible algebraic varieties over an arbitrary ground field κ, of dimension respectively r and ρ, $W \subseteq V$. If $\mathfrak{m}$ is the maximal ideal in the quotient ring $\mathfrak{o}$ of W then the ring $\mathfrak{m}/\mathfrak{m}^2$, regarded as a vector space over the field $\mathfrak{o}/\mathfrak{m}$, is of dimension not less than $r-\rho$. If the dimension is exactly $r-\rho$, W is said to be simple for V. In the first part of the paper this general concept of a simple W is studied by purely local methods. In the second part the global theory is developed. Here the main result is a general Jacobian criterion for simple loci, which reduces to the classical one whenever κ is either of characteristic zero or is a perfect field of characteristic $p \neq 0$. This general criterion implies that the singular manifold of V is always an algebraic proper subvariety of V. An absolutely simple W is defined by the condition that it remain simple under any extension of the ground field. Criteria for an absolutely simple W are: (1) the ordinary Jacobian criterion of the classical case; (2) V is locally, at W, analytically eqivalent to a linear S_r. (Received August 22, 1946.)

Logic and Foundations

394. E. L. Post: *Recursive unsolvability of a problem of Thue.*

Thue's problem (Skrifter utgit av Videnskapsselskapet i Kristiania 1914. I.