BOOK REVIEWS

Étude des sommes d'exponentielles réelles. By Laurent Schwartz. (Actualités scientifiques et industrielles, no. 959.) Paris, Hermann, 1943. 89 pp. 85 fr.

The problems discussed here originate from Weierstrass' polynomial approximation theorem. An introductory section deals with fundamental concepts in vector spaces. Chapter I starts with a discussion of the theorem of Müntz and Szász: If $\lambda_0 = 0$, $\lambda_n \geqq \alpha > 0$ for $n \geqq 1$, then a necessary and sufficient condition for the closure of the sequence $\{x^{\lambda_n}\}$ in $C(0, 1)$ is the divergence of the series $\sum \lambda_n^{-1}$. (It is stated on page 24 that Kaczmarz and Steinhaus gave a new proof; however their proof is the same as Szász's proof.) The author then discusses in detail the case that $\sum \lambda_n^{-1}$ converges. One of the results in this case is: If $\lambda_n + 1/p > 0$ $(p > 1)$, $\lambda_n \to \infty$, $\sum_{\lambda_n > 0} \lambda_n^{-1}$ converges, and if the index of condensation of the sequence $\{\lambda_n\}$ is zero, then every function $f(z)$ belonging to the L_p span of the sequence $\{z^{\lambda_n}\}$ is analytic in $(0, 1)$, it can be continued into the region $|z| < 1$ of the Riemann surface of $\log z$, and it has an expansion $\sum a_n z^{\lambda_n}$ absolutely and uniformly convergent for $|z| < 1 - \epsilon$, $\epsilon > 0$. For the case that the λ_n are integers a similar result appears in a paper by Clarkson and Erdös (Duke Math. J. vol. 10 (1943) pp. 5–11). The author also discusses the problem of completeness for the interval (a, b), where $0 < a < b$.

In Chapter II estimates for the coefficients a_n of generalized polynomials $g(x) = \sum a_n x^{\lambda_n}$ are given when $\int_0^1 |g(x)|^p dx \leqq 1$, $p \geqq 1$. For $\lambda_n = n$ and $p = \infty$ the exact bound was given by S. Bernstein. The author finds the exact bound for $p = 2$, while for arbitrary p asymptotic estimates are given. (Hille, Szegö and Tamarkin gave asymptotic estimates for the exact bound of $|g'(x)|$; see Duke Math. J. vol. 3 (1937) pp. 729–739.)

The book has a bibliography and a table of contents. It would be interesting to consider the analogous problems in several variables.

OTTO SZÁSZ

Mathematical theory of elasticity. By I. S. Sokolnikoff. New York, McGraw-Hill, 1946. 11+373 pp. $4.50.

This book contains approximately the first half of the material of a course given by the author in 1941 and 1942 in the Program of Advanced Instruction and Research in Mechanics, conducted by the

Graduate School of Brown University. In fact, except for the last chapter, the book is a revision of part of a set of lecture notes of this course, which notes have been circulated widely in mimeographed form. A second volume is contemplated which will contain the remainder of the material in these notes. The excellence of these notes has long been recognized, and appearance in printed form of the material contained therein is most welcome.

The first three chapters contain a very complete treatment of the fundamentals of the theory of elastic bodies, that is, the analysis of strain, the analysis of stress, and the stress-strain relations. In the classical treatments of these, many notations have been employed, all of which are cumbersome. In the present volume, a departure is made in this regard, and the tensor notation is used. This results in a great compactness and economy of effort. This volume should hasten the current trend towards an increased use of the powerful methods of tensor calculus in elasticity.

Chapter 4 deals with the application of the fundamental theory to the extension, torsion, and flexure of homogeneous beams. In this one chapter the tensor notation is not employed, in order that the contents of this chapter may be available to persons unfamiliar with the tensor notation but familiar with the fundamental theory. The treatment is quite thorough, and contains among other things a detailed account of the method usually associated with the name N. I. Muschelisvili. In this method, the problems of torsion and flexure, which are reduced to a number of Dirichlet problems, are solved by conformal mapping of the cross section of the beam onto the unit circle. The fundamental equations of elasticity are obtained in terms of general orthogonal curvilinear coordinates, and are used to solve a number of problems. There is also a section on the technical theory of beams, as used by the engineer.

Chapter 5, which is the last chapter, contains material which does not appear in the mimeographed notes on which this book is based. It deals with variational methods in elasticity. Various energy and reciprocity theorems are developed, and there is also a presentation of variational methods of obtaining approximate solutions of boundary value problems. Among these are the Rayleigh-Ritz method, Galerkin's method, and the method of Biezno and Koch. There is also a treatment of the method of finite differences as applied to the approximate solution of boundary value problems, with particular application to the problem of torsion as an example.

This book is written clearly and in a manner which is easy to follow. Also, it contains many interesting exercises. Because of this it is admirably suited to the needs of the beginning student of elasticity. It contains many references to recent papers in elasticity, many of which are in Russian and hence not generally known. Because of this it should prove useful to the more experienced student as a reference book.

G. E. HAY