BOOK REVIEWS

Étude des sommes d'exponentielles réelles. By Laurent Schwartz. (Actualités scientifiques et industrielles, no. 959.) Paris, Hermann, 1943. 89 pp. 85 fr.

The problems discussed here originate from Weierstrass' polynomial approximation theorem. An introductory section deals with fundamental concepts in vector spaces. Chapter I starts with a discussion of the theorem of Müntz and Szász: If $\lambda_0=0$, $\lambda_n \geqq \alpha > 0$ for $n \geqq 1$, then a necessary and sufficient condition for the closure of the sequence $\{x^{\lambda_n}\}$ in $C(0, 1)$ is the divergence of the series $\sum \lambda_n^{-1}$. (It is stated on page 24 that Kaczmarz and Steinhaus gave a new proof; however their proof is the same as Szász's proof.) The author then discusses in detail the case that $\sum \lambda_n^{-1}$ converges. One of the results in this case is: If $\lambda_n + 1/p > 0$ $(p > 1)$, $\lambda_n \to \infty$, $\sum_{\lambda_n > 0} \lambda_n^{-1}$ converges, and if the index of condensation of the sequence $\{\lambda_n\}$ is zero, then every function $f(z)$ belonging to the L_p span of the sequence $\{z^{\lambda_n}\}$ is analytic in $(0, 1)$, it can be continued into the region $|z| < 1$ of the Riemann surface of $\log z$, and it has an expansion $\sum a_n z^{\lambda_n}$ absolutely and uniformly convergent for $|z| < 1-\epsilon$, $\epsilon > 0$. For the case that the λ_n are integers a similar result appears in a paper by Clarkson and Erdös (Duke Math. J. vol. 10 (1943) pp. 5–11). The author also discusses the problem of completeness for the interval (a, b), where $0 < a < b$.

In Chapter II estimates for the coefficients a_n of generalized polynomials $g(x) = \sum a_n x^{\lambda_n}$ are given when $\int_0^1 |g(x)|^p dx \leqq 1$, $p \geqq 1$. For $\lambda_n = n$ and $p = \infty$ the exact bound was given by S. Bernstein. The author finds the exact bound for $p = 2$, while for arbitrary p asymptotic estimates are given. (Hille, Szegö and Tamarkin gave asymptotic estimates for the exact bound of $|g'(x)|$; see Duke Math. J. vol. 3 (1937) pp. 729–739.)

The book has a bibliography and a table of contents. It would be interesting to consider the analogous problems in several variables.

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Mathematical theory of elasticity. By I. S. Sokolnikoff. New York, McGraw-Hill, 1946. 11+373 pp. $4.50.

This book contains approximately the first half of the material of a course given by the author in 1941 and 1942 in the Program of Advanced Instruction and Research in Mechanics, conducted by the