ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

222. H. S. M. Coxeter: *A simple proof of the eight square theorem.*

It is shown, by direct comparison of terms, that the conjugates of Cayley numbers have the properties $\overline{ab}=\bar{b}\bar{a}$ and $\bar{a}a\cdot b=\bar{a}\cdot ab$. Defining $a^{-1}=(Na)^{-1}\bar{a}$, it is deduced that $b=a^{-1}\cdot ab$ and similarly $ab\cdot b^{-1}=a$. As Ruth Moufang remarked, these relations imply $b^{-1}a^{-1}=(ab)^{-1}$. One now has $N(ab)=\overline{ab}\cdot ab=b\bar{a}\cdot ab=Nb\cdot Na\cdot b^{-1}a^{-1}\cdot ab$ $=Na\cdot Nb$. (Cf. Dickson, Ann. of Math. (2) vol. 20 (1919) p. 164.) (Received May 29, 1946.)

223. H. S. M. Coxeter: *Integral Cayley numbers.*

The Cayley numbers $a_0+a_1i+a_2j+a_3k+(a_4+a_5i+a_6j+a_7k)h$, where $h=2^{-1}(i+j+k+e)$ and the a's are arbitrary integers, are shown to be a set of integral elements in the sense of Dickson (*Algebras and their arithmetics*, Chicago, 1923, pp. 141–142). The integral Cayley numbers of norm 1, 2 and 4 are represented by the vertices of the eight-dimensional polytopes 4_{21}, 2_{41} and 1_{42}. (Received May 29, 1946.)

224. Roy Dubisch: *On the direct product of rational generalized quaternion algebras.*

Utilizing the results of Albert (Bull. Amer. Math. Soc. vol. 40 (1934) pp. 164–176) on the integral domains of rational generalized quaternion algebras and the results of Latimer (Duke Math. J. vol. 1 (1935) pp. 433–435) on the fundamental number of a rational generalized quaternion algebra, the author proves that any finite number of such algebras $\mathfrak{Q}_1, \cdots, \mathfrak{Q}_n$ contain a common quadratic subfield $\mathfrak{Z}$ and $\mathfrak{Q}_i=(\mathfrak{Z}, S, -d_i)$ $(i=1, \cdots, n)$ where d_i is the fundamental number of $\mathfrak{Q}_i$. Then $\mathfrak{Q}_1\times \cdots \times\mathfrak{Q}_n\sim\mathfrak{B}=(\mathfrak{Z}, S, \pm\Pi d_i)$. (Received May 27, 1946.)

225. Orrin Frink: *Complemented modular lattices and projective spaces of infinite dimension.*

Garrett Birkhoff (Ann. of Math. vol. 36 (1935) pp. 743–748) showed that every complemented modular lattice of finite dimension is the direct union of the lattices of subspaces of projective geometries. In this paper complemented modular lattices in general, without restriction on the dimension, are characterized as subdirect unions of the subspace lattices of projective planes and irreducible projective coordinate spaces of possibly infinite dimension. It is shown that every complemented modular lattice determines a unique projective space whose points are the maximal dual ideals of the lattice. Conversely, the lattice of subspaces of every projective space is complete, atomic, complemented, and modular. Every irreducible projective space not a projective plane is a coordinate space, and is determined by a division ring and by a cardinal number which may be infinite, namely its dimension. In particular, points, lines, planes and hyperplanes may be added to continuous geometries by the use of lattice ideals. This may permit the introduction of coordinates into continuous geometries by the method of von Staudt. (Received May 11, 1946.)

226. Irving Kaplansky: *Topological rings.*

The paper is devoted to the study of (i) the radical (in the sense of Jacobson, Amer. J. Math. vol. 67 (1945)) of a topological ring, (ii) the structure of compact rings, (iii) rings of functions, (iv) rings of endomorphisms. The radical is closed if the quasi-regular elements are either open or closed, but an example is given of a ring whose radical is not closed. The radical of a compact ring is the union of all (topological) nil ideals. A compact semisimple ring is continuously isomorphic to a complete direct sum of finite simple rings. Compact integral domains are local rings—possibly without chain condition. The ring of functions from a topological space S to a topological ring A is studied; results known when A is the reals or complexes are proved for any simple ring A provided S is totally disconnected. Ideal theory in the ring is also examined. The regular representations of a topological ring and the structure of the ring of continuous endomorphisms of certain topological groups comprise the final section. (Received May 31, 1946.)

227. J. C. C. McKinsey: *On the representation problem for projective algebras.*

C. J. Everett and S. Ulam (*Projective algebra* I, Amer. J. Math. vol. 68 (1946) pp. 77–88) have recently given postulates for projective algebras, and have solved the representation problem for all complete atomic projective algebras. The author shows that every projective algebra is isomorphic to a subalgebra of a complete atomic projective algebra; this result, combined with that of Everett and Ulam, provides a solution of the representation problem for all projective algebras. The method of proof is similar to the method used by M. H. Stone to establish a representation theorem for Boolean algebras. If A is an additive prime ideal, then A_x is defined to be the additive prime ideal which contains all elements of the form a_x, for a in A. If A and B are additive prime ideals, then $A\square B$ is defined to be the class of all additive prime ideals C which contain all elements of the form $a\square b$, where a is in A and b is in B. These definitions are extended in an obvious way so as to permit operations on classes of prime ideals. (Received April 29, 1946.)

Analysis

228. Richard Bellman: *Stability of difference equations.*

Using methods developed in the study of stability of differential equations (cf. the author's paper, *The stability of solutions of linear differential equations*, Duke Math. J. vol. 10 (1943)), the matrix difference equation $X_{n+1}=(A+B)X_n$, where B is a perturbation matrix, is treated, and various hypotheses are made as to the solutions of $X_{n+1}=AX_n$ and the nature of B. Analogues of results of Liapounoff for differential equations are also obtained. (Received April 10, 1946).