

many absolute points (points lying on their polars) as there are points on a line. This minimum may be attained. If there are more than the minimum number of absolute points, then the number of points on a line reduced by one is a square. More detailed information is available for regular polarities which have the property that any two lines which are not absolute, but carry absolute points, carry the same number of absolute points. These results are applied to prove a generalization of a theorem due to Topel which asserts that every geometry of Bolyai-Lobachevskii is infinite. (January 18, 1946.)

90. T. C. Doyle: *Tensor theory of invariants for the projective differential geometry of a ruled surface.*

The differential equations of Wilczynski defining a ruled surface to within a projective transformation are expressed in the tensor form $y_i \dots = (U^2 U_j^r + \rho \delta_j^r) y_r$, and from the tensor coefficients and arguments of these equations there is derived by formal tensor processes the same complete system of invariants and covariants of a ruled surface as first derived by Wilczynski, using integrational methods, in his *Projective differential geometry of curves and surfaces*, Leipzig, Teubner, 1906. One arrives at a systematic procedure for the transition from canonical to general forms and in this way the invariant equations of many of the covariant loci heretofore known only in their canonical forms are displayed. (Received December 17, 1945.)

91. H. W. Eves: *Arc chains and arc necklaces.* Preliminary report.

An arc chain is a sequence of circular arcs (called links), of arbitrary central angles and radii, trailed end to end. If the end points of the chain coincide one has an arc necklace. This paper deals with the elementary geometry of plane and spherical arc chains and necklaces. In addition to a number of new theorems, several well known results of elementary geometry are generalized to hold for arc chains and necklaces. Of particular interest are those arc chains and necklaces in which all the links lie on the circumferences of a two-parameter family of circles, for example, on the circumferences of a family of concurrent circles. Some attention is paid to arc necklaces whose vertices are concyclic. (Received January 27, 1946.)

92. D. P. Ling: *Geodesics on surfaces of revolution.*

The author investigates the number and the distribution of double points of the geodesics on the members of a broad class of surfaces of revolution. A "zoning" of these surfaces is established in a manner dictated by this distribution. It is shown that each surface falls into one or another of three subclasses according as each geodesic has infinitely many double points, a finite number of double points bounded from above for the whole set of geodesics, or a finite number which by a proper choice of the geodesic can be made arbitrarily large. Analytic means of distinguishing between these subclasses is set up, and a particular class of surfaces is given to serve as illustration and counter example. (Received December 3, 1945.)

LOGIC AND FOUNDATIONS

93. G. D. Birkhoff and Garrett Birkhoff: *Distributive postulates for systems like Boolean algebras.*

By slightly strengthening Newman's postulates for direct sums of Boolean algebras and Boolean rings, a simpler proof of sufficiency is obtained. A related set of postulates for distributive lattices is given, together with a discussion of alternative

postulate systems for Boolean algebras and distributive lattices. (Received January 23, 1946.)

94. Archie Blake: *A Boolean derivation of the Moore-Osgood theorem.*

The process of proving a mathematical theorem is represented in symbolic logic by the transformation of logical expressions. This fact is illustrated in the case of the Moore-Osgood theorem, the central features of the derivation of which are shown to be representable in the *Prädikatenkalkül*. (Received January 12, 1946.)

95. Ira Rosenbaum: *Hegel's observations on the differential and integral calculus and its foundations.*

Attention is invited to an extended discussion of the differential and integral calculus and its foundations which appears in Hegel's *Science of logic*. The technical content, character, and interest of Hegel's discussion is indicated by citing authors, texts, methods, and problems with which Hegel dealt. The evidence relating to Hegel's knowledge of mathematics is presented and a picture of the development of Hegel's views is traced; relevant portions of the first and later editions of the *Logic* are compared. Hegel's relation to his contemporaries in mathematics is pointed out. The relevant literature is reviewed critically and after indicating the prevalent neglect of Hegel's discussion, it is concluded that such examination of Hegel's relation to the calculus as does exist is (1) dated, (2) incomplete and/or inadequate, (3) generally independent of earlier and contemporary work in the same field, and (4) limited in scope and point of view. An instance of the unsatisfactory state of the literature on this subject is considered. Hegel's observations on the calculus are examined, placed in their proper historical context, and his views compared with those of his predecessors, contemporaries, and successors. Evaluation from the standpoint of the modern logico-mathematical foundations of analysis is undertaken. (Received February 1, 1946.)

STATISTICS AND PROBABILITY

96. G. W. Brown and J. W. Tukey: *Some distributions of sample means.*

It is shown that certain monomials in normally distributed quantities have stable distributions with index 2^{-k} . This provides, for $k > 1$, simple examples where the mean of a sample has a distribution equivalent to that of a fixed, arbitrarily large multiple of a single observation. These examples include distributions symmetrical about zero, and positive distributions. Using these examples, it is shown that any distribution with a very long tail (of average order greater than or equal to $x^{-3/2}$) has the distributions of its sample means grow flatter and flatter as the sample size increases. Thus the sample mean provides *less* information than a single value. Stronger results are proved for still longer tails. (Received January 14, 1946.)

TOPOLOGY

97. R. H. Bing: *Generalization of a theorem of Janiszewski.*

Suppose that H and K are plane sets neither of which cuts the point A from the point B , that the boundary of H is compact, that the junction of H and K is equal to