a notion which has also been introduced by A. D. Michal this may be re-expressed in the form: If a function on an open set of a complex Banach space to a complex Banach space has a derivative, it possesses a Fréchet differential. (Received January 22, 1946.)

Applied Mathematics

79. R. J. Duffin: *Nonlinear networks.* II.

A system of n nonlinear differential equations is studied and shown to have a unique asymptotic solution; that is, all solutions approach each other as the independent variable becomes infinite. The interest of these equations is that they describe the forced vibration of electrical networks. Consider an arbitrary linear network of inductors, resistors, and capacitors which has no undamped free modes of vibration. A given impressed force may give rise to more than one response but as time goes on there is a unique association between impressed force and response. This, of course, is well known. The main result of this note states that if the linear resistors of such a network are replaced by quasi-linear resistors then there still is this unique asymptotic association. A quasi-linear resistor is one in which the potential drop across it and the current through it are increasing functions of one another. No other sort of nonlinearity besides this type of nonlinear damping is considered. The proof is made to rest solely on well known properties of the Laplace transform and Hermitian forms. (Received January 19, 1946.)

80. Herbert Jehle: *Transformation of hydrodynamical equations of stellar dynamics.*

In abstract 51-9-170, the author pointed out a transformation of continuity equation and Bernoulli equation into a Schroedinger equation. The presence of $\bar{\sigma}$ (replacing $\hbar/m$ of wave mechanics) implies no modification of classical equations of motion, but a statement about residual velocities or "pressure function." Assume that the distribution (numbers and intensities) of excited ψ_{nlm} states (n goes up to about 10^5 for the author's choice of $\bar{\sigma}$) corresponds statistically to the distribution (in numbers and masses) of statistically independent elements of a system. It is known that if all stationary ψ_{nlm} states are filled up to a certain frequency limit with one element (particle) per state there will be an average of one element per phase space volume $(2\pi\bar{\sigma})^3$ for the inner regions. The above assumption is therefore equivalent to the assumption of an upper limit for the expectation value of density (of numbers) of statistically independent elements in six-dimensional phase space. This is a plausible assumption for systems close to statistical equilibrium; it means that too great densities in position space without large residual velocities cause aggregations of formerly independent masses into larger independent units. (Received January 21, 1946.)

81. R. S. Phillips: *rms error criterion in servo system design.*

A servomechanism is required to follow a signal from a knowledge of only the error in following. This error signal is usually a mixture of the true following error and some sort of random disturbance. The servo must make a compromise between following the original signal and not following the noise. This paper presents a method by which this compromise can be made. Assuming that the spectra of the signal and noise are known, one can then determine that servo system which minimizes the rms error in following. Actually the paper limits itself to determining the best values of control parameters when the type of control is given. It is assumed that the servo can be represented by a linear differential equation with constant coefficients and that the spectra can be approximated by rational fractions. The problem is then reduced to the evaluation of integrals of the type $I_n = (2\pi i)^{-1}\int_{-\infty}^{\infty}(g_n(x)/f_n(x)\cdot f_n(-x))dx$ where $f_n(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$, $g_n(x) = b_0x^{2n-2} + b_1x^{2n-4} + \cdots + b_{n-1}$, and all the roots of f_n lie in the upper half-plane. The integrals are evaluated in terms of the coefficients of the polynomials f and g. The paper concludes with a table of integrals for $n \leqq 7$. (Received January 18, 1946.)

82. Harry Polachek: *Solution of the differential equations of motion of a projectile in a medium of quasi-Newtonian resistance.*

A solution is obtained in closed form for the differential equations of motion of a projectile in a medium of quasi-Newtonian resistance. The theory of differential variations as developed by Bliss, Moulton and Gronwall is applied. Use of this result may be made to solve a large class of ballistic problems, which are now being solved by indirect numerical methods. It is especially applicable to the case of projectiles traveling with velocities which are entirely outside the trans-sonic region. (Received January 14, 1946.)

83. Eric Reissner: *Stresses and small displacements in shallow spherical shells.*

A system of two simultaneous equations for two unknown functions F and w is derived for the bending and stretching of shallow segments of thin, elastic, spherical shells. In these equations F is Airy's stress function for the direct stress resultants and w the normal component of displacement. Writing $F_n = \cos n\theta f_n(r)$, $w_n = \cos n\theta g_n(r)$ the equations are such that f_n and g_n result as combinations of elementary functions and of Bessel functions of order n and argument proportional to $i^{1/2}$. The formulas obtained, particularly for the nonsymmetrical case, are appreciably simpler than the known results for spherical shells without the assumption of shallowness. (Received January 17, 1946.)

84. P. A. Samuelson: *A connection between the Bernoulli and Newton iterative processes.*

By the Bernoulli method, the root of a polynomial $f(X) = \sum_0^n a_j X^{n-j} = \prod_1^n (X - X_i) = 0$ is approximated by $Y_{t+1}/Y_t = (C_1X_1^{t+1} + \cdots + C_nX_n^{t+1})/(C_1X_1^t + \cdots + C_nX_n^t)$, where $\sum_0^n a_j Y_{t+n-j} = 0$, $Y_0 = b_0, \cdots, Y_{n-1} = b_{n-1}$, and the C's depend upon the X's and the b's. Weights w_k are sought such that $Z_{t+1}/Z_t = \sum_0^n w_k Y_{t+1-k}/\sum_0^n w_k Y_{t-k}$ is a good approximation to X_i, by virtue of the fact that coefficients of the *other* exponential terms are made to vanish. Clearly the w_k should be coefficients of $P(X)/(X - X_i)$ to give the exact root, X_i, in one step. If, instead, one uses the coefficients of $P(X)/(X - X_i^0)$, where X_i^0 is an approximate root, and sets $b_j = (X_i^0)^j$, then the newly calculated root $\overline{X}_i = Z_n/Z_{n-1} = X_i^0 - [f(X_i^0)/f'(X_i^0)]$, which is identical with the Newton approximation to a root. (Received February 1, 1946.)

85. P. A. Samuelson: *Computation of characteristic vectors.*

Wayland has shown (Quarterly of Applied Mathematics vol. 2 (1945) p. 277) that the method of Danielewsky and the method of elimination are the two most efficient known ways of computing the coefficients of the characteristic equation. This note shows: (1) the Danielewsky reduction can be interpreted as a special case of the method of elimination; (2) the characteristic vectors of a companion matrix with simple roots are given by a Vandermonde matrix and its easily derived inverse; (3) the characteristic vectors of any matrix can therefore be derived with about $2n^3$ multiplications all told, by applying the Danielewsky transformations to the vectors of the corresponding companion matrix. (Received February 1, 1946.)

86. P. A. Samuelson: *Generalization of the Laplace transform for difference equations.*

The Laplace transformation has standardized operational methods in the field of ordinary differential equations. Its efficacy hinges on the fundamental relation $L(s;\ Df)_D = sL(s;f)_D - f(0)$ where $L(s;f)_D = \int_0^\infty \exp(-st)f(t)dt$. The Laplace transform has been applied to difference equations, but it is a clumsy tool there by virtue of the fact that it does *not* satisfy a similar fundamental relation with respect to the shifting operator E. One can easily verify that the linear functional $L(s;\ f)_E = \sum_0^\infty f(i)s^{-i-1}$ does have the fundamental property $L(s;\ Ef)_E = sL(s;f)_E - f(0)$. This generalized transform can also be easily inverted by the calculus of residues and extended by suitably defined "convolution." Consequently, after a table of "generalized" transform pairs has been drawn up, the solution of ordinary difference equations can be derived by operational methods *exactly* like those of differential equations. The most important of these transform pairs is $y(t) = t(t-1), \cdots, (t-n+1)a^{t-n}$ and $\bar{y}(s) = (s-a)^{-n}(n-1)!$, $|s| > |a|$. (Received February 1, 1946.)

87. C. A. Truesdell: *On Sokolovsky's "momentless shells."*

V. V. Sokolovsky (Applied Mathematics and Mechanics n.s. vol. 1 (1937) pp. 291–306) has given expressions for the membrane stress resultant Fourier coefficients for surfaces of revolution whose meridians may be expressed in Cartesian coordinates in the forms: $f = kz^\mu$; $f = a \sin^c \phi$, $z = -ca\int \sin^c \phi d\phi$; $f = a \sec^c \phi$, $z = -ca\int \sec^c \phi \tan^2 \phi d\phi$. The first family has already been treated and generalized by the author. In the present note the author shows that a slight modification of his previous treatment of Nemenyi's stress functions enables us quickly to find solutions in terms of hypergeometric functions for the family of surfaces whose meridian is $f = a \sin^p \xi$, $z = -pb\int \sin^p \xi \tan^q \xi d\xi$, including Sokolovsky's second and third families of surfaces as special cases. Surfaces having meridians given by an error integral curve, $z = ap!\int_0^{kf} \exp(-t^p)dt$, are shown by the same means to have solutions in terms of Whittaker functions. (Received January 29, 1946.)

88. Alexander Weinstein: *On Stokes' stream function and Weber's discontinuous integral.*

It is shown that the stream function ψ corresponding to sources distributed with the density one over a circumference C is a many-valued function with the period $4\pi a$, where a denotes the radius of C. This fact, combined with the divergence theorem, yields a new proof for Weber's formula (J. Reine Angew. Math. vol. 75 (1873) p. 80) for the discontinuous integral $\int_0^\infty J_0(as)J_1(bs)ds$, which is equal to $1/b$ for $b > a$, and to 0 for $a > b$. (Received January 17, 1946.)

Geometry

89. Reinhold Baer: *Polarities in finite projective planes.*

It is shown that every polarity in a finite projective plane possesses at least as