ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

53. A. A. Albert: *On Jordan algebras of linear transformations.*

Consider any linear space A of linear transformations $a, b, \cdots$ over a field F of characteristic not two such that $a \cdot b = 1/2(ab+ba)$ is in A for every a and b of A where ab is the associative product. Then A is called a *Jordan algebra* with respect to the operation $a \cdot b$. The radical of A is defined to be its maximal solvable ideal N and it is shown to be solvable if and only if all transformations in A are nilpotent. A trace criterion is derived in case F is nonmodular, and a Pierce decomposition relative to an idempotent is obtained. The trace condition is then used to derive the analogues of the associative algebra theorem in the case of a principal idempotent. If $N=0$, A is called semisimple and it is shown that A is a direct sum of simple algebras. All simple Jordan algebras over an algebraically closed field are obtained and the results may be combined with those of a Chicago dissertation of G. Kalisch to yield all simple Jordan algebras over any nonmodular field. (Received December 21, 1945.)

54. B. L. Brown and N. H. McCoy: *Radicals and subdirect sums.*

Suppose that in each ring $\mathfrak{R}$ there is assigned a mapping F of $\mathfrak{R}$ into the set of two-sided ideals of $\mathfrak{R}$ in such a way that if $a \rightarrow \bar{a}$ is a homomorphism of $\mathfrak{R}$ on $\overline{\mathfrak{R}}$, then $F(\bar{a}) = \overline{F(a)}$. An element a of $\mathfrak{R}$ is *F-regular* if and only if $a \in F(a)$, and an ideal is *F-regular* if and only if each of its elements is F-regular. The F-radical $\mathfrak{N}$ of $\mathfrak{R}$ is to consist of the elements of $\mathfrak{R}$ which generate F-regular two-sided ideals. It is shown that $\mathfrak{N}$ is a two-sided ideal in $\mathfrak{R}$, that $\mathfrak{R}/\mathfrak{N}$ has zero F-radical and that, under certain conditions, the F-radical of the matrix ring $\mathfrak{R}_n$ is $\mathfrak{N}_n$. The vanishing of the F-radical is necessary and sufficient that $\mathfrak{R}$ be isomorphic to a subdirect sum of subdirectly irreducible rings of zero F-radical. Various choices of F are discussed, a case of special interest being $F_1(a) = \{x + ax + \sum y_i z_i + \sum y_i a z_i;\ x,\ y_i,\ z_i \in \mathfrak{R}\}$. In the presence of the descending chain condition for right ideals, the F_1-radical coincides with the radical as defined by Jacobson (Amer. J. Math. vol. 67 (1945) pp. 300–320) and also with the various other definitions. The vanishing of the F_1-radical is necessary and sufficient that $\mathfrak{R}$ be isomorphic to a subdirect sum of simple rings, each with unit element. (Received January 30, 1946.)

55. Marshall Hall: *The sum of continued fractions.*

By proving a general theorem on Cantor point sets, it is shown that every real number modulo 1 is the sum of two continued fractions whose partial quotients do not exceed 4. The number 4 is the best possible value for this theorem. As an application it is shown that every number less than $1/32^{1/2}$ is the minimum of a quadratic form of discriminant 1, a result which contrasts with the Markoff theorem that the number of minima exceeding any number greater than 1/3 is finite. (Received January 29, 1946.)

56. L. K. Hua: *Orthogonal classification of Hermitian matrices.*

The paper is concerned with the orthogonal conjunctiveness of the Hermitian matrices H and K with complex elements. H and K are said to be conjunctive orthogonally if $K=\bar{P}HP'$ for $PP'=I$. Then $\bar{H}H$ and $\bar{K}K$ are similar and have the same elementary divisors. The nature of the elementary divisors of a matrix T expressible as such a product $\bar{H}H$ is determined and a discussion is given of the existence of an H, unique apart from orthogonal conjunctiveness, such that $\bar{H}H$ has a prescribed set of elementary divisors. Canonical forms are given. (Received December 21, 1945.)

57. R. E. Johnson and Fred Kiokemeister: *The endomorphisms of the total operator domain of an infinite module.*

Let P be a division ring, and let Ξ be a P-module with a countable P-basis. The total operator domain Δ of Ξ can be identified with an infinite matrix ring over a division ring $\bar{P}$ anti-isomorphic with P. The present paper is a study of the endomorphisms of Δ. It is shown that the only nonzero endomorphisms of Δ are the meromorphisms. Any meromorphism is complete in the sense that it preserves certain infinite sums which are defined in Δ. A representation is found for the class of all meromorphisms. The $\bar{P}$-meromorphisms are shown to be expressible as a direct sum of inner meromorphisms, while any $\bar{P}$-automorphism is itself inner. (Received January 23, 1946.)

58. G. K. Kalisch: *Completion of topological rings and fields.*

This paper deals with the possibility of completing topological rings and fields, the principal result being a proof that any topological ring can be completed (cf. in this connection D. van Dantzig, *Zur topologischen Algebra*, I, Math. Ann. vol. 107 (1932) pp. 587–626, in particular TR. 21 and TR. 23 on p. 620, his "Ringkomplettierungsaxiom"). (Received January 21, 1946.)

59. D. H. Lehmer: *Some non-existence theorems on partitions.*

According to theorems due to Euler and Rogers-Ramanujan, every positive integer n may be partitioned into distinct parts in as many ways as into odd parts; and n may be partitioned into parts differing by two or more in as many ways as into parts congruent to 1 or 4 modulo 5. The present note proves that there exists no set S of positive integers with the property that every positive integer may be partitioned into parts differing by d or more in as many ways as into parts taken from the set S either with or without repetitions allowed, except for the above mentioned cases where $d=1$ and 2. A similar theorem holds for compositions. (Received January 21, 1946.)

60. J. M. Thomas: *Division sequences.*

Formulas are developed for the Euclid and Sturm sequences defined by polynomials of $R[x]$, where R is an integrity domain. The polynomials of the sequences are expressed by certain determinants formed from the resultant by recursion relations. The results are complete in the sense that they apply either to indeterminate or to numerical coefficients for arbitrary degree differences. (Received January 16, 1946.)

61. Leonard Tornheim: *A method for determining the number of compositions of certain types.*

In the remainder on division of x^m by $x^n - a_1x^{n-1} - \cdots - a_n$, the coefficient of x^{n-1} is $\sum a_{i_1}a_{i_2}\cdots$, where the sum is over all compositions $(i_1, i_2, \cdots)$ of $m-n+1$ with every element $i \leqq n$. By assigning to $a_1, \cdots, a_n$ certain numerical values the total number of compositions with elements $i \leqq n$ of the following types are obtained: (1) all such compositions; (2) all with certain elements absent; (3) all containing certain elements a given number of times; and (4) all with (2) and (3) holding jointly. Also it is possible with this method to find the total number of partitions with parts not greater than n, to list them, and to determine how many compositions correspond to a given partition. (Received January 17, 1946.)

Analysis

62. Gertrude Blanch: *On the computation of Mathieu functions.*

If a characteristic value of Mathieu's differential equation is known to within an error λ, it is possible, by fairly simple means, to correct the characteristic value and the approximate values of the Fourier coefficients defining the solution, to within an error proportional to λ^2. The precise formulas for the corrections, a systematic method of carrying out the computations, and two illustrative examples are given in this paper. Although Mathieu functions only are dealt with, the method is applicable to solutions of other types, where the coefficients of the Fourier (or power) series are determined by a three-term recurrence formula. (Received January 12, 1946.)

63. R. C. Buck: *On a class of entire functions.* I.

Let $K(a, c)$ denote the class of entire functions of order 1 and of type at most a on the whole real axis and type c on the whole imaginary axis, with $c<\pi$. This class has the property that a function $f(z)$ belonging to it is determined completely by the sequence $f(n)$, $n=1, 2, \cdots$. The author first proves a necessary and sufficient condition that for a given sequence w_n there exist a function $f(z)$ of $K(a, c)$ such that $f(n)=w_n$. Using this, a wide variety of theorems is obtained. For example, if $f(n)=r_ne^{i\theta_n}$ and if there is an α, $c<\alpha\leqq\pi$, such that $\lim\inf\{\cos(\theta_n+n\alpha)\}^{1/n}>0$, then $f(z)\equiv 0$, if $f(z)$ belongs to $K(a, c)$. As a special case, if $\mathcal{D}_1$ and $\mathcal{D}_2$ are two disjoint closed convex sets and if $f(n)\in\mathcal{D}_1$ for $n\in\mathcal{A}_1$, $f(n)\in\mathcal{D}_2$ for $n\in\mathcal{A}_2$ where $\mathcal{A}_1\cup\mathcal{A}_2$ has density 1, and if $\mathcal{A}_1$ has density 1/2 and satisfies an additional condition, then $\lim\sup\log|f(iy)|/|y|\geqq\pi$. In case the w_n are close enough to integral values, more can be said. Thus, for example, it is proved that no function of $K(0, 0)=K_0$ can take prime values at the integers without being constant. (Received January 17, 1946.)

64. R. C. Buck: *On a class of entire functions.* II.

The results of the previous paper are extended to more general sequences than $\{f(n)\}$, treating instead numbers F_n expressible as linear combinations of the numbers $f(n)$. Most of the theorems proved for the sequence $f(n)$ hold also for F_n. Thus, for example, if $F_n=\sum_0^n g(k)f(k)$ where $g(z)$ belongs to K_0 and $f(z)$ to $K(a, c)$ for $c<\pi$, then for any $\epsilon>0$, $\log|F_n|>(h(0)-\epsilon)n$ for a set of maximum density at least $1-c/\pi$. These theorems also have corollaries concerned with the solution of certain