index or bibliography, but there are historical and bibliographical notes at the ends of the first two chapters. This is in every way a welcome and useful addition to the literature of logic.

ORRIN FRINK, JR.

*Mathematical theory of optics.* By R. K. Luneberg. (Supplementary notes by M. Herzberger.) Providence, Brown University, 1944. 17+6+401+93 pp. $4.00. (Mimeographed.)

In this book, geometrical optics and diffraction are both treated as special applications of the electromagnetic theory of light. This treatment has advantages in some respects and disadvantages in others. Some types of problems, for instance those of polarized light, are handled easily, while the development of the problems of geometrical optics proper becomes somewhat involved.

The author has made several ingenious contributions in this connection. The idea of treating geometrical optics as a special limiting case of wave optics has never been carried out before in so much detail. The author has nowhere restricted himself to homogeneous media, following in the path of W. R. Hamilton. He covers first-order theory in general systems, third-order image-error theory in rotationally symmetrical systems, and the diffraction theory of spherical and near-spherical waves. A special chapter is devoted to media with parallel layers of constant refractive index.

Luneberg's treatment of diffraction optics is an important step forward. His fundamental contribution may be described as follows:

The light distribution is assumed to be known in a plane of infinite extent, and represented by the function $f(x_0, y_0)$. The light distribution may be only sectionally continuous inside a finite area, but outside this area the function must be small and continuous, and grow smaller with increasing distance from the center of the plane. Specifically, outside the limited area, $f$, $\partial f/\partial x_0$, and $\partial f/\partial y_0$ must all be continuous and smaller than $B(x_0^2+y_0^2)^{-1/2}$, where $B$ is a constant. A further condition is that the resulting light distribution in space, represented by the function $u(x, y, z)$, shall be indistinguishable from that of a spherical wave at great distances from the plane, since at great distances the finite area of the plane is indistinguishable from a point. Mathematically, this condition is that, beyond a certain large distance from the center of the plane represented by $R=(x^2+y^2+z^2)^{1/2}$, the absolute values of $u$ and $\partial u/\partial R$ shall be smaller than $C/R$, and the absolute value of the expression $\partial u/\partial R - iku$ shall be smaller than $D/R^2$, where $D$ and $C$ are constants.

The solution for the light distribution in space under these conditions is given as

$$u(x, y, z) = -\frac{z}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x_0y_0)\left(\frac{ik}{r^2} - \frac{1}{r^3}\right)e^{ikr}dx_0dy_0$$

where $r=((x-x_0)^2+(y-y_0)^2+z^2)^{1/2}$. The solution is found by the application of a mathematical technique which is reminiscent of a method used by A. Sommerfeld (Math. Ann. vol. 47 (1896)) for the solution of another problem of diffraction.

Luneberg proves that this solution fulfills all the conditions imposed, and he proves furthermore that all of these conditions are necessary to guarantee that the solution shall be unique.

It should be noted that this solution demands only the knowledge of $u$ in the plane $z=0$, and not its normal component. Thus are avoided the difficulties encountered by Kirchhoff and Abbe in their early attempts to solve the problem.

The book has some of the advantages and also some of the shortcomings of lecture notes. A new edition might be more helpful to the reader if certain formal changes were made. For instance, the reviewer found it very disturbing that the author does not distinguish between the symbols for scalars and those for vectors, especially in view of the fact that the author uses vector integration.

At many points the author might with advantage have stated the physical reasons for his assumptions before proceeding with the mathematical argument. It is also regrettable that the book quotes little of the recent literature in the fields of geometrical optics and diffraction.

Nevertheless, we may say in conclusion that the book contains important new material, interestingly presented. It is highly recommended as a source of valuable information.

M. Herzberger

It was very fortunate that Herzberger cared to add some supplementary notes to Luneberg's book. The first supplement deals with so-called electron optics and gives an excellent account of this theory in close analogy to ordinary optics. Starting from a properly generalized Fermat principle and the associated canonical equations, the author leads up to the set of propositions that correspond to the Gaussian approximation and to the third-order error theory in the usual treatment of optical instruments. Greatly at variance with many papers and even textbooks, the presentation is logically complete and self-consistent, not leaving out necessary intermediate steps. At some places the reviewer would have liked more explicit references to plain geometrical facts. But a reader well versed in the language of analysis and familiar with the elements of differential geometry will never be at a loss. This is probably the best succinct presentation of electronic optics so far published in any language.

In the second supplement Herzberger discusses the physical properties of optical glass. Based on his long standing experience in all parts of optical practice, he supplies a most useful collection of facts and numerical data. In a brief note headed Mathematics and geometrical optics, he examines the relationship between different starting points in the presentation of the usual material of geometrical optics, comparing Hamilton's approach with Lagrange's theory, and so on.

Of greatest interest is the last supplement on Symmetry and asymmetry in optical images. Here Herzberger gives some very brief hints of a new error theory of which he is the author. This theory is based on the notion of "diapoint," that is, the point in which the image ray intersects the meridian plane of the object point. By studying the manifolds of diapoints, one arrives at a new classification and thus at a new treatment of optical errors. Although it cannot be expected that this conception will final y supersede the classical Seidel theory, it presents new, surprising, and very useful aspects which might become of great value in the design of instruments. It is to be hoped that Herzberger will soon find the opportunity to give a full account of his results in a book of his own.

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