most general result known and to discuss all other results as special cases. This procedure is an ideal one for a survey of the present character. As a result the reader is able to get a clear picture of the whole subject and to see in proper perspective the interrelations of the various special parts. In view of the nature of the book the authors have not included proofs of all theorems. But enough of these have been given to give the reader a real understanding of the methods. The choice of what to include and what to omit is generally in excellent taste. The reviewer would prefer to have seen included more of the proofs of the results of N. I. Achyeser and M. Krein, for their treatise on the subject is written in Russian. There are very few misprints. Those discovered by the reviewer are of a trivial nature and can be corrected in an obvious way by the reader. The book is certainly a very valuable addition to mathematical literature. The American Mathematical Society is to be congratulated on this auspicious initiation of its series of surveys.

D. V. WIDDER

Introduction to mathematical logic. Part I. By Alonzo Church. (Annals of Mathematics Studies, no. 13.) Princeton University Press, 1944. 6+118 pp. $1.75.

This booklet contains the material of the first half of a graduate course given by the author at Princeton and is, in fact, the revision of a set of lecture notes of that course. It is hoped that Part II, covering the second half of the course, which exists now in rough draft in the form of lecture notes, will some day appear in print.

The title of the book is somewhat misleading. As the author says, this is a monograph rather than a text. Its aim is not to give a broad survey of recent developments in symbolic logic but to present formally and rigorously, and with all the latest improvements, the theory of one of the oldest branches of the subject, namely, the calculus of propositional functions. In this aim the author has succeeded admirably. The difficult points are emphasized instead of being avoided. The care and precision for which the author is noted are in evidence throughout. The end result is, for the qualified beginner, a compact presentation of an important theory which can serve also as a model of present-day standards of rigor. The book is of value to the specialist also in bringing together in one place and in one notation rigorous proofs of important basic theorems which are otherwise only to be found scattered throughout the literature.

There are four chapters, of which the first deals with the calculus of propositions, the second and third with two different formulations of the functional calculus of first order, and the fourth with functional calculi of higher order.

The first chapter presents various alternative but equivalent formulations of the propositional calculus as a formal system. The now familiar distinctions between object language and syntax language, between axioms and rules of procedure, and between theorems of the calculus and theorems about the calculus (syntactic theorems) are explained. The highlights of this chapter are the deduction theorem and the solution of the decision problem, both syntactic results.

The second chapter presents one formulation of the functional calculus of first order, called by the author the system F^1. By using infinite axiom schemata instead of finitely many axioms, the author avoids the necessity of including the complicated substitution rules for individual variables, propositional variables, and functional variables among the primitive assumptions. These substitution rules, however, are obtained as derived rules. Other important results of this chapter are the deduction theorem for the functional calculus, and the theorem on reduction to prenex normal form.

The third chapter takes up another formulation called the pure functional calculus of first order F^1_p. This system has only a finite number of axioms and no constants but it has complicated rules of substitution. This chapter contains some of the deepest results of the book. Among them are the Gödel completeness theorem and the Löwenheim-Skolem theorem. It is noted that the decision problem for the functional calculus is unsolvable in the general case. A solution of the decision problem for formulas in certain special forms is given and the problem of finding simple forms with only a few quantifiers to which every well formed formula is reducible is treated.

The last chapter is a short one. It discusses informally without proofs the extension of the previous theory to the functional calculi of second order F^2 and of infinite order F^ω. Also discussed are the axiom of infinity, the axioms of extensionality, the theory of types, and various forms of the axiom of choice. The difficulties of proving consistency when various combinations of these assumptions are made is pointed out. In this connection the famous Gödel incompleteness theorem is mentioned but discussion of it is postponed to Part II. However, Gödel's conditional consistency theorem is discussed.

It is interesting to note that, although the title of the book refers to mathematical logic, the subject matter is mostly logic with very little mathematics. This is because the applications to number theory and to set theory have been postponed to Part II. The book has no index or bibliography, but there are historical and bibliographical notes at the ends of the first two chapters. This is in every way a welcome and useful addition to the literature of logic.

ORRIN FRINK, JR.

Mathematical theory of optics. By R. K. Luneberg. (Supplementary notes by M. Herzberger.) Providence, Brown University, 1944. 17+6+401+93 pp. $4.00. (Mimeographed.)

In this book, geometrical optics and diffraction are both treated as special applications of the electromagnetic theory of light. This treatment has advantages in some respects and disadvantages in others. Some types of problems, for instance those of polarized light, are handled easily, while the development of the problems of geometrical optics proper becomes somewhat involved.

The author has made several ingenious contributions in this connection. The idea of treating geometrical optics as a special limiting case of wave optics has never been carried out before in so much detail. The author has nowhere restricted himself to homogeneous media, following in the path of W. R. Hamilton. He covers first-order theory in general systems, third-order image-error theory in rotationally symmetrical systems, and the diffraction theory of spherical and near-spherical waves. A special chapter is devoted to media with parallel layers of constant refractive index.

Luneberg's treatment of diffraction optics is an important step forward. His fundamental contribution may be described as follows:

The light distribution is assumed to be known in a plane of infinite extent, and represented by the function $f(x_0, y_0)$. The light distribution may be only sectionally continuous inside a finite area, but outside this area the function must be small and continuous, and grow smaller with increasing distance from the center of the plane. Specifically, outside the limited area, f, $\partial f/\partial x_0$, and $\partial f/\partial y_0$ must all be continuous and smaller than $B(x_0^2+y_0^2)^{-1/2}$, where B is a constant. A further condition is that the resulting light distribution in space, represented by the function $u(x, y, z)$, shall be indistinguishable from that of a spherical wave at great distances from the plane, since at great distances the finite area of the plane is indistinguishable from a point. Mathematically, this condition is that, beyond a certain large distance from the center of the plane represented by $R=(x^2+y^2+z^2)^{1/2}$, the absolute values of u and $\partial u/\partial R$ shall be smaller than C/R, and the absolute value of the expression $\partial u/\partial R-iku$ shall be smaller than D/R^2, where D and C are constants.