

is the case, for example, for the series to sequence transform:  $A_n = \sum_{\nu=0}^n u_\nu x_n^\nu$ , where  $x_n \uparrow 1$  as  $n \rightarrow \infty$  so slowly that  $n^\nu x_n^n \rightarrow 0$  for all integers  $\nu$ . (Received March 17, 1945.)

92. F. T. Wang: *Strong summability of Fourier series.*

Let  $S_n(x)$  be the partial sum of the Fourier series  $f(t) \sim a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$  at  $t=x$ , and let  $\phi(t) = (1/2) \{f(x+t) + f(x-t) - 2S\}$ . The following result gives the solution of a problem proposed by Hardy and Littlewood (Fund. Math. vol. 25 (1935)): If  $\int_0^t |\phi(u)| \{1 + \log^+ |\phi(u)|\} du = o(t)$  as  $t \rightarrow 0$ , then  $\sum_{\nu=0}^n |S_\nu(x) - S|^2 = o(n)$  as  $n \rightarrow \infty$ . (Received March 12, 1945.)

93. F. T. Wang: *Tauberian theorem of oscillating series.*

Let  $\sigma_n^{(r)}$  be the  $r$ th Cesàro mean of the series  $\sum_{n=0}^{\infty} a_n$ ; and  $\sigma_n^{(r)} - s = o(n^{\delta-1})$  as  $n \rightarrow \infty$  for  $r > 0$ ,  $0 < \delta < 1$ ; and  $a_n > -Kn^{-\delta}$ ; then the series  $\sum_{n=0}^{\infty} a_n$  converges to  $S$ . An example shows that the order in the above inequality is the best possible in its kind. (Received March 12, 1945.)

94. H. J. Zimmerberg: *A class of definite boundary value problems.*

This paper is concerned with an extension of the results of Reid (Trans. Amer. Math. vol. 52 (1942) pp. 381-425) to differential systems consisting of the vector differential equation  $y' = A(x)y + \lambda B(x)y$ , and the two-point boundary conditions  $(M_0 + \lambda M_1)y(a) + (N_0 + \lambda N_1)y(b) = 0$ , in which the elements of the coefficient matrices of the system are allowed to be complex-valued. It is shown that under suitable assumptions of definiteness such systems possess fundamental properties similar to those previously established for real-valued, definitely self-adjoint problems by Bobonis (doctoral dissertation, Chicago, 1939; *Contributions to the calculus of variations*, 1938-1941, pp. 99-138). In particular, this study yields new results for the definitely self-adjoint systems considered by Bobonis. It is also shown that certain important types of boundary value problems associated with the second variation of an isoperimetric problem of Bolza in the calculus of variations which are not definitely self-conjugate adjoint do belong to this new class of problems. (Received March 19, 1945.)

#### GEOMETRY

95. L. K. Hua: *Geometries of matrices. I. Generalizations of von Staudt's theorem.*

A geometry is studied whose points are defined as the symmetric matrices  $Z$  of degree  $n$ ; a class of points at infinity is to be added. The group of transformations of this geometry consists of all mappings  $Z = (AZ^* + B)(CZ^* + D)^{-1}$  where the matrices  $A, B$  of degree  $n$  form the upper half of a symplectic matrix  $S$  of degree  $2n$  while  $C, D$  form the lower half of  $S$ . The question of equivalence of systems of points with regard to this group is investigated. A generalization of von Staudt's theorem is obtained. Finally, some other geometries of a similar nature are discussed. (Received February 10, 1945.)

96. L. K. Hua: *Geometries of matrices. I<sub>1</sub>. Arithmetical construction.*

The paper forms an illustration and a supplement to the first part. The geometry of two-rowed matrices is studied in more detail. It is shown that one of the conditions appearing in the generalization of von Staudt's theorem is redundant. (Received February 10, 1945.)

97. H. F. DeBaggis: *A simplified projective theory of order and parallelism in the hyperbolic plane.*

The point  $B$  is said to lie between  $A$  and  $C$  if every line through  $B$  intersects at least one of each pair of intersecting lines through  $A$  and  $C$  (F. P. Jenks, Reports of a Mathematical Colloquium vol. 1, p. 46). On the basis of this definition the entire theory of linear and planar order in the hyperbolic plane can be derived from Jenks' Postulates I, V–VIII and the following assumptions: II'. Each line contains at least two points; III'. There are three collinear and three noncollinear points; IV'. Through a point outside of two intersecting lines  $l_1$  and  $l_2$  there is a line intersecting exactly one  $l_i$ . In particular, the "postulate" of Pasch can be proved. Postulate IV', which is one-half of Jenks' Postulate IV, is satisfied in the hyperbolic plane enlarged by "ends" (pairs of parallel lines) whereas IV is not. The theory of hyperbolic parallelism can be developed by proving that lines and points including ends satisfy I, II', III', IV', V–VIII. (Received February 21, 1945.)

98. Karl Menger: *A projective definition of hyperbolic perpendicularity without reference to parallelism.*

Abbott's projective definition of hyperbolic perpendicularity (Reports of a Mathematical Colloquium vol. 4, p. 22) is based on parallelism. R. Baer asked whether perpendicularity could be defined in hyperbolic planes not containing parallel lines. If  $l$  is a finite line,  $Q$  a finite or ideal point not on  $l$ , then for a finite point  $X$  let  $\text{sgn}(X; Q, l)$  be equal to 1 if  $(X+Q) \cdot l$  is a finite point, and equal to 0 otherwise. The finite line  $l'$  is perpendicular to the finite line  $l$  if and only if  $l'$  contains an ideal point  $P'$  such that  $\text{sgn}(X; P', l) = 1$  for each finite point  $X$ . The ideal point is a pair of non-intersecting finite lines. Equality of such points, as well as their joins with finite points, are defined by means of Pappus constructions. (Received February 21, 1945.)

99. Karl Menger: *A projective definition of hyperbolic congruency independent of parallelism.*

Let  $Q$  and  $Q'$  be two distinct finite points on distinct lines  $l$  and  $l'$ , respectively. There is exactly one finite line  $h(Q, Q'; l, l') = h$  such that (1)  $t(X) = \text{sgn}(X; Q, l') + \text{sgn}(X; Q', l)$  is 0 or 2 for each finite  $X$  on  $h$ , and (2) there exist three finite points on  $h$  in the order  $ABC$  for which  $t(A) = t(C) \neq t(B)$  if  $l$  and  $l'$  are not parallel or (2') there exist two finite points for which  $t(A) \neq t(B)$  if  $l$  and  $l'$  are parallel. Two non-collinear pairs of finite points  $Q, R$  and  $Q', R'$  are congruent if and only if  $h(Q, Q'; l, l') = h(R, R'; l, l')$  where  $l = Q+R$  and  $l' = Q'+R'$ . Two collinear pairs of points are congruent if they are congruent with the same noncollinear pair. (Received February 21, 1945.)

100. Peter Scherk: *On differentiable arcs and curves.* V. Preliminary report.

The terminology used here is that of the preceding paper of this series [Ann. of Math. vol. 46 (1945) pp. 68–82]. Each point  $s$  of a  $K^{n+1}$  is mapped on the (possibly empty) set  $t_m^n(s)$  of those points of the  $K^{n+1}$  which are projected from the osculating  $m$ -space of  $s$  into singular points ( $0 \leq m < n-1$ ). The existence, continuity and direction of the components of these mappings are studied outside of their fixed points. As a corollary the finiteness of the numbers  $N_{ik}^n$  is obtained ( $0 \leq i, 0 \leq k, i+k \leq n-2$ ; cf. *ibid.* pp. 78–81). (Received March 12, 1945.)