ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

73. A. P. Hillman: *On identities for differential polynomials.*

The author generalizes his previous theorems (see Bull. Amer. Math. Soc. vol. 49 (1943) pp. 711–712) as follows. Let $F\neq 0, C_1, \cdots, C_s$ be differential polynomials, and let $C_1P_1+\cdots+C_sP_s=0$, where the P_i are distinct power products of degree d_i and order p_i in F and its derivatives. Also let $p_j>p_k$ whenever $d_j<d_k$. Then each C_i holds F. There is an analogue for partial differential polynomials. (Received March 29, 1945.)

74. A. P. Hillman: *Theorems obtained from the Newton polygon process for differential polynomials.*

Two of Ritt's theorems (see §8, Amer. J. Math. vol. 60 (1938) pp. 1–43) are generalized to include the following. Let A and B be forms (ordinary or partial differential polynomials), let A have constant coefficients, and let B hold A. Then the form B^* composed of the terms of B for which $ad+bw$ is least holds the corresponding form A^*. Here d stands for degree, w stands for weight, and a and b are any real numbers. (Received March 29, 1945.)

75. Bjarni Jonsson: *On unique factorization problem for torsionfree abelian groups.*

The problem has been raised whether the unique factorization theorem holds for all groups which are (finite) direct products of indecomposable groups (cf. A. Kurosch, Math. Ann. vol. 106). The solution is negative, even for the class $\mathfrak{H}$ of torsionfree abelian groups of finite rank. In fact, let a, b, c, d be rationally independent real numbers. Let A, B, C, D be additive groups of real numbers, defined as follows. A and B consist of all numbers $5^n ia$ and $5^n ib+7^n jc+11^n kd+(l/3)(b-c)+(m/2)(b+d)$ respectively; i, j, k, l, m, n being integers. C and D consist of those members of $A\times B$ which are of the form $t(3a-b)+uc$ and $t(2a-b)+ud$ respectively; t and u being rational numbers. Then $A\times B=C\times D$, all four groups being indecomposable and non-isomorphic. However, a unique factorization theorem in a weaker form applies to $\mathfrak{H}$. The two groups A, B are called equivalent, $A\equiv B$, if A is isomorphic with a subgroup of B and conversely. A group A, of positive rank, is strongly indecomposable if $A\equiv B\times C$ never holds unless $A\equiv B$ or $A\equiv C$. Then every group in $\mathfrak{H}$ is equivalent to a direct product of strongly indecomposable groups; and if it is equivalent to two such products, then the number of factors in both products is the same, and the factors are (apart from order) respectively equivalent. (Received March 24, 1945.)

76. G. K. Kalisch: *A spectral theory for symmetric operators over generalized Hilbert spaces.*

In a previous paper the author defined generalized Hilbert spaces S over fields F with non-Archimedean valuations. In the present paper he considers symmetric bounded linear operators R over S, such that $(Ra, b) = (a, Rb)$ for all a and b in S. Besides a spectral theorem for totally continuous operators which is exactly the same as in the classical case, the principal result is a spectral theorem valid for symmetric operators R all of whose characteristic manifolds are proper in the terminology of the above-mentioned paper. Characteristic manifolds are manifolds $M(\lambda, \eta)$ for λ and η in F which reduce R, and on which $|Rx - \lambda x| < |\eta|$, and $(R-\lambda I)^n/\eta^n \to 0$. Any such operator can be decomposed into $R = U + V$. U has a pure point spectrum (countable) and a corresponding canonical form as in the classical case, and V has a pure continuous spectrum and can be represented as $V = \int \lambda dE_\lambda$ which is an abbreviation for a limiting process akin to the Riemann-Stieltjes integral, based on a decomposition of the identity, E_λ, which in turn is based on the decomposition of S by the (orthogonal) characteristic manifolds with properly chosen λ's and η's. Also the nature of the continuous spectrum is discussed, and some consequences of the spectral theorem are drawn, such as $f(R) = \int f(\lambda)dE_\lambda$ for polynomials $f(\lambda)$ and certain limits thereof. (Received February 3, 1945.)

77. Irving Kaplansky: *A contribution to von Neumann's theory of games.*

Let $\|a_{ij}\|$ be the matrix of a zero-sum two-person game G, as defined by von Neumann (*Theory of games and economic behaviour*, Princeton, 1944). The author gives a method of finding the value of v' of G in a finite number of steps. If G is "completely mixed" in the sense that all strategies must actually be used, then $\|a_{ij}\|$ is square and $v' = |a_{ij}| / \sum A_{ij}$, where A_{ij} is the cofactor of a_{ij}. If G is not completely mixed (for which a criterion is given), one passes to matrices of lower order. (Received April 2, 1945.)

78. Irving Kaplansky: *A note on groups without isomorphic subgroups.*

A theorem of R. A. Beaumont asserts that an abelian group of finite rank with elements of finite order has no isomorphic subgroups. It is shown that the theorem extends to vector spaces over a ring R provided that R is a principal ideal ring all of whose proper residue class rings are finite. (Received April 2, 1945.)

79. Irving Kaplansky: *Maximal fields with valuations.* II.

Schilling showed that a maximal field with discrete value group of finite rank is a power series field (Ann. of Math. vol. 38 (1937) pp. 551–576). In this paper the result is carried over to (suitably defined) discrete groups of infinite rank. Together with other known results, this virtually completes the determination of the structure of maximal fields, a problem proposed by Krull. (Received April 2, 1945.)

80. N. H. McCoy: *Subdirectly irreducible commutative rings.*

A special case of a theorem of Garrett Birkhoff (Bull. Amer. Math. Soc. vol. 50 (1944) pp. 764–768) states that every ring is isomorphic to a subdirect sum of subdirectly irreducible rings. Birkhoff also showed that a subdirectly irreducible commutative ring without nilpotent elements is a field. The purpose of the present paper is to make some further characterizations of subdirectly irreducible commutative rings. Let R be a commutative ring, not all elements of R being divisors of zero, and denote by D the set of all elements of R which are divisors of zero. Then R is subdirectly irreducible if, and only if, it has the following four properties: (i) the set of all elements x of R such that $Dx=0$ is a principal ideal $J=(j)\neq 0$, (ii) the set of all elements y of R such that $Jy=0$ is precisely D, (iii) R/D is a field, (iv) if d_1 is any element of D, not in J, there exists an element d_2 of D, not in J, such that $d_1d_2=j$. Some additional related results are also obtained. (Received February 20, 1945.)

81. Kathryn A. Morgan: *Representation of a positive binary form by a positive quaternary form.*

The conditions for representing a positive binary form as a sum of squares of linear forms were discussed by Mordell (Quart. J. Math. Oxford Ser. vol. 1 (1930) pp. 276–288) and Chao Ko (Quart. J. Math. Oxford Ser. vol. 8 (1937) pp. 81–98). This paper presents a method for finding the representation of a positive primitive binary form by a positive quaternary form and especially the number of representations of a primitive binary form as the sum of four squares. (Received March 3, 1945.)

82. F. E. Satterthwaite: *Error control in matrix calculation.* II.

The arithmetic calculation of the inverse of a matrix or of the solution of a set of simultaneous equations is often complicated by a serious magnification of rounding errors. The proposed method is as follows: (1) Each equation (or line) of the Doolittle solution is expressed approximately as an exact linear function of the original equations. (2) The discrepancy between the approximation and the ideal is adjusted by the same type of process as is used to adjust the original equations in a standard Doolittle method. (3) If the approximation is close enough, the coefficients in the adjustment will be small enough so that they can not cause any significant carrying forward of errors. A second approximation is sometimes necessary to satisfy this condition. The advantages of this method are: (1) It works for any matrix or set of equations. (2) It does not require an advance approximate solution. (3) Any number of decimal places may be carried with complete assurance that errors are never greater than one or two in the last decimal place. (4) Each step is self-checking. (5) The method is ideally suited for use with modern high speed calculating machines. (6) The routine can be easily taught to the average clerk. (Received February 5, 1945.)

83. J. E. Wilkins: *A generalization of the Euler ϕ-function.*

One defines the function $\phi_n(k)$ so that exactly $\phi_n(k)$ of the k arithmetic progressions $mk+l$ ($m=0, 1, \cdots$; $l=0, 1, \cdots, k-1$) contain infinitely many numbers not divisible by an nth power greater than 1. It is shown that $\phi_n(k)$ is that multiplicative function for which $\phi_n(p^r)=p^r$ if $r<n$ and p is prime and $\phi_n(p^r)=p^r-p^{r-n}$ if $r\geqq n$ and p is prime. Thus $\phi_1(k)$ is the Euler ϕ-function. It is also shown that the function $\phi_n(k)/k$ is uniformly almost periodic in the sense of Bohr for $n\geqq 2$, and its asymptotic mean value is $1/\zeta(2n)$. For $n=2$, these results are due to Haviland, whose discussion, however, is not free from errors. (Received March 20, 1945.)

Analysis

84. R. P. Agnew: *Spans of translations of peak functions.*

A peak function $F(x)$ is defined, in terms of positive constants a and b, by