BOOK REVIEWS

Infinite series. By J. M. Hyslop. New York, Interscience, 1942. 120 pp. $1.75.

This textbook gives a concise presentation of the classic theory of convergence of real sequences and series. It is intended for students having had an elementary course in calculus. The brevity of the text precludes extensive motivation of the definitions and theorems, but numerous examples worked out in the text provide illumination. Problems, with answers given, appear at the end of each chapter.

Chapter 1 (17 pages) treats limits of functions by the ϵ-δ methods, and introduces the σ-O notation. Chapter 2 (7 pages) gives Taylor formulas, with remainders, for several elementary functions. Chapter 3 (11 pages) gives definitions, examples, and fundamental properties of convergence, including the Cauchy-sequence criterion. Chapter 4 (22 pages) gives the integral, comparison, ratio, Cauchy nth root (Cauchy condensation, sic), Kummer, and Raabe tests for convergence of series of nonnegative terms. For positive integers n, the Stirling formula $n! = (2n\pi)^{1/2}n^n e^{-n}[1+O(1/n)]$ is derived. Chapter 5 (10 pages) treats absolute and conditional convergence, giving the Abel and Dirichlet tests and Riemann's theorem on rearrangements. Chapter 6 (18 pages), on series of functions, treats uniform convergence, termwise integration and differentiation, and power series. Chapter 7 (7 pages) treats convergence of "the product series." The product series, defined without motivation, is that of Cauchy; one may feel that the author should have called the series by its familiar name so that the possibility of other interesting product series (Dirichlet, for example) would not be summarily dismissed. Chapter 8 (12 pages) treats infinite products, including those of the sine, cosine, and factorial (gamma) functions.

Chapter 9 (11 pages) treats double series. Summation "by triangles," "by squares," "by rows" and "by columns" are defined; to correct the formulation of the second of these definitions (p. 107, formula 2), replace N by k inside the parentheses and insert $\sum_{k=1}^{N}$ before the first parenthesis. The definition of convergence and divergence of a double series $\sum a_{mn}$ runs as follows: "Naturally, we wish our definition of the sum of a double series to conform as closely as possible to the definition of the sum of a single series. This analogy may be preserved by starting at the top left-hand corner of the array and taking successive groups of terms, where each group consists of only a finite number of terms of the series and contains all the elements of the preceding group, and then examining the limit of the pth group as p tends to infinity. These successive groups correspond in fact to successive partial sums in the case of single series. Generally speaking, the limit of the pth group will depend on the system by means of which the groups are formed. When, however, the limit is finite and independent of the system of grouping, we say that the double series is CONVERGENT and that the limit in question is the sum of the series. In all other cases, the series is said to be DIVERGENT." This unorthodox definition of "sum" of a double series seems to be inadequate. The author must have intended that each term a_{mn} of the series would appear in the union of the groups; without such an understanding, no double series would be convergent unless all of its term were 0. Unless one adds a further requirement limiting the characters of the groups, one can arrange the terms of the double series into a simple series in any way one chooses and then determine the groups so that the pth group will consist of the first p terms of the simple series; this implies that $\sum u_{mn}$ cannot be convergent unless it is absolutely convergent. Since the chapter gives the classic theorems pertaining to Pringsheim convergence, giving the customary relations between convergence and absolute convergence, it would seem that "convergence" should be replaced by Pringsheim convergence in this chapter.

The text is carefully written, with due regard to such details as the distinction between positive and nonnegative. It is, as a whole, precise and accurate. Some lapses are to be found. There are a few places where $<$ should be replaced by $\leqq$, and a few where $=$ should be replaced by $\leqq$. There are a few places where statements involving "any" are ambiguous; when these statements are removed from their contexts, one cannot tell whether the "any" means "each" or "at least one." Sometimes the intended meanings of statements are destroyed by unfortunate wordings. In the process of defining the least upper bound K of a function $f(x)$, the author is guilty of saying "there is at least one value of x for which $f(x) > K-\epsilon$, where ϵ is any positive number" when he should (and doubtless does) mean that "to each $\epsilon > 0$ corresponds at least one value of x for which $f(x) > K-\epsilon$." A student who reads the author's statement and concludes that $f(x) \geqq K$ is not to be condemned; under the author's definition the set of numbers x for which $0 < x < 1$ does not have a least upper bound. Several theorems in which an equation involving θ is said to hold "where $0 < \theta < 1$" could be made more precise and intelligible by saying that there is at least one θ such that $0 < \theta < 1$ and the equation holds.

One who has finished a first course in calculus should find this book to be an intelligible introduction to basic methods in modern mathematical analysis as well as to the classic theory of convergence of series.

RALPH P. AGNEW

Miscellaneous physical tables. Planck's radiation functions and electronic functions. New York, Work Projects Administration, 1941. 7+61 pp. $1.50.

In the first part of this volume of the well known W.P.A. series, values are given for the spectral radiancy of a black body, the rate of photon emission, and the indefinite integrals (from 0 to λ in wavelength) of the functions. These are tabulated at constant intervals of $\lambda \cdot T$ (wavelength multiplied by Kelvin temperature), the size of the intervals changing occasionally. Auxiliary tables give values of some of the preceding quantities as tabular functions of wavelength alone at a few definite temperatures.

The second part tabulates relativistically correct values of the following properties of an electron, as functions of its velocity (entered with constant intervals in fractions of c, the velocity of light): the energy, in units of the electron rest energy m_0c^2; the momentum, in units of m_0c; the energy in electron-kilovolts; and the $H \cdot \rho$ values (ρ is the radius of a circle in which the electron would move if projected with the tabulated velocity normal to a magnetic field of H oersteds).

Each part of the volume was inspired by suggestions of well known scientists, as stated in the introductory discussions. The first part has been published previously in the February 1940 number of the Journal of the Optical Society of America. The introduction contains brief descriptions of the functions, examples illustrating use of and interpolation in the tables, a summary of the methods used in assembling or computing the entries, and a bibliography. In the major tables of the first part, first differences and second central differences are given for greater ease and accuracy in interpolation. A complication not met in purely mathematical tables exists here due to the fact that all the functions tabulated depend on measured physical constants: the velocity of light, Planck's constant, and electronic properties. The tables were based on the present best values of these constants, but are carried to extra figures. Directions are given for making corrections in case these are necessitated by newer values of the basic physical constants.

The tables are in the usual format of the W.P.A. series. They