BOOK REVIEWS

Fourier series. By G. H. Hardy and W. W. Rogosinski. (Cambridge Tracts in Mathematics and Mathematical Physics, no. 38.) New York, Macmillan; Cambridge University Press, 1944. 8+100 pp. $1.75.

The object of the authors, as stated in their preface, was to write a book on Fourier series "in a modern spirit, concise enough to be included in the (Cambridge) series" of tracts, yet complete enough to serve as an introduction to Zygmund's *Trigonometrical series*. How well their object has been achieved is readily seen on examining the wealth of material covered in some ninety pages. At the outset they define what is expected of the reader and what aspects of the subject they propose to omit. The reader is expected to have a knowledge of Lebesgue integral theory as treated in Chapters X–XII of Titchmarsh's *Theory of functions*; and while it is recognized "unofficially" that the reader will undoubtedly have some knowledge of the theory of trigonometrical series, officially the tract is self-contained.

Certain topics are frankly omitted: for example, the inequalities of Young and Hausdorff, M. Riesz's theorem on conjugate series, theorems on Cesàro summability of general order, and uniqueness theorems for summable series. Nor is Denjoy's theory of general trigonometrical series considered.

Chapter I, entitled *Generalities* lays the groundwork for all subsequent developments. At the outset trigonometrical series and trigonometrical Fourier series are defined, and at the same time their relation to harmonic and analytic functions is brought out. The emphasis on this relationship is reinforced at various points throughout the tract by the application of Fourier series theorems to analytic function theory. The essential results of Lebesgue integral theory and the theory of the spaces L^p which are needed for the sequel are stated succinctly.

Chapter II is directed toward the classical L^2 theory of Fourier series. First the situation for general normal orthogonal systems is considered. Here the Riesz-Fischer theorem, the Parseval theorem, and the relation between closure and completeness find their natural setting. From this point on, attention is focussed on trigonometrical series. The completeness of the trigonometrical system (T): $1/2$, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, $\cdots$ in $L(0, 2\pi)$ is demonstrated; and the completeness of (x^n) for $n=N, N+1, \cdots$ in $L(a, b)$ and the Weierstrass approximation theorems appear as corollaries.

The theorems of Chapter III fall into two main categories. The first category is concerned with general properties of Fourier series: the theorems which relate the Fourier constants of a given function of $L(-\pi, \pi)$ and the Fourier constants of functions associated with it in certain simple ways; the Riemann-Lebesgue Theorem; the theorem on the term-by-term integration of a Fourier series; the Gibbs phenomenon. The second category of theorems consists of two classes, the first dealing with properties of the Fourier constants which appear as consequences of special hypotheses on the function; the second dealing with trigonometrical series whose coefficients are positive and decreasing to zero. Properties of Fourier constants are given for functions f which satisfy one of the following conditions: f is non-negative in $(0, 2\pi)$, is odd and non-negative in $(0, \pi)$ (the results in these two cases yield Carathéodory's inequalities for the coefficients of a function analytic and with non-negative real part in the interior of the unit circle and the theorem of Dieudonné and Rogosinski on schlicht functions), is monotone in $(0, 2\pi)$, is convex in $(0, 2\pi)$, satisfies a uniform Hölder condition in $(0, 2\pi)$, is of bounded variation, and so on. In the case of functions of bounded variation the example of Hille and Tamarkin is given of a function which is continuous and of bounded variation and yet has Fourier coefficients which are not $o[|n|^{-1}]$ where n is the index.

Chapter IV is concerned with the convergence of a Fourier series and its associated conjugate series. Various known sufficient conditions for the pointwise convergence of a Fourier series are given, such as Dini's test, Jordan's test, Lebesgue's test. Related problems for the conjugate series are considered. An example is given for a function whose Fourier series diverges at a point of continuity.

The summability of a Fourier series is treated in Chapter V, which is introduced by a brief discussion of linear regular methods of summation. General methods of summation of Fourier series are discussed and the $(C, 1)$ and A methods are developed as special cases of the general theory.

Chapter VI deals with applications of the results of Chapter V. Here the theorem is proved that there are Fourier series which diverge almost everywhere. Other topics treated are Fourier series with positive coefficients, gap theorems, the Gibbs set, the existence of the conjugate function.

The final chapter is devoted to the theory of general trigonometrical series. The classical uniqueness theorem for trigonometrical series associated with the names of Riemann, Cantor, and du Bois-Reymond is established, and the remainder of this chapter is devoted to proving that if a trigonometrical series converges except in an enumerable set to a finite and integrable function, then it is the Fourier series of this function.

M. H. HEINS

The advanced theory of statistics. Vol. 1. By Maurice G. Kendall. Philadelphia, Lippincott, 1944. 12+457 pp. $16.00.

Modern statistics is built around sampling theory. It is not well presented by books in the tradition of a quarter-century ago which exalted an extreme and sterile empiricism and ignored or deprecated probability and mathematics in general, even when such books are revised to mention modern developments. Neither is it adequately presented in the books, now becoming available to research workers in an increasing number of fields, which give sound practical advice and formulae, but without the derivations. For the serious student of statistics nothing is wholly satisfactory short of a treatment starting from first principles and proceeding by fully stated definitions and derivations to the methods needed for the entire array of situations with which statistics deals. To get on as far as possible with this program has been the object of only a few of the many books on statistics. Of these, Mr. Kendall's is the largest, and covers in fullest detail the subjects treated.

In spite of the title, this book is not "advanced" in the sense of requiring of the reader a prior knowledge of statistics. It does call for more of a mathematical background than is possessed by most students of statistics. Anyone who has mastered advanced calculus should get along fairly well with it, though there are occasional uses of such relatively advanced mathematics as complex integration and evaluation of integrals by residues, the Euler-Maclaurin sum formula, Stirling's formula (which is used without proof or explicit statement), the integral form of the remainder in Taylor's theorem, and definite quadratic forms. For a reader who is not troubled by these purely mathematical matters the style is unusually clear and explicit. There are many illustrations drawn from actual observations. Each chapter ends with a collection of problems; these are of a superior quality, suitable for testing mathematical skill and mastery of the material, not merely numerical data to be substituted in formulae. The work therefore possesses qualifications as an excellent textbook for suitably prepared students in the hands of a suitable teacher. The teacher, however, should not only be prepared to help the students over the mathematical hurdles noted and others, but should also be enough