the integral $\iint F dxdy$ is equivalent to the condition of irrotationality. Hilbert conjectured that every regular variational problem is solvable. A. Haar showed (Math. Ann. vol. 97, p. 124) that this is indeed the case for the integral $\iint f(\psi_x, \psi_y)$. Haar's theorem cannot be applied directly for our case because ρ is a double valued function of τ. Furthermore τ has a maximum value τ_s at the speed of sound. However one can introduce a hypothetical "semi-compressible" fluid such that (a) for $\tau < \tau_s - \epsilon$, $p\rho^{-k}$ is (approximately) constant; (b) for $\tau > \tau_s - \epsilon$ the density is (approximately) constant; (c) the associated variational problem is regular. As a consequence of Haar's theorem *the interior boundary value problem is always solvable for such a semi-compressible fluid. The necessary and sufficient condition for the existence of a subsonic solution of a real compressible fluid is that for the (always existing) "semi-compressible" solution* $\tau < \tau_s$. It is believed that the theorem can be extended for the exterior problem too. (Received June 27, 1944.)

GEOMETRY

235. Herbert Busemann: *Local metric geometry.*

The author studies systematically (not necessarily symmetric) metric spaces in which extremals exist. The local properties of such spaces are investigated first, for instance, segments are constructed which may replace the line elements of differential geometry. Convergence of extremals as point sets, sets of line elements, and curves is analyzed. Then a theory of parallelism between infinite rays in unbounded spaces is developed which is new also for the differentiable case. The theory of locally isometric spaces proves to be analogous to the topological theory of covering spaces with one noteworthy exception: compact and locally isometric spaces are congruent. Finally some of the fundamental theorems of differential geometry on spaces with constant curvature are derived without any differentiability assumptions. (Received June 17, 1944.)

236. Claude Chevalley: *Intersections of algebraic and algebroid varieties.*

The paper contains a local theory of intersections for algebroid and algebraic varieties. The definition of intersection multiplicities is based on the notion of multiplicities of a local ring with respect to a system of parameters as introduced in the author's paper on *The theory of local rings*, Ann. of Math. vol. 44, no. 4. (Received July 1, 1944.)

237. John DeCicco: *Conformal maps with isothermal systems of scale curves.*

Let a surface Σ be mapped conformally upon a plane Π. The scale function $\sigma = ds/dS$ depends only upon the position of the point. A scale curve is the locus of a point along which the scale σ does not vary. Under any conformal map of Σ upon Π there are ∞^1 scale curves (whereas in the nonconformal case there are ∞^2 scale curves). Any family (isothermal or not) of ∞^1 curves may represent the scale curves of a conformal map of some surface Σ upon the plane Π. The author considers the surfaces Σ of non-constant gaussian curvature which are applicable upon a surface of revolution for which there exists a conformal map of Σ upon Π with isothermal families of scale curves. Of course, any such surface Σ may be so mapped. In that case, the scale curves must be parallel straight lines or concentric circles. It is proved that there

are essentially seven types of surfaces Σ applicable upon a surface of revolution for which there exist conformal maps upon a plane Π with isothermal systems of scale curves which are neither parallel lines nor concentric circles. (Received July 5, 1944.)

238. H. W. Eves: *Applications of some new matrix products to geometry.*

A symbolic product, such as the various products of vectors and of matrices, is useful to the geometer only when the combination defining the product occurs with fair frequency in geometrical investigation, and this degree of frequency is, to the geometer, the gauge justifying the product's existence. This paper partially exploits as new symbolic products certain combinations of the elements of pairs of matrices which are quite frequent in an analytical treatment of the projective geometry of conics and in the differential invariant theory of surfaces in 3-space. Only the former applications, however, are considered in this paper. The products are closely associated with the invariant and covariant theory of conics. After briefly developing the needed algebra of the new products certain analytical criteria, in terms of the products, are obtained for some projective relationships, and several classical results are proved with these products as new tools. (Received June 10, 1944.)

239. H. W. Eves: *Skew curves setting up a null system in space.*

An n-curve is defined as a skew curve such that (A) the points of osculation of all osculating planes through a point are coplanar with the point, (B) the osculating planes of a set of coplanar points of the curve are copunctual at a point on this plane. The author develops, by vector methods, a number of projective and metrical properties of n-curves. Some necessary and sufficient conditions for a curve to be an n-curve are given, and the relation of n-curves to linear complexes is dealt with. It is shown that every n-curve satisfies a differential equation of the form $|r, x, x'| = s \cdot x'$, where r, x, s are vectors. This differential equation is integrated and special n-curves, such as the twisted cubic, the circular helix, and so on, are noted. (Received June 21, 1944.)

240. C. C. Hsiung: *A study on the theory of conjugate nets.*

Let an analytic non-ruled surface S in ordinary space be referred to a conjugate parametric net N_x, and C_λ be any curve on S through a general point P_x. As P_x moves along C_λ the parametric tangents u, v generate respectively two non-developable ruled surfaces $R^{(u)}$, $R^{(v)}$. In this paper the author first discusses the correspondence between the line joining any two points T, $\overline{T}$ on the parametric tangents u, v through P_x and the intersection of the tangent planes of $R^{(u)}$, $R^{(v)}$ at T, $\overline{T}$, respectively; and then introduces two quadrics, one has contact of the second order with $R^{(u)}$ at T and contact of the first order with $R^{(v)}$ at $\overline{T}$, and the other has similar properties with the roles of u, v interchanged. Finally, by means of the projections of the two parametric curves from any point on a line through P_x onto their osculating planes at P_x and a one-parameter family of quadrics associated with P_x of N_x obtained by the author in a previous paper (*Theory of conjugate nets on a surface*, to appear in the American Journal of Mathematics) he derives two one-parameter families of lines, of which two may be regarded as generalizations of the canonical edges of Green of the asymptotic net of a surface to a conjugate net. (Received June 8, 1944.)

241. C. C. Hsiung: *New geometrical characterizations of some conjugate nets.*

Let N_x be a conjugate net with parameters u, v on an analytic proper surface S in ordinary space, and let x_{-1}, x_1 be respectively the ray-points, or Laplace transformed points, of a general point P_x of S with respect to the v-curve and the u-curve of N_x. As u, v vary the loci of x_{-1}, x_1 are two surfaces S_{-1}, S_1 on which the parametric curves form two conjugate nets N_{-1}, N_1, which are called the minus-first and first Laplace transformed nets of N_x, respectively. The purpose of this paper is to give new geometrical characterizations of the conjugate net N_x in some special cases by introducing two quadrics, one of which has contact of the second order with S_{-1} at x_{-1} and contact of the first order with S_1 at x_1, and the other has similar properties with the roles of u, v interchanged. (Received July 20, 1944.)

242. C. C. Hsiung: *The contact of conics and quadrics with two surfaces in space of n dimensions.*

In this paper the author establishes, among other things, the following theorem: Let S, S^* be two proper analytic surfaces with general relative position in a linear space S_n of $n(>2)$ dimensions, and O_1, O_2 be two ordinary points on them respectively. Let F, F^* be respectively the sections of S, S^* made by a variable linear space S_ν of ν $(n>\nu\geqq 2)$ dimensions through a given linear space $S_{\nu-1}$ of $\nu-1$ dimensions which contains O_1, O_2. As S_ν varies through $S_{\nu-1}$ the locus of the quadric in S_ν with second order contact with F at O_1 and first order contact with F^* at O_2 is a unique quadric in S_n which has contact of the second order with S at O_1 and contact of the first order with S^* at O_2. (Received June 8, 1944.)

243. Edward Kasner: *Conformal symmetry and satellite theory for algebraic curves.*

Schwarz defined symmetry with respect to any real analytic curve, and this concept is used for analytic prolongation. The author gave the theory a purely intrinsic geometric formulation (Proceedings of the Fifth International Congress of Mathematicians, Cambridge, 1912). In the present paper he studies the case where the base curve is any algebraic curve (real or imaginary) in the complete complex (4-dimensional) plane. Symmetry is then many-valued. The image of the given curve is in general reducible and the new component is called the satellite curve. The satellite of a conic is a conic. The satellite of a potential algebraic curve is itself. Use is made of the induced satellite transformation. (Received June 19, 1944.)

244. Sister Ingonda von Mezynski: *Projective description of some plane sextic curves derived from conics as base curves.*

Following a general method described by Pettit (Tôhoku Math. J. vol. 28 (1927) pp. 72–79), corresponding rays of two pencils A and C meet a single ray of a pencil B in points of two conics M_A, M_B. The two pencils, A and C, through this correspondence, generate an octive curve. By a suitable choice of the positions of points A, B and C relative to the base conics M_A, M_B, the octic may be made to degenerate into two straight lines and a sextic. Thus 159 types of sextics with one quadruple point and three double points, 143 with one quadruple point and four double points, 32 with nine double points and 20 having ten double points were derived. (Received July 6, 1944.)

245. Peter Scherk: *On the number of certain singularities of differentiable curves of order $n+1$ in real projective n-space.*

The order of a curve in real projective n-space R_n is the maximum number of its points of intersection with a linear $(n-1)$-space. A K^{n+1} is a closed curve of order $n+1$ in R_n which satisfies certain differentiability assumptions. The singular points of a K^{n+1} can be described and provided with multiplicities. Let N_ν^n be the number of $(n-\nu)$-fold singular points and let $N_{\mu 0}^n$ be the number of osculating μ-spaces of the K^{n+1} that meet it again. In a previous paper it was proved that $\sum_0^{n-1}(n-\nu)N_\nu^n \leqq n+1$ and that the $N_{\mu 0}^n$ are finite and even bounded for a given n $(0 \leqq \mu \leqq n-2)$. Its methods would yield the estimate $\sum_0^{n-2} N_{\mu 0}^n \leqq \exp(O(n))$. In this paper, the following estimate is proved: $\sum_0^{n-1}(n-\nu)N_\nu^n + \sum_0^{n-2}(n-\mu-1)N_{\mu 0}^n \leqq n^2/4+3/2n$. (Received June 23, 1944.)

246. Y. C. Wong: *Family of totally umbilical hypersurfaces in Einstein 4-space.*

A U_3 (or $\overline{U}_3$) is a Riemannian 3-space V_3 imbeddable in an Einstein 4-space as a member of a family of totally umbilical (or totally geodesic) hypersurfaces. The purpose of this paper is to study those U_3 and $\overline{U}_3$ for which the elementary divisors of the Ricci and fundamental tensors are all simple. Let $\omega_1, \omega_2, \omega_3$ be the Ricci invariants, then there are three cases. (1) $\omega_1=\omega_2=\omega_3$. Every V_3 of constant curvature is a U_3 and a $\overline{U}_3$, and conversely. (2) $\omega_1=\omega_2\neq\omega_3$. The existence of some simple equations characterizing both the U_3 and the $\overline{U}_3$ of this type proves that these U_3 and $\overline{U}_3$ are identical. These equations have an immediate geometric meaning and lead to the fundamental forms of the U_3 and $\overline{U}_3$ in certain privileged coordinates. A particular case was previously considered by Levi-Civita. (3) $\omega_1, \omega_2, \omega_3\neq$. Only the particular case where the ω's are all constant is considered. It is proved that no such U_3 exists and that the only $\overline{U}_3$ of this class is imbedded in an Einstein 4-space previously obtained by Kasner in another connection. A consequence of this is the non-existence of U_3 and $\overline{U}_3$ with non-null geodesic Ricci congruences. (Received July 5, 1944.)

247. Y. C. Wong: *Quasi-orthogonal ennuple of congruences in Riemannian space.*

In the investigation of those problems in a Riemannian n-space V_n with fundamental tensor g_{ij} where the principal congruences of a certain symmetric tensor h_{ij} play a part, it is customary to suppose that the elementary divisors of these two tensors are all simple, so that there exists an orthogonal ennuple of unit principal congruences and Ricci's method of the coefficients of rotation may be used. When the elementary divisors of h_{ij} and g_{ij} are not all simple, however, null principal congruences appear; for this case Ricci's method is powerless and a deadlock is reached. In this paper it is shown that this difficulty can be overcome by using a special ennuple of congruences, called a *quasi-orthogonal* ennuple, which consists partly of null and partly of non-null congruences. First, some general observations are made on this ennuple, including its use to express a symmetric tensor in normal form. Then an invariant theory of the ennuple is developed, analogous to that of Ricci's coefficients of rotation. Finally, the quasi-orthogonal ennuple in a V_3 is studied in some detail; and, as an application, a complete solution is given to the problem of finding pairs of V_3's with corresponding geodesics. (Received July 5, 1944.)