BOOK REVIEW

Vector and tensor analysis. By H. V. Craig. New York and London, McGraw-Hill, 1943. 14+434 pp. $3.50.

During the last decade, there has been considerable emphasis on the presentation of subjects in textbooks from the axiomatic viewpoint. This approach implies carefully worded definitions, axioms, and theorems. Further, in this point of view, the stress is placed on the analytical and logical rather than the geometrical and physical aspects of a subject. The type of presentation associated with this view has furnished interesting and valuable textbooks in such introductory subjects as college algebra and calculus. Previously, no such presentation had been attempted for a senior-graduate level text in vector and tensor analysis. Craig has admirably presented vector and tensor analysis in the light of this analytical-logical viewpoint.

In writing such a text on vector and tensor analysis, a subject whose origins and developments are closely connected with geometry and physics, an author faces many problems. Perhaps one of the most difficult of these problems is concerned with the author's treatment of differentials. From the logical viewpoint, differentials are non-essential tools since all their functions may be performed by derivatives. However, differentials act in two very important roles in vector and tensor analysis. First, the literature, both past and present, of the subject abounds in the use of differential notation. Second, differentials are useful to both the geometer and the physicist in interpreting his results. From these interpretations, many new results have been obtained, often by proofs which are incorrect from the modern point of view. However, rigor and concise thinking are fundamental to the author's approach to the subject. Hence, he has chosen to omit the treatment of differentials. Another important problem is the treatment of coordinate transformations and invariants. Very few introductory texts to vector and tensor analysis furnish an adequate treatment of this important topic. The principal difficulty is that a proper presentation of this topic requires that the student possess a knowledge of determinants and linear transformations. Because of the present author's analytical approach, he is able to offer an excellent account of this subject. In fact, the concept of invariance dominates the greater part of the book.

The book is divided into four parts; Part A deals with advanced calculus; Part B with elementary vector analysis; Part C with tensors and extensors; Part D with applications.

Part A is concerned with those parts of advanced calculus which are frequently used in the remainder of the text. Chapter I points out some common errors in reasoning and offers suggestions on reading to various types of students. The real number system, continuity, differentiability and integrability of one function of one real variable are carefully treated in the next chapter. In chapter III, the author considers the analogous properties of a function of two or more real variables. Implicit functions, the distinction between iterated and multiple integrals and the relation of these integrals are discussed. Geometry is introduced into the text in the next chapter via a discussion of the notion of parameterized arcs. In particular, Theorem (A 15.1) should be mentioned. This interesting result is neglected in most texts on advanced calculus. The chapter closes with a treatment of some topics in the calculus of variations. In particular, the author discusses the Euler equations in order to provide a base for a later consideration of the theory of geodesics. An excellent account of the e-systems and δ-systems, determinants and coordinate transformations, brings Part A to a close. Although the group concept is completely developed in the treatment of coordinate transformations, unfortunately, the specific term "group" is nowhere defined nor used. The summation convention is introduced in Part A and the role of the Jacobian in integral transformations is clearly presented. Further, the problems in this part and the following parts of the book are well chosen and serve to supplement the theorems of the text proper.

Part B deals principally with the standard topics of vector analysis. The presentation differs considerably from that found in most texts in that: (1) definitions, axioms, theorems are clearly stated; (2) considerable stress is placed upon invariants. In chapter VI, the properties of vectors are defined, the scalar and vector products are introduced as invariants, and the triple scalar and vector products are discussed. It should be noted that with the aid of the e-system, the author offers an easy proof of the usually difficult triple vector expansion. Chapter VII contains an axiomatic approach to n-dimensional vector spaces and linear manifolds. Following the modern approach to geometry, the author, next, introduces a metric in his n-space. An interesting exposition of the Schmidt orthogonalization process for positive definite and indefinite metrics concludes the chapter. Chapter VIII contains a discussion of the elements of the differential geometry of curves and surfaces, the gradient, divergence, and curl, and an introduction to tensor and extensor transformation theory. In concluding Part B, the author discusses the important integral transformations of vector analysis—the Gauss, Stokes, and Green theorems. These topics—particularly Stokes' theorem—are well treated. This section contains only one troublesome typographical error: on page 159, the expression $(\gamma-\omega\times R)$ should read $\gamma-(\omega\times R)$. Further, the term "unitary orthogonal" on page 174 appears to be misapplied. In the literature, this term is reserved for vectors which are functions of a complex variable.

Part C deals with the theory of weighed tensors and extensors. Since the author has made important research contributions to extensor theory, his presentation of this subject should prove valuable to potential research workers in this field. Chapter X deals with the algebra of weighed tensors and extensors—the addition, multiplication, and contraction laws. The next chapter presents the differential calculus of tensors and extensors. Fundamental extensors, the fundamental tensors, and the Christoffel symbols are introduced in terms of derivatives of the radius vector in a Euclidean n-space. By a contraction of these fundamental extensors, the author obtains the ordinary covariant derivative. This procedure gives the combined theory of extensors and tensors considerable unity. A treatment of geodesics, geodesic coordinates, and the Riemann-Christoffel tensor concludes this part of the text. There is one minor point in notation which may cause some confusion. The author uses a pair of brackets around any two indices to indicate their antisymmetric nature. However, in a few instances, brackets are inserted about three indices without adequate explanations. Aside from this omission, part C is clearly written and constitutes a very good introduction to the theory of tensors and extensors.

Part D applies the material developed in parts B and C to classical dynamics and relativity. The first chapter in this section discusses such topics as moving coordinate systems, kinetic and potential energy of a particle and a system of particles, and the mechanics of continuous media. An interesting feature in the treatment of this last topic is the introduction of the Newtonian (mass constant) form and special relativity (mass variable) form of the equations of motion. Chapter XIII furnishes a discussion of the basis of special relativity, the Lorentz-Einstein transformation, applications of this transformation to velocity, acceleration, force, and mass, and finally a discussion of the energy-momentum tensor. The final chapter gives an introduction to General Relativity. In particular, the computation of the Schwarzchild line element is given in considerable detail.

This book presents vector and tensor analysis from an interesting point of view. It should provide a valuable addition to mathematical literature.

N. COBURN