however abstract, which may not some day be applied to phenomena of the real world."

The book is written in an elegant, almost poetic, style that makes it delightful to read, and its many references conveniently placed at the bottom of the page near the related subject matter serve as a lure to further reading.

J. WALTER WILSON

Partial differential equations. By Frederick H. Miller. New York, Wiley; London, Chapman and Hall, 1941. 9+259 pp. $3.00.

According to the author the book is intended to be a text in a first course in partial differential equations. The chapter on ordinary differential equations is intended for review and reference purposes and not as a first course in the subject. The author finds it advisable to include a chapter on direction cosines and partial derivatives, probably because so many exercises in the book are taken from geometry. In the main the book is concerned with the quest for solutions depending on arbitrary constants and arbitrary functions. The examples of Chapter III show very clearly why this viewpoint of the subject is much more complicated in the case of partial differential equations than it is in the case of ordinary equations. In ordinary equations the solution of an nth order equation depends on n arbitrary constants, and conversely, the elimination of n arbitrary constants leads to an equation of nth order. In general, the number of partial derivatives of a given order is higher than the number of independent variables. The elimination of two arbitrary functions may lead to a pair of third order equations in one unknown. Since the first order partial differential equation behaves more like an ordinary equation than do those of higher order, Chapter IV on the linear equation of first order and Chapter V on nonlinear equations of first order are almost entirely devoted to the quest for solutions depending on arbitrary functions and arbitrary constants.

Chapter VI on Fourier series and the boundary value problems in Chapter VII furnish an exception to the above viewpoint. In this work the author is, of course, not seeking solutions depending on arbitrary functions. Chapter VI on Fourier series contains a statement of the expansion theorem for a function continuous except for a finite number of jump discontinuities. Chapter VII on the linear equation of higher order is devoted largely to the consideration of operator methods, undetermined coefficients, and variation of parameter methods for obtaining the particular solution. In case the nth order differential operator can be factored into linear factors a complimentary function depending on n arbitrary functions can be obtained. As mentioned above, Chapter VII also contains separating variables and expansion of boundary conditions in Fourier series. The equations treated include the vibrating string, the cable, fluid flow, and heat flow. These equations have already been derived from physics in Chapter III.

Chapter VIII on nonlinear equations of second order is devoted largely to obtaining solutions of Monge's equation depending on arbitrary functions.

The book contains very few examples from physics except in the boundary value problems solved with Fourier series. However, the book contains an exceptionally large number of problems in which the student is asked to find a solution depending on arbitrary constants or functions. In a large proportion of these the partial differential equation is proposed as a problem in geometry.

The multiple integration approach to partial differential equations is not touched upon. The concept of characteristics of a second order equation and the classification of second order equations does not appear. Also there is no mention of successive integrations.

EDWIN W. TITT

The calculi of lambda-conversion. By Alonzo Church. (Annals of Mathematics Studies, no. 6.) Princeton, Princeton University Press; London, Humphrey Milford and Oxford University Press, 1941. 2+77 pp. $1.25.

This is a brief and attractively written introduction to the remarkable formal systems discovered by Church and called by him calculi of lambda-conversion. These systems were developed by Church in collaboration with his students, S. C. Kleene and J. B. Rosser. The present booklet, which is lithoprinted, is in most respects a considerable improvement over the same author's mimeographed Princeton lecture notes of 1936, of which it may be considered a revision. The notation has been simplified and improved, and the treatment of Gödel numbers is much simpler than in the former version. The proof of the fundamental Church-Rosser consistency theorem is now given in full detail, and the section on recursive arithmetic has been considerably expanded.

Nevertheless the text of the present version totals only 71 pages. This brevity is partly accounted for by the plan which the author has wisely adopted of considering only the calculus of lambda-conversion proper in full detail. In fact, the first four chapters are devoted to this, the most elementary of the lambda-calculi. The more com-