plication and addition) on indeterminates (which first appear as indeterminate coefficients). The situation is studied from several points of view. I (Abstract point of view): One set of relations may imply another. A set may be contradictory. Examples are given of complete sets; a set is complete when any relation compatible with it is implied by it. II (Realization): Here a linear vector space is considered and linear operators on it which satisfy the same relations as those that are given. The connection with I is given by the fact that relations satisfied in an invariant subspace imply relations in the whole space. In III a ring is considered generated (with the aid of a field) by operators satisfying given relations. The special case when the relations involve multiplication only correspond to a group algebra. IV deals with relations satisfied by operators as a result of their being operators on a vector space of a given dimensionality. (Received October 23, 1943.)

16. H. E. Salzer: *New tables and facts involving sums of four tetrahedral numbers.*

The author has a second empirical theorem about tetrahedral numbers, that is, $(n^3-n)/6$ for integral n. Every tetrahedral number greater than 1 is the sum of four other non-negative tetrahedrals. This theorem has been verified for the first 200 cases in a table expressing every tetrahedral from 4 through 1373701 as a sum of four non-negative tetrahedrals. With the exception of 153, the first 200 triangular numbers $n(n+1)/2$ can each be expressed as the sum of four non-negative tetrahedrals. There are only 45 integers less than or equal to 1000 which cannot be expressed as the sum of four non-negative tetrahedrals. All numbers ending in 0, 5, or 6 which are less than or equal to 2006 are expressible as a sum of four non-negative tetrahedrals. This includes the first 201 cases of each type. It is interesting to note that the smallest example of a number ending in 4 which is not expressible as a sum of four non-negative tetrahedrals occurs at 1314. Thus here is an instance where a statement is true in the first 131 cases, but fails in the 132nd. (Received October 13, 1943.)

17. L. R. Wilcox: *Modularity in Birkhoff lattices.*

The following theorem connecting G. Birkhoff's upper semi-modular lattices with the author's M-symmetric lattices is proved. Let a lattice be called upper semi-modular if $a+b$ covers a, b when a and b cover ab; let a lattice be called M-symmetric if $(a+b)c=a+bc$ for every $a\leqq c$ implies $(d+c)b=d+cb$ for every $d\leqq b$; finally, let a lattice be called of finite dimensional type if every a, b with $a<b$ have a finite principal chain connecting them. Then a lattice of finite dimensional type is upper semi-modular if and only if it is M-symmetric. The purpose of this theorem is to replace the condition of Birkhoff, forceful only when some chain condition is assumed, by a strictly algebraic condition which is suitable for use in the infinite dimensional case. (Received October 19, 1943.)

Analysis

18. Stefan Bergman: *The determination of singularities of functions satisfying a partial differential equation from the coefficients of their series development.*

Let $U(z, \bar{z})=A_{00}+\sum_{m=0}^{\infty}\sum_{m=1}^{\infty}A_{mn}z^m\bar{z}^n$ be a (complex) solution of the equation $L(U)\equiv U_{z\bar{z}}+a_1U_z+a_2U_{\bar{z}}+a_3U=0$ where a_k, $k=1, 2, 3$, are entire functions of two variables $z=x+iy$, $\bar{z}=x-iy$, x, y real. Using the results of the papers Rec. Math.

(Mat. Sbornik) N.S. vol. 2 p. 1169, and Trans. Amer. Math. Soc. vol. 53 (1943) p. 130, §7, the author shows that the positions (that is, the coordinates x, y) of singularities of $U(z, \bar{z})$ are determined by the sequence $\{A_{m0}\}$, $m=0, 1, 2, \cdots$, and are independent of the a_k, $k=1, 2, 3$. He investigates further the connections between other subsequences of $\{A_{mn}\}$ and the positions of singularities. For instance the A_{m1} and the $\alpha_{m0}^{(\nu)}$, $\nu=1, 2, 3$, $m=0, 1, 2, \cdots$, where $a_\nu=\sum \alpha_{mn}^{(\nu)} z^m \bar{z}^n$, determine the positions of the singularities. If $a_2=\bar{a}_1$ and a_3 is real and $u=\sum D_{mn} z^m \bar{z}^n$ is a real solution of $L(u)=0$, then the D_{m0} and the $\alpha_{0m}^{(1)}$, $m=0, 1, \cdots$, determine the positions of singularities. Finally the author shows that many properties of the singularities are determined by the A_{m0} and are independent of the a_k, $k=1, 2, 3$. (Received October 26, 1943.)

19. Lipman Bers and Abe Gelbart: *On a class of functions defined by certain partial differential equations.*

This paper contains a detailed mathematical treatment of the results presented in the Quarterly of Applied Mathematics vol. 1 (1943) pp. 168–188, and several new results on the properties of the functions satisfying the system of equations $\sigma_1(x)u_x=\tau_1(y)v_y$, $\sigma_2(x)v_y=-\tau_2(y)v_x$. (Received October 4, 1943.)

20. Lipman Bers and Abe Gelbart: *On functions satisfying certain systems of partial differential equations.*

This paper continues the study of the class of complex-valued functions $f(z)=u(x, y)+iv(x, y)$ where u and v satisfy the system of differential equations (1) $u_x=\tau_1(y)v_y$, $u_y=\tau_2(y)v_x$, τ_i being positive and analytic. With the aid of the function $E(\alpha, y)$ belonging to the class and corresponding to the analytic function exp (αy) (cf. Quarterly of Applied Mathematics vol. 1 (1943) pp. 168–188), a "Laplace transformation" $f(z)=\int_0^\infty E(-\alpha, z)g(\alpha)d\alpha$ is defined. Under suitable conditions $f(z)$ belongs to the class. If $g(z)\equiv 1$, $f(z)=z^{(-1)}(z)$ corresponds to the analytic function $1/z$. Its nth Σ-derivative corresponds to $1/z^n$, its Σ-integral to log z. Properties of these functions are investigated. (Received October 4, 1943.)

21. D. G. Bourgin and C. W. Mendel: *Orthonormal sequences.*

Let $f(x)\in L_2$ be odd and periodic of period 2π. Suppose $\{f(nx)\}$, $n=1, 2, \cdots$, is an orthonormal sequence on $0\leqq x\leqq 2\pi$. The investigation of functions satisfying these conditions leads to certain natural subclasses. Explicit examples are given other than the trivial solutions $f(x)=\sin kx$. Properties of these solutions are developed and various subsidiary conditions are formulated under which the trivial solutions alone are possible. (Received October 22, 1943.)

22. Alfred Brauer: *On certain limits.*

The following theorem is proved: Let k and l be positive integers and $m=k+l-2$. Suppose that the functions $f(x)$ and $g(x)$ and their first $m+2$ derivatives exist and are bounded in the interval $0<x\leqq h$. Suppose further that $f(x)$ is positive there. Finally suppose that $\lim f(x)=\lim f'(x)=\cdots=\lim f^{(k-1)}(x)=0$, but $\lim f^{(k)}(x)\neq 0$, and $\lim g(x)=\lim g'(x)=\cdots=\lim g^{(l-1)}(x)=0$, but $\lim g^{(l)}(x)\neq 0$. Then $\lim f(x)^{g(x)}=1$. In all limits $x\rightarrow +0$. (Received November 19, 1943.)

23. W. B. Caton and Einar Hille: *On the class of functions $H_p(1/2)$.*

One says that $f(z)\in H_p(1/2)$, $1<p\leqq 2$, if $f(z)$ is holomorphic in $R(z)>1/2$ and if $\int_{-\infty}^{\infty}|f(x+iy)|^p dy < M$ for all $x>1/2$. In this note a new proof is given of the well known theorem which states that $f(z) \in H_p(1/2)$ implies $f(z) = \int_0^\infty e^{-uz}F(u)du$, $F(u)e^{-u/2} \in L_q(0, \infty)$, $1/p+1/q=1$. The proof follows a general plan outlined in a paper of Hille (Compositio Math. vol. 6 (1938) pp. 93–102). (Received October 19, 1943.)

24. B. H. Colvin: *The expansion problem associated with a third order ordinary differential system of highly irregular type.*

The expansion problem considered is that associated with the third order ordinary differential system $y'''(x)+\lambda^3y(x)=0$, $H_i(y) \equiv h_{i1}(\lambda)y''(a)+h_{i2}(\lambda)y'(a)+h_{i3}(\lambda)y(a)+h_{i4}(\lambda)y''(b)+h_{i5}(\lambda)y'(b)+h_{i6}(\lambda)y(b)=0$ $(i=1, 2, 3)$, in which the boundary conditions are of highly irregular type. The variable x is restricted to the real interval $a \leqq x \leqq b$: λ is an unbounded complex parameter and the coefficients $h_{ij}(\lambda)$ are polynomials in λ with complex coefficients. The general mode of attack is that recently employed by R. E. Langer in connection with highly irregular systems of the second order. By the use of some rather general notions of summability (first introduced by R. E. Langer) an expansion theory associated with such irregular systems is developed for integrable vectors $f(x)$ which fulfill certain conditions customarily imposed in such theories. The third order boundary systems considered include as special cases those discussed previously by J. W. Hopkins and L. E. Ward. The present theory, however, provides summable expansions for a class of functions far wider than that for which they obtained any results. (Received October 30, 1943.)

25. Paul Erdös and S. M. Ulam: *Some combinatorial problems in set theory.* Preliminary report.

The following problems seem of some interest: 1. Given a class K of subsets of a set E of power m, the class closed with respect to the operation of addition of less than m sets belonging to the class, and with respect to complementation, and with this additional property: For every subdivision of E into disjoint sets containing each more than one element, there exists a set in K which contains exactly one element of each of the sets of the subdivision. Must K under these conditions coincide with the class of all subsets of E? If $m=\aleph_0$, the answer is negative; otherwise, unsolved. 2. Let E be a set of power m. Is it possible to define $n<2^m$ completely additive measure functions on the class of all subsets of E so that each subset will be measurable in at least one measure function? 3. If the continuum hypothesis is true, every subset of the continuum can be obtained by Borel operations effected on sets that are additive subgroups of the real numbers. The statement that every subset can be obtained by Borel operations on sets consisting of rationally independent numbers is equivalent to the continuum hypothesis. (Received October 27, 1943.)

26. R. C. James: *Orthogonality and differentiability in normed linear spaces.*

Several definitions can be given of orthogonality in normed linear spaces. In this paper, it is said that an element x is orthogonal to y if and only if $\|x+ky\| \geqq \|x\|$ for all k. Such orthogonality is neither symmetric ($x \perp y$ does not imply $y \perp x$) nor additive ($x \perp y$ and $x \perp z$ do not imply $x \perp y+z$). However, this orthogonality is never vacuous, since for any x and y there exists an a such that $x \perp (ax+y)$. An equivalent definition of orthogonality is: "$x \perp y$ if $\lim_{n\to\infty}\|nx+y\|-\|nx\|=0$," provided $x \neq 0$, and it follows that this orthogonality is additive if and only if the norm is linearly Gateaux differentiable at each nonzero point. Furthermore, if this differential exists at a point x, it is equal to $-a\|x\|$, where $x \perp (ax+y)$. This gives rise to a condition for the existence of a scalar product, and to a means of investigating the effect on a normed linear space of the existence of the differential of the norm. (Received October 29, 1943.)

27. Anne L. Lewis: *Sufficiency proofs for the problem of Bolza in the calculus of variations.*

This paper is mainly concerned with the establishment of sufficient conditions for a strong relative minimum for the general isoperimetric problem of Bolza in non-parametric form. The proof involves an expansion of the Hilbert integral and the use of a so-called quasi Mayer field, which is defined in terms of the variations of the slope functions and multipliers generally associated with a Mayer field. This procedure combines the field theory methods of Hestenes (Trans. Amer. Math. Soc. vol. 42 (1937) pp. 141–153; Duke Math. J. vol. 5 (1939) pp. 309–324) and the expansion methods of Reid (Trans. Amer. Math. Soc. vol. 42 (1937) pp. 183–190; Ann. of Math. (2) vol. 38 (1937) pp. 662–678) and utilizes certain dominant properties of the Weierstrass E-function. There are obtained, moreover, various results concerning the Weierstrass E-function which yield a Lindeberg theorem for the isoperimetric problem considered. (Received October 22, 1943.)

28. A. T. Lonseth: *The propagation of error in linear problems.*

Suppose that A is a bounded linear transformation in Hilbert space H which possesses a bounded inverse A^{-1}, and that C is in H. Then the solution $X = A^{-1}C$ of $AX = C$ is also in H, its norm $N(X)$ satisfying $N(X) \leqq M(A^{-1})N(C)$, where $M(A^{-1})$ is the bound of A^{-1}. The error ξ induced in X by errors α in A and γ in C is considered; it is found that $N(\xi) \leqq M(A^{-1})\{N(\gamma)+M(\alpha)N(X)\}/\{1-M(A^{-1})M(\alpha)\}$, provided that $M(A^{-1})M(\alpha)<1$. (This inequality generalizes readily to linear functional equations in a Banach space.) A case of special interest is that in which $A = I+K$, where I is the identity and $M(K)<1$; then (E. Hilb) A has a unique inverse A^{-1} and $M(A^{-1})<1/\{1-M(K)\}$. In this case bounds are also found for the components of ξ. The inequalities provide limits to the errors incurred in certain approximate methods for solving linear problems: infinite systems of linear equations in infinitely many unknowns (method of segments) and the related linear integral equations of first and second kinds. (Received October 20, 1943.)

29. Morris Marden: *A recurrence formula for the solutions of certain partial differential equations.*

Given the second order linear partial differential equation $L(U) = U_{z\bar{z}} + AU_z + BU_{\bar{z}} + CU = 0$, where A, B and C are analytic functions of the two complex variables $z = x+iy = re^{i\theta}$ and $\bar{z} = x-iy = re^{-i\theta}$. As proved by Bergman (Rec. Math. (Mat. Sbornik) N.S. vol. 44 (1937) pp. 1169–1198), there exists a function $E(z, \bar{z}, t)$ such that the operator $P(f) = \int_{-1}^{1} E(z, \bar{z}, t) f[(1/2)z(1-t^2)](1-t^2)^{-1/2}dt$, acting upon an arbitrary analytic function $f(z)$, produces a solution U of the equation $L(U) = 0$. As the solutions so produced may be expanded in terms of the particular solutions $U_n = P(z^n)$, the study of the solutions U would be aided by the existence of recurrence relations among the solutions U_n. In this paper it is proved that when $\log E(z, \bar{z}, t)$ is an even or odd polynomial in t, there can be found $q+1$ functions $k_j(r, \theta, n)$ such that $\partial U_n/\partial r = k_0 U_n + k_1 U_{n+1} + \cdots + k_q U_{n+q}$, where q is the degree of $\log E$ as a polynomial in t if $\log E$ is an odd polynomial in t, but is only half this degree if $\log E$ is an even polynomial in t. The result is a generalization of a recurrence relation for Bessels' functions J_n. (Received October 7, 1943.)

30. K. L. Nielsen: *On operators for linear partial differential equations.*

Bergman (Trans. Amer. Math. Soc. vol. 53 (1943) pp. 130–155) has considered the relation between functions $U(z, \bar{z})$ satisfying certain partial differential equations $L(U) = U_{z\bar{z}} + aU_z + bU_{\bar{z}} + cU = 0$ and an associated analytic function $f(z)$ in terms of which the solution of $L(U) = 0$ can be expressed. He has introduced an operator $P(f) = \int_{-1}^{+1} E(z, \bar{z}, t) f(z[1-t^2]/2)(1-t^2)^{-1/2} dt$ which transforms the class of analytic functions of one complex variable into a certain class of functions $C(E)$ which represents solutions of $L(U) = 0$. The author and Ramsay (Bull. Amer. Math. Soc. vol. 49 (1943) pp. 156–162) have found simple forms of $E = \exp\,[N(z, \bar{z})t^n + M(z, \bar{z})t^m]$ for given partial differential equations. In this paper the case of $E = \exp\,[\sum_{v=1}^{n} p_v(z)t^v]$ is developed and limiting theorems on m and n are found for the above expression of E. (Received October 22, 1943.)

31. W. T. Reid: *Expansion theorems for boundary problems of the calculus of variations.*

In a previous paper (Amer. J. Math. vol. 54 (1932) pp. 769–790) the author treated a two-point boundary problem associated with the general problem of Bolza in the calculus of variations, and established certain expansion theorems in terms of the characteristic solutions of the considered problem. The present paper is concerned with the proof of more refined expansion theorems for such a boundary problem. In particular, the general theorems herein proved provide improvements of the expansion theorems of Kamke (Math. Zeit. vol. 45 (1939) pp. 759–787; vol. 46 (1940) pp. 231–250 and pp. 251–286) for self-adjoint definite problems involving a single differential equation of even order. The paper contains results of the Riesz-Fischer type which include as very special cases the results of Krein (Rec. Math. (Mat. Sbornik) N.S. vol. 2 (1937) pp. 923–933) for a self-adjoint problem involving a single differential equation of even order, and the more extensive results of this type for a self-adjoint problem involving a second order differential equation that have been established by Galbraith and Warschawski (Duke Math. J. vol. 6 (1940) pp. 318–340). (Received October 21, 1943.)

32. A. R. Schweitzer: *On functional equations with solutions containing arbitrary functions.* III.

When the method of solving functional equations outlined in previous reports in this Bulletin (abstracts 49-9-212, 213) is not applicable, the author notes that a procedure of restricting variables, or generalizing functions or variables, frequently leads to equations with solutions containing arbitrary functions. Perhaps the simplest example is offered by the associative equation $\phi\{x, \phi(y, z)\} = \phi\{\phi(x, y), z\}$. The equation $\phi\{x, \phi(y, x)\} = \phi\{\phi(x, y), x\}$ has the solution $\phi(x, y) = \alpha(x+y)$ and the equation $\phi\{x, f(y, z)\} = f\{\phi(x, y), z\}$ has the solution $\phi(x, y) = \alpha(x) \cdot y$, $f(x, y) = x \cdot \beta(y)$. Also the equation $\phi\{x_1, x_2, \cdots, x_n, \phi(y_1, y_2, \cdots, y_{n+1})\} = \phi\{\phi(x_1, x_2, \cdots, x_n, y_1), y_2, y_3, \cdots, y_{n+1}\}$ has the solution $\phi(x_1, x_2, \cdots, x_{n+1}) = x_1 \cdot \alpha(x_2, x_3, \cdots, x_n) \cdot x_{n+1}$ $(n > 1)$. The latter equation has the inverse $f(u, x_2, \cdots, x_n, v) = f(x_1, x_2, \cdots, x_{n+1})$ where $u = f(x_1, t_1, t_2, \cdots, t_n)$ and $v = f(x_{n+1}, t_1, t_2, \cdots, t_n)$. In the above the functions α and β are arbitrary. (Received October 21, 1943.)

33. Wolfgang Sternberg: *On difference equations.*

The author gives a solution of the often treated fundamental difference equation

$F(t+1)-F(t)=\phi(t)$. It is supposed that the given function $\phi(t)$ can be expanded in a Fourier series in every finite interval of t and, for simplicity, that $\phi(t)$ is continuous. The solution is $F(t)=-\phi(t)/2+\int_a^t\phi(\tau)d\tau+2\sum_{k=1}^{\infty}\int_a^t\phi(\tau)\cos 2\pi k(t-\tau)d\tau$ where a is an arbitrary real constant. The above series is not a Fourier series in the usual sense, because the upper limit of the integrals is not constant, but the variable t itself. Some other solutions are given besides the above one. Finally the general linear difference equation of order n with constant coefficients $a_nF(t+n)+\cdots+a_1F(t+1)+a_0F(t)=\phi(t)$ is reduced to n equations of order 1 by the following theorem: If the characteristic equation $P(u)=a_nu^n+\cdots+a_1u+a_0=0$ has n different roots α_i $(i=1, 2, \cdots, n)$ and if $F_i(t)$ is an arbitrary solution of $F_i(t+1)-\alpha_iF_i(t)=\phi(t)/P'(\alpha_i)$, then $F_1(t)+F_2(t)+\cdots+F_n(t)$ is a solution of the above equation of order n. The case of multiple zeros of $P(u)$ can be dealt with in a similar way. (Received October 30, 1943.)

34. W. J. Thron: *Sets of convergence for continued fractions.* Preliminary report.

Let A be a set of complex numbers. Corresponding to every set A, a set $Z(A)$ is defined as follows: $1\in Z$; for every positive integer k, $1+a_1/1+a_2/1+\cdots+a_k/1\in Z$ if $a_i\in A$ $(i=1, 2, \cdots, k)$; the set Z contains no other elements. The following theorem has been established: for every set A the set $Z(A)$ either contains the point $z=0$ or has no point in common with the set $1-Z(A)$ $(v\in 1-Z$ if $1-v\in Z)$. If $0\in Z(A)$ there exists a continued fraction $1+K(a_n/1)$, all of whose elements a_n are in A, which diverges by oscillation. Hence such a set A cannot be a set of convergence, even if convergence is understood to mean convergence in the wider sense. Thus a necessary condition for a set A to be a set of convergence for continued fractions $1+K(a_n/1)$ is that the sets $Z(A)$ and $1-Z(A)$ have no point in common. (Received November 17, 1943.)

35. W. J. Trjitzinsky: *Problems of representation and uniqueness for functions of a complex variable.*

The leading idea in the present extensive work (as in the author's earlier related memoirs in Ann. École Norm. (3) vol. 55 (1938) Fasc. 2 pp. 119–191, and in Acta Math. vol. 70 (1939) pp. 63–163) is that functions are studied which are not necessarily analytic but which at the same time are sufficiently specialized so as to come within the scope of classical analytic tools (like Lebesgue-Stieltjes integration). It appears that conditions of monogeneity in one sense or another (in a rather general sense) over sets in the complex plane, which may be without interior points, are the conditions which give the desired degree of specialization. On the other hand, the classes of functions so obtained are of such great vastness and the possibility of classifying these functions according to various uniqueness properties and subsequently studying them is so wide that there appears to be on hand a very extensive field for new investigation. (Received October 23, 1943.)

36. S. M. Ulam: *Theory of the operation of product of sets.* III. Preliminary Report.

Two subsets A, B of a product set E^n are called product isomorphic (abstract 47-9-406), if there exists a one-to-one transformation $T(x)$ of E into itself such that the transformation T^n: $x_1\rightarrow T(x_1), \cdots, x_n\rightarrow T(x_n)$ maps A into B: $T^n(A)=B$. Starting with a class K of subsets of E one obtains the projective class containing it by considering in the product set E^n sets of the form $Z_1 \times Z_i \times Z_n$, where Z_i belongs to K, the Borel field over such sets and using the operation of projection and complementation a finite number of times. Necessary and sufficient conditions are found in terms of a special mapping of E into E^n preserving a given class of sets in E^n, for the product isomorphism in case of sets A, B in the projective class. The classical theories of Borel sets and of projective sets correspond to the case where K is the sequence of rational intervals. Many results of these theories hold for the case of a general K and thus show a purely combinatorial and not topological origin. This permits the formulation of an analogous theory in the even more general case of projective algebras (abstract 49-5-151). (Received October 22, 1943.)

37. S. E. Warschawski: *On Theodorsen's method of conformal mapping of nearly circular regions.*

The paper deals with the problem of determining the mapping function of a circle onto a nearly circular region. This problem has some practical importance in the theory of airfoils. Let C be a nearly circular closed Jordan curve: $\rho = \rho(\phi)$, $0 \leqq \phi \leqq 2\pi$, (ρ, ϕ polar coordinates) where $(1/\rho)(d\rho/d\phi)$ is continuous and $1 \leqq \rho(\phi) \leqq 1+\epsilon$ for some ϵ, $0<\epsilon<1$. Suppose that $w=f(z)(f(0)=0, f'(0)>0)$ maps the circle $|z| = |re^{i\theta}| < 1$ conformally onto the interior of C. If $\arg f(e^{i\theta}) = \phi(\theta)$, the $\log (f(z)/z)$ for $z = e^{i\theta}$ may be written as $\log \rho[\phi(\theta)] + i(\phi(\theta)-\theta)$. Hence by Fatou's formula, $\phi(\theta)-\theta = (1/2\pi)\int_0^\pi \{\log \rho[\phi(\theta+t)] - \log \rho[\phi(\theta-t)]\} \cot (t/2)dt \equiv H[\phi(\theta)]$, and the function $\phi(\theta)$ may thus be determined by solving this integral equation. Theodorsen (National Advisory Committee for Aeronautics, Report 411, 1932) and Theodorsen and Garrick (National Advisory Committee for Aeronautics, Report 452, 1933) developed a practical method for computing a solution of this equation by successive approximations. In the present paper the theoretical basis of this method is studied. Sufficient conditions for the curve C are established under which the approximations $\phi_0(\theta) \equiv \theta$, $\phi_n(\theta) = H[\phi_{n-1}(\theta)]$ and their derivatives $\phi_n'(\theta)$ converge to $\phi(\theta)$ and $\phi'(\theta)$ respectively, and the errors $|\phi_n(\theta) - \phi(\theta)|$ and $|\phi_n'(\theta) - \phi'(\theta)|$ are estimated. (Received October 27, 1943.)

Applied Mathematics

38. L. W. Cohen and S. M. Ulam: *On the algebra of systems of vectors and some problems in kinematics.*

The properties of equivalence for systems of vectors as postulated for the mechanics of rigid bodies are studied in linear spaces. It is proved that any finite system of vectors in n-space is equivalent to a unique system of vectors collinear with the edges of an n-simplex. It is also proved that any such system is equivalent to a system of $[n/2]+1$ vectors. Similar theorems hold for infinitely many vectors and for spaces of infinitely many dimensions. The problem of topological invariants of trajectories of systems of n points with respect to arbitrary motions of the coordinate system is formulated and results are obtained for the case of three points. (Received October 22, 1943.)

39. A. H. Copeland: *The nature of turbulence.*

There are exhibited in this paper a number of flows which consist of series of disturbances distributed temporally but unfortunately not specially at random, and which satisfy the hydrodynamic equations together with appropriate boundary con-