EDWARD BURR VAN VLECK—IN MEMORIAM

The death of Edward Burr Van Vleck on June 2, 1943, at almost eighty years of age, in Madison, Wisconsin, will bring regret to a wide circle of friends and to American mathematicians generally. His creative mathematical work, his important role in the development of American mathematics since 1890, and his qualities of personality were such as to make all feel that something of inestimable and characteristic value has departed and yet remains in treasured remembrance.

Van Vleck was born in Middletown, Connecticut, on June 7, 1863, of Knickerbocker stock, for the first Van Vleeck came from Amsterdam, Holland, to New Amsterdam (New York) in 1658. His father was a distinguished American mathematical astronomer, Professor John Monroe Van Vleck, in whose memory has been erected the Van Vleck Observatory at Wesleyan University, given by a brother. The scientific tradition, thus doubly established in the Van Vleck family, has been notably continued to a third generation by his son, Professor John Hasbrouck Van Vleck, now Professor of Mathematical Physics in Harvard University.

The general account of Edward Burr Van Vleck's life is simple. As a boy, college student, and instructor, he developed in the typical New England environment of Middletown, living at home with his parents and three sisters (one of whom survives him) until about twenty-five years of age, obtaining an A.B. degree from Wesleyan University at twenty-one years of age and an A.M. degree three years later. Then he spent two years as a graduate student at Johns Hopkins University, where he veered from the field of physics to that of mathematics. After this came three further years at the University of Göttingen, Germany, where he was inspired by the great mathematician Felix Klein. He obtained his Ph.D. degree in 1893 at Göttingen at about thirty years of age. Thus he entered upon his mathematical research work somewhat later than usual, a fact which he always felt to be regrettable.

His first college post was that of instructor for two years at the University of Wisconsin, after which he taught for ten years at his alma mater, Wesleyan University, becoming associate and full professor there. In 1906 he went to the University of Wisconsin as Chairman of the Department of Mathematics, and, except for a half-year (1919–20) when he was visiting lecturer in the Department of Mathematics at Harvard University, and for various periods of travel in this country and abroad, remained in Madison thereafter. He became professor emeritus in 1929.

Behind this academic outline there was always a full and gracious life. Van Vleck married Miss Hester Laurence Raymond of North Lyme, Connecticut, shortly after he returned from his studies in Germany. In both Middletown and Madison, the Van Vlecks added much to the college and community life. Their lovely home in Madison, overlooking Lake Mendota, will be remembered by many as a center from which radiated kindness, friendship, and delightful hospitality to colleagues and their families, to students, and indeed to all those about them.

One may conjecture that Van Vleck was a very thoughtful and rather shy boy. Throughout his life he disliked all disputes, so that, even to his own disadvantage, he would avoid controversies with others. He liked urbane living and achieved it to a notable extent. However, when questions of principle arose, he stood his ground firmly and said precisely what he thought, although in polite form.

To those who knew him during his maturer years, he appeared as a clearsighted and liberal-minded idealist never deviating from the highest standards for himself, and yet fully appreciating the qualities of others who had a different outlook. His life was built around the motives of friendship and genuine scientific interest to a greater degree than is usually the case in the American academic scene.

Van Vleck's principal outside interests were reading and travel, both of which he shared with Mrs. Van Vleck. During the last half of his life he collected Japanese prints, and grew to be a connoisseur in this field.

On the mathematical side, Van Vleck owed much, for stimulus and inspiration, to Felix Klein. His thesis treated the development in continued fractions of integrals, forming solutions of certain linear differential equations of the second order, such as Lamé's integral. These integrals fall into natural families, any three "contiguous" members being connected by homogeneous linear difference equations. These integrals had appeared earlier in classical developments in continued fractions due to Gauss and others. This was a field which lay at the very heart of that part of analysis in which Klein was most interested. In his thesis Van Vleck extended considerably the types of such developments in a manner somewhat analogous to that devised by Pade in the similar development of arbitrary analytic functions. His thesis showed that not only was the young Van Vleck a master of the field involved but that he possessed definite creative power.

The question of the convergence of these continued fractions was left untouched in the thesis. For continued fractions of special form, this was taken up by him in three papers published in the second and fourth volumes of the Transactions of the American Mathematical Society (1901, 1904). A well known and general theorem there proved is that if in the continued fraction

$$\frac{1}{\alpha_1 + i\beta_1 +} \frac{1}{\alpha_2 + i\beta_2 +} \cdots \qquad i = (-1)^{1/2}$$

the real quantities α_i have the same sign while the real quantities β_i are alternately positive and negative, then the continued fraction will converge if the series $\sum(\alpha_u + i\beta_u)$ converges.

In his Colloquium Lectures of 1903, *Divergent series and continued fractions*, he first reviews the recent work of Poincaré, Stieltjes and others on divergent series (Part I) and then passes on to consider continued fractions (Part II). The two subjects are intertwined because "Both historically and prospectively one of the most suggestive and important methods of investigating power series is by the instrumentality of algebraic continued fractions." Van Vleck shows clearly how, from the broad point of view, the theory of continued fractions and their generalizations is closely related to the theory of linear homogeneous difference equations whose coefficients are polynomials in a complex variable. He wrote only one paper directly concerned with linear difference equations (Trans. Amer. Math. Soc., 1912) in which he extended a classic theorem of Poincaré. Doubtless the reason for this limited publication was the rapidity with which appeared new developments in the field by Nörlund and Galbrun abroad and by Carmichael, myself, and others in this country. However, while instructor at Wisconsin I had attended a graduate course given by Van Vleck in which he discussed various open problems concerning linear difference equations in a suggestive and stimulating way; while Carmichael shortly thereafter was a graduate student with me at Princeton. One must therefore look upon Van Vleck as an essential factor in the American contributions to linear homogeneous difference equations.

I have no doubt that when Van Vleck saw this promising field in which he had begun to work taken up so quickly by others, there was no tinge of regret on his part, for he would know that there were many other beautiful fields to be explored, and would feel that it was relatively unimportant from the larger point of view what the formal assignment of credit might be, so long as the subject itself was developed. It was enough for him that all should work together earnestly and sincerely for the increase of mathematical knowledge.

For a while Van Vleck seriously contemplated writing a book on functional equations, but the apparent lack of mathematical unity in the field prevented him from doing so. He wrote interesting papers dealing with special functional equations, in particular those satisfied by the elliptic ϑ-functions (Trans. Amer. Math. Soc., 1916); this last was the joint work of himself and one of his most gifted students, F. T. H'Doubler.

The range covered by Van Vleck in his more than thirty published articles was wide. He had a very keen analytic mind well adapted to the study of the niceties of point set theory, as manifested for instance in his article on non-measurable sets (Trans. Amer. Math. Soc., 1916). He also occupied himself with questions concerning the roots of algebraic equations and of the hypergeometric series, with special functions defined by differential equations, with the combination of substitutions, linear substitutions, and so on. In my opinion he resembled Bôcher in the respect that as the years went on, the self-imposed demand for elegance and simplicity exercised an inhibitive influence upon his production.

His general articles, *The influence of Fourier's series upon the development of mathematics* (Science, 1912), *The role of the point-set theory in geometry and dynamics* (Presidential Address, Bull. Amer. Math. Soc., 1915), *Current tendencies of mathematical research* (ibid., 1916), and *On the location of roots of polynomials and entire functions* (ibid., 1928) were thought-provoking essays and showed not only a wide knowledge and balanced perspective but an unusual ability to find the important open problems in the fields concerned. For example, in the second of these articles he states that Poincaré's definition of (dynamical) probability is equivalent to one in terms of the Lebesgue integral, although he does not give his proof. Carathéodory's well known proof of this equivalence appeared only in 1919.

In the teaching of undergraduates and of graduate students Van Vleck was clear and stimulating. At Wisconsin he gave himself unsparingly to the direction of the thesis work of undergraduates specializing in mathematics. As Chairman of the Department of Mathematics at Wisconsin for many years, he was deeply considerate of the members of the staff, completely democratic in spirit, and absolutely impartial. The friendship between him and his colleague Professor Ernest Skinner was one of great depth on both sides.

The mathematical reputation of Van Vleck was an international one. His relations with French mathematics were particularly close and sympathetic. Moreover his affection and admiration for France were profound. He was decorated "Officier de l'instruction publique" in 1920 in recognition of his services as teacher and investigator and for his work during the First World War. The University of Groningen on the three hundredth anniversary of its founding made him an Honorary Doctor in Mathematics and Physics in 1914. He, together with E. H. Moore and W. F. Osgood, received an honorary LL.D. at Clark University in 1909. At the twenty-fifth anniversary celebration of the University of Chicago in 1916, he received the honorary degree of Doctor of Science. His election to membership to the National Academy of Sciences came in 1911.

Van Vleck was President of the American Mathematical Society for the period 1913–1915. In earlier days he had been an effective and conscientious Editor of the Transactions. E. H. Moore, Maxime Bôcher, and H. S. White were others among his near contemporaries who were similarly honored by election to both of these important offices. All of these men served American mathematics faithfully and well. It will be remembered by many that, at one moment when the unity of the American Mathematical Society was somewhat in jeopardy, Van Vleck exerted an especially constructive influence.

It would have been interesting to have had Van Vleck himself formulate his outlook upon the world. Of his own volition he would of course never have done so. But, had he been somehow persuaded to give his philosophy of living, it would have expressed a large measure of tolerance towards others. As a young colleague of his at Wisconsin, I recall asking him about a great writer whose personal life had been far from admirable. Van Vleck replied very quickly: "Don't you think that what he has done for the world of literature more than made up for any personal defects?" Above all, he would have emphasized the virtues of kindness and friendship, of disinterested devotion to intellectual and other ideals, and of a due regard for the amenities of our civilization.

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