

which the stress does not vary along a normal. The significance of the assumptions is discussed. Finally it is shown that solutions of the equations of the membrane theory lead to a state of strain which does not satisfy the conditions of compatibility, so that displacements calculated in the membrane theory violate geometry. (Received August 6, 1943.)

295. D. V. Widder: *Positive temperatures on an infinite rod.*

In this paper it is proved that any solution  $u(x, t)$  of the heat equation  $\partial^2 u / \partial x^2 = \partial u / \partial t$  which is non-negative for positive  $t$  and which vanishes for  $t=0$  is identically zero. By use of this result it is shown that any solution which is non-negative for  $t > 0$  has the Poisson-Stieltjes representation  $u(x, t) = \int_{-\infty}^{\infty} k(x-y, t) d\alpha(y)$ . Here  $\alpha(y)$  is nondecreasing and  $k(x, t)$  is the familiar source solution  $(4\pi t)^{-1/2} \exp(-x^2/4t)$ . As a consequence any such solution is analytic in  $x$  and in  $t$ . (Received August 24, 1943.)

### GEOMETRY

296. R. C. Buck: *Partition of space.*

By an application of elementary topology, it is shown that  $n$  hyperplanes, with general intersection, partition Euclidean  $r$ -space into  $M_r(p, n)$   $p$ -dimensional regions, where  $M_r(p, n) = \sum_{r-p}^r C_{n,k} C_{k,r-p}$ , of which  $C_{r,p} C_{n,r+1} (r+1) / (n+p-r)$  are bounded. The problem is also solved for projective  $r$ -space, yielding  $\sum_0^{\lfloor (r-p)/2 \rfloor} C_{r-2k,r-p} C_{n,r-2k}$  as the number of  $p$ -dimensional regions. This completely solves the well known "cheese slicing" problem. (Received September 11, 1943.)

297. John DeCicco: *Dynamical and curvature trajectories in space.*

Kasner has studied the geometry of dynamical trajectories in the plane and in space in the Princeton Colloquium (Amer. Math. Soc. Colloquium Publications, vol. 3). This paper considers the problem of determining all quintuply-infinite systems of curves in space which are at once dynamical and curvature trajectories. In the plane, Kasner has shown that the appropriate families are the trajectories of all central or parallel fields of force. It is shown that the systems of  $\infty^5$  curves which are simultaneously dynamical and curvature trajectories are the dynamical trajectories of the following three distinct types of fields of force: (I) Those whose lines of force all lie in a pencil of planes. (II) Those whose lines of force are orthogonal to a family of  $\infty^2$  circular helices, all of which possess the same axis and the same period. (III) Those of the central or parallel type. Each of these types is projectively invariant. (Received August 11, 1943.)

298. Edward Kasner and John DeCicco: *Union-preserving transformation of space.*

Sophus Lie showed that the only lineal-element transformations of the contact type are the extended point transformations. This result is extended by studying transformations from differential curve-elements of order  $n$ :  $(x, y, z, y', z', \dots, y^{(n)}, z^{(n)})$ , where  $n$  is 2 or more, into lineal-elements  $(X, Y, Z, Y', Z')$ . The entire class of the union-preserving transformations is determined. Any general union-preserving transformation from curve-elements of order  $n$  into lineal-elements is completely determined by a new directrix equation  $\Omega(X, Y, Z, x, y, z, y', z', \dots, y^{(n-2)}, z^{(n-2)}) = 0$ . The only available union-preserving transformations (in the whole domain of

curve-elements) are the group of point transformations and the union-preserving transformations from curve-elements of order  $n$  into lineal-elements, or the extensions of these two types. An additional theorem is that any transformation from curve-elements of the  $(x, y, z)$ -space into lineal-elements of the  $(X, Y, Z)$ -space, by which any union of the  $(X, Y, Z)$ -space corresponds to exactly  $\infty^{2(n-1)}$  curves of the  $(x, y, z)$ -space, is union-preserving. (Received August 11, 1943).

299. L. A. MacColl: *Geometrical characterizations of some families of dynamical trajectories.*

This paper deals with a certain five-parameter family of curves, which can be regarded as the family of trajectories of an electrified particle in an arbitrary static magnetic field. A set of geometrical properties is given which completely characterizes the family of curves. Certain other related families of curves, including the four-parameter family of trajectories of the particle moving with an arbitrarily prescribed value of the energy, are also discussed and characterized by sets of geometrical properties. (Received October 1, 1943.)

300. Alice T. Schafer: *Two singularities of space curves.*

This paper uses the methods of projective differential geometry to study an analytic space curve in the neighborhood of an inflexion point and, second, a planar point. Canonical power-series expansions representing the curve in the neighborhood of each singular point are deduced by suitably choosing the projective coordinate system. These canonical expansions are then used to study properties of the curve in this neighborhood. Particular emphasis is placed on the surfaces osculating the curve, sections of the tangent developable of the curve made by the faces of the tetrahedron of reference, and projections of the curve onto the faces of the tetrahedron of reference. (Received October 1, 1943.)

### STATISTICS AND PROBABILITY

301. W. K. Feller: *On a general class of "contagious" distributions.*

This paper is concerned with some properties of a class of contagious distributions which contains, among others, some distributions studied by Greenwood and Yule, Polya, and Neyman, respectively. (Received August 3, 1943.)

302. H. B. Mann and Abraham Wald: *On the statistical treatment of linear stochastic difference equations.*

For any integer  $t$  let  $x_{1t}, \dots, x_{rt}$  be a set of  $r$  random variables which satisfy the system of linear stochastic difference equations  $\sum_{j=1}^r \sum_{k=0}^{p_{ij}} \alpha_{ijk} x_{j,t-k} + \alpha_i = \epsilon_{it}$  ( $i=1, \dots, r$ ). The coefficients  $\alpha_{ijk}$  and  $\alpha_i$  are (known or unknown) constants and the vectors  $\epsilon_t = (\epsilon_{1t}, \dots, \epsilon_{rt})$  ( $t=1, 2, \dots$ ) are independently distributed random vectors each having the same distribution. It is assumed that  $E(\epsilon_{it})=0$ . The problem dealt with in this paper is to estimate the unknown coefficients  $\alpha_{ijk}$  and  $\alpha_i$  on the basis of  $Nr$  observations  $x_{it}$  ( $i=1, \dots, r; t=1, \dots, N$ ). The statistics used as estimates of the unknown coefficients are identical with the maximum likelihood estimates if  $\epsilon_t$  is normally distributed. The joint limiting distribution of these estimates is obtained without assuming normality of the distribution of  $\epsilon_t$ . (Received August 7, 1943.)