tive, real-valued, non-negative, finite measure function on E, vanishing only for ϕ, the zero of E. E is given the star-topology of G. Birkhoff, and it is shown that E will have such a measure if and only if it is metrizable in a certain way in this topology. It is next shown that E is a topological space if and only if it satisfies a certain distributive law. A basis for the neighborhoods of ϕ can then be characterized algebraically, making it possible to state simple algebraic equivalents for the various separation axioms. If E satisfies the countable chain condition, T_3 implies metrizability, which gives an outer measure on E. A measure is obtained by means of an additional algebraic requirement. Thus, if E satisfies the distributive law referred to, the countable chain condition, the algebraic equivalent of T_3, and the additional requirement, there exists a measure-function on E. These conditions are easily seen to be necessary. It is not known whether they are independent. (Received October 2, 1943.)

250. R. M. Thrall: *On the decomposition of modular tensors.* II.

Let G be the n-rowed full linear group over a field k of characteristic p. A representation of G is called a tensor representation if its space is a direct sum of subspaces and factor spaces of tensor spaces. A main result of the present paper is that for a finite field k, the k-group ring of G has a faithful tensor representation. In paper I the representations afforded by all tensors of rank $m<2p$ were determined subject to the condition that k has more than p elements. In this paper the same is done for the field k with p elements. A main tool in this investigation is the construction of a representaton of G from each irreducible representation of the non-modular full linear group, and a corresponding extension of the Brauer-Nesbitt modular character theory to this case. The presence of zero divisors in the ring of polynomial functions over a finite field enters into the treatment of the case of tensors of rank $2p-1$ over a two-dimensional vector space, and the situation in that case should help point the way to the general decomposition theory. (Received October 1, 1943.)

ANALYSIS

251. Stefan Bergman: *Fundamental solutions of partial differential equations of the second order.*

As was previously shown, for every differential equation $L(U)=U_{Z\bar{Z}}+H(Z,\bar{Z})U=0$, $Z=X+iY$, $\bar{Z}=X-iY$, there exists a function $\mathrm{E}(Z,\bar{Z},t)=1+Z\bar{Z}t^2\mathrm{E}^*(Z,\bar{Z},t)$ such that $U=\mathrm{P}(f)\equiv\int_{-1}^{+1}\mathrm{E}(Z,\bar{Z},t)f(Z(1-t^2)/2)dt/(1-t^2)^{1/2}$, where f is an arbitrary analytic function, is a solution of $L(U)=0$ (see Duke Math. J. vol. 6 (1940) p. 537). The author shows that a fundamental solution $\Gamma(z,\bar{z},\zeta,\bar{\zeta})$ of the equation $S(v)=v_{z\bar{z}}+F(z,\bar{z})v=0$ is given by $\mathrm{P}(1/2\pi)\log|Z|+G(Z,\bar{Z})$. Here $Z=z-\zeta$, $\bar{Z}=\bar{z}-\zeta$, and P is the operator introduced above for the equation $L(U)=0$ with $H(Z,\bar{Z})=F(Z+\bar{\zeta},\bar{Z}+\bar{\zeta})$. $G(Z,\bar{Z})=-\int_0^Z\int_0^{\bar{Z}}D(Z,\bar{Z})dZd\bar{Z}+\int_0^Z\int_0^{\bar{Z}}H(Z,\bar{Z})(\int_0^Z\int_0^{\bar{Z}}D(Z,\bar{Z})dZd\bar{Z})dZd\bar{Z}+\cdots$ where $D(Z,\bar{Z})=(1/2)\int_{-1}^{+1}t^2[2\mathrm{E}^*+\bar{Z}\mathrm{E}^*_{\bar{Z}}+Z\mathrm{E}^*_Z]dt/(1-t^2)^{1/2}$. Using the representations of functions $v(z,\bar{z})$, $S(v)=0$, in the form of a line integral over a closed curve in terms of Γ, $\partial\Gamma/\partial n$, v and $\partial v/\partial n$ the author studies the growth of v and $\partial v/\partial n$ along circles $|z|=r$, $r\rightarrow\infty$. The existence of an analogous function $\mathrm{E}(X,Y,t)$ for every equation $H(U)=U_{XY}+H(X,Y)U=0$ (of hyperbolic type) has been established (see above reference). $(1/2\pi)\int_{-1}^{+1}\mathrm{E}(X,Y,t)dt/(1-t^2)^{1/2}$ is now shown to be the Riemann function of the equation $v_{xy}+F(x,y)v=0$, where $X=x-\xi$, $Y=y-\eta$ and $H(X,Y)=F(X+\xi,Y+\eta)$. Analogous relations hold for more general equations. (Received September 11, 1943.)

252. B. H. Bissinger: *Generalizations of continued fractions.*

The simple continued fraction for a real number x, $0<x<1$, may be written in the form $f(a_1+f(a_2+\cdots$, where the a's are the partial denominators and $f(t)=1/t$. The present paper generalizes the simple continued fractions by using functions $f(t)$, other than $1/t$. When $f(t)$ belongs to an appropriate class F of real functions, defined for $t\geqq 1$, and $a_1, a_2, \cdots$ is a sequence of positive integers, the "f-expansion" $f(a_1+f(a_2+\cdots$ converges to a number x, $0<x<1$. The "f-expansion of x," $0<x<1$, is given by an algorithm equivalent to the euclidean algorithm when $f(t)=1/t$. The main result of the paper is the identity of the f-expansion of x with the f-expansion which converges to x. The remainder of the paper is concerned with generalizations of certain results in the theory of simple continued fractions, established by Borel and F. Bernstein. One of these results is that, for almost all x, $0<x<1$, the a's form an unbounded sequence. This and theorems of a similar nature are proved for generalized continued fractions, provided that $f(t)$ belongs to a certain subclass of F. (Received August 5, 1943.)

253. Fritz Herzog and B. H. Bissinger: *Generalization of Borel's and F. Bernstein's theorems on continued fractions.*

Let $f(a_1+f(a_2+\cdots$ be the f-expansion of x, $0<x<1$, as defined by Bissinger in his paper *Generalizations of continued fractions* (abstract 49-11-252). For the case $f(t)=1/t$ (simple continued fractions) Borel and F. Bernstein have proved: (a) the set of x, $0<x<1$, for which the a's are bounded has measure zero; (b) the set of x, $0<x<1$, for which all a's are greater than unity has measure zero. In the paper referred to above the statements (a) and (b) were proved to hold when $f(t)$ belongs to a certain class of polygonal functions. The present paper is concerned with the problem of characterizing analytically a class of functions which does include $f(t)=1/t$ and such that statements (a) and (b) hold when $f(t)$ belongs to that class. This investigation at the same time reveals by which analytic properties of the function $f(t)=1/t$ the original statements of Borel and F. Bernstein may be proved. (August 5, 1943.)

254. R. C. Buck: *A note on subsequences.*

Starting from a result of H. Steinhaus (Prace Matematyczno-Fizyczne vol. 22 (1911) p. 129), it is shown that a sequence s_n, limitable by a regular matrix method T, is convergent if and only if each of its subsequences is also limitable T. This general tauberian theorem gives rise to a similar result for series. If there exists a regular matrix method T which sums every series obtained from $\sum a_n$ by bracketing blocks of terms, then $\sum a_n$ is itself convergent. (Received September 11, 1943.)

255. R. C. Buck: *Multiple sequences.*

An r-multiple sequence of points of a limit space L is a single value function $p(j)$ from $\mathfrak{J}$ to L, where $\mathfrak{J}$ is the r-fold cartesian product of the integers with themselves. Under an inclusion ordering, $\mathfrak{J}$ is a directed system; a subsequence may then be defined in terms of a monotonic function $\lambda(j)$, defined on $\mathfrak{J}$ to $\mathfrak{J}$. In terms of a product measure (C. Visser, Studia Mathematica vol. 7, p. 143) it is shown that the set of all convergent subsequences of $p(j)$ form a measurable set, with measure 0 or 1. By means of a lemma on measure preserving translations and sets of measure 1, it is shown that if $p(j)$ is divergent, so are almost all of its subsequences. This is a generalization of a previous result of H. Pollard and the author. (Received September 29, 1943.)

256. R. C. Buck and Harry Pollard: *Convergence and summability properties of subsequences.*

The authors discuss the relation of the convergence or summability of a sequence to that of its subsequences. The set of subsequences of a sequence can be mapped in a 1:1 fashion on the interval (0, 1). In terms of Lebesgue measure on this set, it is shown that a sequence is convergent if and only if almost all of its subsequences are convergent. Similarly, for bounded sequences, a sequence is $(C, 1)$ summable if and only if almost all of its subsequences are $(C, 1)$ summable. For unbounded sequences, $(C, 1)$ summability of s_n follows from that of almost all of its subsequences, but not conversely. The principal tools are the properties of homogeneous sets, and the Rademacher functions. These results are intimately connected with probability; in particular, a theorem of Birnbaum and Zuckerman (Amer. J. Math. vol. 62 (1940) pp. 787–791) follows easily. (Received September 11, 1943.)

257. Nelson Dunford and D. S. Miller: *On the ergodic theorem.*

Let S be a space with measure $|e|$ in a σ-field F, and $|S|$ finite. Define ϕ to be a 1-1 map of S into all of itself with $\phi^{-1}e\in F$ when $e\in F$. For every real function f on S let $Tf=g$ where $g(t)=f(\phi t)$. In this paper it is shown that a necessary and sufficient condition for the mean ergodic theorem to hold for such an operator T is that $n^{-1}\sum_{\nu=0}^{n-1}|\phi^{-\nu}e|\leqq M|e|$, $e\in F$, $n=1, 2, \cdots$. It is also proved that in spite of the fact that ϕ may not be a measure preserving transformation it is nevertheless still true that the mean ergodic theorem implies the point ergodic theorem. The two above results are also obtained for the n-parameter continuous case. (Received August 4, 1943.)

258. Paul Erdös and Hans Fried: *On the connection between gaps in power series and the roots of their partial sums.*

Let $f(z)=1+a_1z+\cdots+a_nz^n+\cdots$ be a power series with the radius of convergence 1. It has Ostrowski gaps if there exists a $\rho<1$ and a pair of infinite sequences m_k and n_k with $\lim_{k\to\infty}m_k/n_k>1$, such that $|a_n|<\rho^n$ if $m_k\leqq n\leqq n_k$. It has infinite Ostrowski gaps if $\lim_{k\to\infty}m_k/n_k=\infty$. The number of roots of $f_n(z)=1+a_1z+\cdots+a_nz^n$ in the circle of radius $1+r$ is denoted by A_n^{1+r}. The following theorems are proved. If $f(z)$ has Ostrowski gaps, then there exists an $r>0$ such that $\lim\inf_{n\to\infty}A_n^{1+r}/n<1$. This converse is also true. If there exists an $r>0$ such that $\lim\inf_{n\to\infty}A_n^{1+r}/n<1$, then $f(z)$ has Ostrowski gaps. This theorem is not new. Mr. Bourion (*L'Ultraconvergence dans les Séries de Taylor*, Actualités Scientifiques et Industrielles, no. 472) proved it but his proof is quite different from that given here. Further, the following theorem is proved. If $f(z)$ has infinite Ostrowski gaps, then there exists an $r>0$, such that $\lim\inf_{n\to\infty}A_n^{1+r}/n=0$. The converse is also true. One of the main tools is the following lemma which may be of interest in itself. Let $F(\rho)$, where $\rho<1$, be the family of all polynomials of the form $1+a_1z+\cdots+a_mz^m+\cdots+a_nz^n$ where $|a_k|<\rho^k$ if $m\leqq k\leqq n$, and if $r<1/\rho$, then there exists a c_r such that the number of roots of every polynomial belonging to $F(\rho)$ outside the circle with the radius r is greater than $c_r(n-m)$. (Received August 4, 1943.)

259. Herbert Federer: *Surface area.* I.

Suppose $m\leqq n$ are positive integers For each subset S of n-space let $\gamma(S)$ be the superior of the m-dimensional Lebesgue measures of the perpendicular projections of

S into all m-dimensional subspaces of n-space. $\gamma_r(S)$ is the inferior of numbers of the form $\sum_{i=1}^{\infty}\gamma(A_i)$ where $A_1, A_2, A_3, \cdots$ are open connected subsets of n-space of diameter less than r whose union contains S. $\Phi(S)=\lim_{r\to 0+}\gamma_r(S)$ Φ is an m-dimensional measure over n-space which agrees with Carathéodory linear measure in case $m=1$ and with Lebesgue measure if $m=n$. The author is interested in the relation of Φ to m-dimensional surfaces in n-space. He proves: If $m=2$, $n=3$, R is a rectangle and f is a continuous numerically valued function on R, then the Lebesgue surface area of f equals the Φ measure of the set of all points of the form $(u, v, f(u, v))$ with (u, v) in R. (Received September 23, 1943.)

260. Herbert Federer: *Surface area. II.*

Retaining the conventions stated in abstract 49-11-259 the author investigates the validity of the formula $\int N(f, T, y)d\Phi y=\int_T Jf(x)dx$ where f is a Lebesgue measurable function on m-space to n-space and T is a Lebesgue measurable subset of m-space Here $N(f, T, y)$ is the number (possibly ∞) of points x in T for which $f(x)=y$; and $Jf(x)=(\det(\bar{L}L))^{1/2}$ where L is the approximate differential of f at x and $\bar{L}$ its conjugate The formula holds for general m whenever lim sup $\text{approx}_{z\to x}|f(z)-f(x)|/|z-x| < \infty$ for every x in T. In the special case $m=2$ (here $Jf=(EG-F^2)^{1/2}$) the formula holds if corresponding to each point x in T there are three distinct unit vectors $\bar{x}^1, \bar{x}^2, \bar{x}^3$ such that $\lim\sup_{t\to 0+}|f(x+t\bar{x}^j)-f(x)/t| < \infty$ for $j=1, 2, 3$. Finally if $m=n=1$ and if the upper right-hand derivative of f is finite at each point of T, then the formula is likewise true and Jf may be replaced in it by the absolute value of the Dini derivative. (Received September 23, 1943.)

261. M. H. Heins: *On a problem of Walsh concerning the Hadamard three circles theorem.*

The present paper consists of a contribution to the solution of the following problem proposed to the author by J. L. Walsh: Let $\mathfrak{A}$ consist of the class of functions $f(z)$ which satisfy the following requirements; (a) $f(z)$ is analytic for $z<R(>0)$, (b) $|f(z)|<M(>0)$ for $|z|<R$, (c) $|f(z)|\leqq m(<M)$ for $|z|\leqq r(<R)$. Under these circumstances it is required to determine l.u.b. $M(f, \rho)$ with $f\in\mathfrak{A}$, where $r<\rho<R$ and $M(f, \rho)=\max_{|z|=\rho}|f(z)|$, and the associated extremal functions. The main concern of the paper is with the descriptive properties of the extremal functions. A method highly suggestive of the typical Tchebycheff argument is used to show that the extremal functions define $(1, k)$ directly conformal maps of $|z|<R$ onto $|w|<M$ and that they are unique when suitably normalized. Under the assumption that r is sufficiently small, the degree k is determined and the associated extremal functions are calculated for the special case where $r^2\leqq m/M$. Related questions are treated for the class of functions satisfying (a) and (b) and (c) modified by replacing "$|z|\leqq r$" with "$z\in E$" where E belongs to and is closed relative to $-1<x<+1$, and l.u.b. $E<1$. (Received August 6, 1943.)

262. M. H. Heins: *On the problem of Milloux for functions analytic throughout the interior of the unit circle.*

Let E denote a point set lying in and closed relative to $|z|<1$ and having the property that it has a non-void intersection with $|z|=r$ for $0\leqq r<1$. Let m denote a positive number less than one. Finally let $f(z)$ denote a function analytic and of modulus less than one for $|z|<1$ which satisfies the further condition that there exists an E-set E_f such that $|f(z)|\leqq m$ for $z\in E_f$. It is desired to determine l.u.b. $\{\max_{|z|=\rho}|f(z)|\}$ $(0<\rho<1)$ and the corresponding extremal functions. This problem is an extremal problem associated with the original Milloux problem. The extremal value and the corresponding extremal functions are explicitly determined in terms of the Jacobi sn-function. Two other methods are given for treating the problem. One employs the theory of Blaschke products, the other gives an algorithm in terms of a process involving the composition of (1, 2) directly conformal maps of the interior of the unit circle onto itself. (Received August 14, 1943.)

263. M. R. Hestenes: *The isoperimetric problem of Bolza in parametric form.*

The purpose of the present paper is to establish a conjecture due to McShane that for a strong relative minimum it is sufficient that for each admissible variation y there is a set of multipliers with which the arc under consideration satisfies the Euler-Lagrange equation, the transversality condition, the Weierstrass condition, non-singularity and such that the second variation is a modification of those used by McShane (Trans. Amer. Math. Soc. vol. 52 (1942) pp. 344–379) and Myers (Duke Math. J. vol. 10 (1943) pp. 73–97). An interesting feature of the method is that one obtains the sufficiency theorem for the isoperimetric problem without transformation of the problem to a non-isoperimetric problem. Moreover, one also obtains simultaneously the analogues of the theorems of Osgood. (Received August 4, 1943.)

264. Norman Levinson: *On a nonlinear differential equation of the second order.*

In the equation $\ddot{x}+f(x)\dot{x}+x=e(t)$, $e(t)$ is continuous and periodic of period L; $f(x)$ is continuous and greater than 0 except possibly at a finite number of points. If the integral of $f(x)$ over (x_0, ∞) diverges then the above differential equation has a periodic solution of period L and *all* other solutions tend to this solution as $t\rightarrow\infty$. (Received September 25, 1943.)

265. Brockway McMillan: *Random point patterns.* Preliminary report.

X is a space, X^k the k-fold Cartesian product of X by itself. W is a space whose elements are countable subsets $w=\{x_i\}\subseteq X$. N. Weiner (Amer. J. Math. vol. 60 (1938) pp. 925, 928) exhibited a probability measure P in W such that if $F(w)=f_0+\sum_k\sum' f_k(x_1, \cdots, x_k)$, where the $\sum'$ is over all k-tuples of distinct points $x_i\in w$, then (i) $\int F(w)dP=f_0+\sum_k\int f_k(x_1, \cdots, x_k)dm_k=L(F)$, where m_k is a suitable measure in X^k. Such P's are characterized by moment conditions on the number of points in Aw, $A\subseteq X$. An inverse problem has been treated by N. Wiener and A. Wintner (Abstract 46-11-480): Given $L(F)$ defined by (i), does a probability measure P exist with $L(F)=\int F(w)dP$? The present paper solves this inverse problem by applying previous results about absolutely monotone functions of sets (Abstract 47-11-471). Any completely monotone (CM) function of sets $A\subseteq X$ defines a measure in W. P exists whenever $1+\sum_k(-1)^k(k!)^{-1}m_k(A^k)$ converges absolutely to a CM function of A. (Received August 3, 1943.)

266. A. P. Morse: *A theory of covering and differentiation.*

A covering theory is developed which embraces a variety of theorems of the Vitali type. By substituting a new and frequently useful concept of regularity for the classical notion the author arrives at a general theory of differentiation. This includes and supplements the Lebesgue theory and, at the same time, encompasses results in the differential theory of nets. Use is made of the closed subset theorems alluded to in the abstract which follows. (Received September 11, 1943.)

267. A. P. Morse and J. F. Randolph: *The ϕ rectifiable subsets of the plane.*

Suppose A is a plane set, ϕ is a measure with $\phi(A) < \infty$, and suppose that, at ϕ almost all points of A, the upper linear ϕ density of A is less than 1.01 times the lower linear ϕ density of A. Under these circumstances it is shown that all except a ϕ small part of A lies on a rectifiable arc. A systematic study of the relations between density and ϕ rectifiability is made. In fact by setting ϕ equal to Carathéodory linear measure, one of our theorems becomes in essence a compendium of the results obtained by Besicovitch in his paper *On the fundamental geometrical properties of linearly measurable plane sets of points.* II, Math. Ann. vol. 115 (1938) pp. 296–329. Some closed subset theorems are obtained. These involve measure and are apparently heretofore unknown. The paper under discussion together with the paper whose abstract precedes will appear in an early issue of the Transactions. (Received September 11, 1943.)

268. F. J. Murray: *On the existence of quasi-complements in Banach spaces.*

Let B be a reflexive Banach space. Let M and N be closed additive subsets of B. N is a quasi-complement of M if $M \cdot N = (0)$ and $M \cdot + \cdot N$ is dense in B. It is proved that for every closed additive M, such a quasi-complement exists. (Received September 23, 1943.)

269. H. E. Newell: *On the asymptotic forms of the solutions of a linear matric differential equation in the complex domain.*

In an earlier paper (*The asymptotic forms of the solutions of an ordinary linear matric differential equation in the complex domain*, Duke Math. J. vol. 9 (1942)) the author discussed the existence of solutions to the matric differential equation $(d/dx)Y(x, \lambda) = \{\lambda(\delta_{ij}r_j(x)) + (q_{ij}(x, \lambda))\} Y(x, \lambda)$, in which x and λ are complex variables. Under certain conditions of suitability imposed upon the coefficient functions and the domains of variation of x and λ, it was possible to show that regions of existence can be constructed in which the differential equation possesses solutions of the form $P(x, \lambda)E(x, \lambda)$, where $E(x, \lambda) = (\delta_{ij} \exp\{\lambda \int^x r_j(x)dx\})$, and $P(x, \lambda)$, analytic in x, reduces uniformly in x to the identity matrix when λ becomes infinite. The author applied the elegant and basic concepts of associated and fundamental regions originated by R. E. Langer (*The boundary problem of an ordinary linear differential system in the complex domain*, Trans. Amer. Math. Soc. vol. 46 (1939) pp. 151–162, and correction, p. 467) to some of the cases in which the functions $r_j(x)$ have poles and the differences $r_i(x) - r_j(x)$, $i \neq j$, have poles or zeros on the boundary of the x region in question. In adapting the concept of associated regions to the cases considered, the condition was imposed upon the differences $r_i(x) - r_j(x)$, $i \neq j$, that those having simple poles be of the form $a^{ij}(x - x_0)^{-1}$, where $a^{ij}(+0)$ is a constant. The present paper removes that restriction. (Received August 4, 1943.)

270. C. E. Rickart: *An abstract Radon-Nikodym theorem.*

Let $\mathfrak{M}$ be a σ-field of subsets of an abstract set M and let $m(e)$ be a non-negative measure function defined on $\mathfrak{M}$. This paper contains a proof of the following generalization of the Radon-Nikodym theorem: *If M is the union of a countable number of sets of finite measure and $X(e)$ is any completely additive function defined on $\mathfrak{M}$ to an arbitrary Banach space, then a necessary and sufficient condition that $X(e)$ be representable as an integral, with respect to $m(e)$, is that $X(e)$ be absolutely continuous relative to* $m(e)$ (that is, $m(e)=0$ implies $X(e)=0$). The integral used is the Pettis integral (B. J. Pettis, Trans. Amer. Math. Soc. vol. 44) extended (R. S. Phillips, Trans. Amer. Math. Soc. vol. 47) so that it is defined for certain "contractive" functions (that is, multivalued set functions such that $e_1 \subseteq e_2$ implies $f(e_1) \subseteq f(e_2)$). (Received August 6, 1943.)

271. H. M. Schwartz: *Riemann integration with respect to additive interval functions.* Preliminary report.

It is shown first that in order to have finite upper Darboux integrals for continuous integrands, the integrator $g(i)$ (defined and additive over the closed intervals i lying in a fixed finite many-dimensional interval) must be of bounded variation. It is then shown that when $g(i)$ is continuous, all the essential properties of ordinary Riemann integration remain valid. Since every $g(i)$ can be decomposed into a continuous function and a function of jumps (in an extended sense), the essentially new properties are obtained by studying the integration with respect to the latter function. It is also shown that the class of Riemann integrable functions (taken with the Moore-Smith limit) can be generated from the class of simple step functions by employing a natural notion of limit of functions, which is closely related to the one employed by Arzelà and Hobson (cf. E. W. Hobson, *Theory of functions of a real variable*, vol. 2, 3d edition, pp. 312–314). (Received August 7, 1943.)

272. Max Shiffman: *Isoperimetric inequalities and continuity of area for classes of surfaces.*

The isoperimetric inequality $A \leqq L^2/4\pi$ has been extended to minimal surfaces by Carleman, and to sufficiently regular surfaces of nonpositive curvature by Beckenbach and Radó. Advanced methods were used, involving analytic functions or conformal mapping and subharmonic functions. Here it is shown by purely elementary methods that the isoperimetric inequality applies to polyhedra all of whose interior vertices have the sum of face angles greater than or equal to 2π. The result can then be extended to surfaces of nonpositive curvature through approximation by such polyhedra. The isoperimetric inequality can be used as a decisive analytical tool in the estimation of areas of boundary strips. For example, one can establish general theorems involving the continuity of area of surfaces of nonpositive curvature. See, for example, Shiffman, *Unstable minimal surfaces with any rectifiable boundary*, Proc. Nat. Acad. Sci U.S.A. vol. 28 (1942) pp. 103–108, where minimal surfaces are considered. (Received October 1, 1943.)

273. Dorothy M. Stone: *The representation of abstract measure functions.*

In this paper a standard form is obtained for general abstract measure algebras in which the measures have values in abelian semi-groups. Incidentally it is shown that the theory of such abstract measures can be reduced to the theory of ordinary numerical measures. Let E be a non-atomic Boolean σ-algebra satisfying the countable chain condition. Let λ be an abstract measure-function on E; λ is assumed to be countably additive and non-atomic. Writing $a \sim b$ if $\lambda(a)=\lambda(b)$, an equivalence relation between certain pairs of elements of E is defined, which has four simple properties. Conversely, any equivalence relation with these properties determines such a measure λ, uniquely to within isomorphism. Working only with the equivalence relation it is shown that there exists a finite isomorphism ϕ of E onto a certain subalgebra of a direct sum of measure algebras E_α, each having a numerical-valued measure μ_α, in such a way that $a \sim b$ if and only if $\mu_\alpha[\phi_\alpha(a)]=\mu_\alpha[\phi_\alpha(b)]$ (for all α), where $\phi_\alpha(x)$ denotes the projection of $\phi(x)$ onto E_α. A σ-representation can be deduced from this by reducing modulo a certain σ-ideal in the direct sum. (Received October 2, 1943.)

274. J. V. Wehausen: *A remark concerning a set of completely continuous transformations.*

Let E be a Banach space, K a closed convex subset of E, $\mathfrak{T}$ the set of all completely continuous transformations defined in E with $T(K)\subset K$, $\mathfrak{T}'$ the subset of $\mathfrak{T}$ consisting of transformations with unique fixed points in K, and $\mathfrak{T}''$ the subset consisting of those with more than one fixed point in K. From the fixed point theorem of Schauder $\mathfrak{T}=\mathfrak{T}'+\mathfrak{T}''$. Let $\mathfrak{T}$ be metrized by $\|T_1-T_2\|=\sup_K\|T_1x-T_2x\|$. Then $\mathfrak{T}$ is complete and the following theorem holds. If $\mathfrak{T}'$ is dense in $\mathfrak{T}$, then $\mathfrak{T}'$ is of the second category and $\mathfrak{T}''$ of the first category of $\mathfrak{T}$. This generalizes a result of Orlicz (Bulletin international de l'Académie des sciences de Cracovie A, Nr. 8/9 (1932) pp. 221–228) concerning the differential equation $y'=f(x, y)$. (Received September 24, 1943.)

275. J. E. Wilkins: *Multiple integral problems in parametric form in the calculus of variations.*

This paper discusses the problem of minimizing a multiple integral in a class of varieties defined parametrically. In order that the multiple integral be independent of the parametric representation, it is necessary and sufficient that the integrand function satisfy a certain homogeneity condition. The implications of this homogeneity condition for the derivatives of the integrand of arbitrary order are investigated. The usual necessary conditions known in the non-parametric case for multiple integrals are extended to the parametric case. A discussion of the form of the Weierstrass condition is given. Finally, as a first step in obtaining a field theory, necessary and sufficient conditions that an integral be invariant are given. (Received August 5, 1943.)

276. Antoni Zygmund: *On certain integrals.*

(a) Given a function $\phi(z)$ regular for $|z|<1$ and of the class H^r, $r>1$, let $g^*(\theta)=\{\int_0^1(1-\rho)\chi^2(\rho, t)d\rho\}^{1/2}$, where $\chi^2(\rho, \theta)=\pi^{-1}\int_0^{2\pi}|\phi'(\rho e^{i(\theta+t)})|^2P(\rho, t)dt$, $P(\rho, t)$ denoting the Poisson kernel. Then the ratio $M_r(g^*(\theta))/M_r(\phi(e^{i\theta}))$ is contained between two positive numbers depending on r only. (b) Let $f(\theta)$ be of period 2π and of the class L^r, $r>1$. Let $F(\theta)$ be the indefinite integral of f, and $\mu(\theta)=\{\int_0^\pi t^{-3}[F(\theta+t)+F(\theta-t)-2F(\theta)]dt\}^{1/2}$. The ratio $M_r(\mu)/M_r(f)$ is again comprised between two positive numbers depending on r only. (c) Every lacunary series $\sum c_k$ which is absolutely summable A must be absolutely convergent. (Received September 10, 1943).