Applied Mathematics

155. H. R. Branson: *On the difference equation of a general quantum mechanical problem.*

The problems of quantum mechanics may be derived from the classical functions either in the q-language or the p-language. In the q-language $F(q, p) = F(q, hi^{-1}\partial/\partial q)$; in the p-language $F(q, p) = F(-hi^{-1}\partial/\partial p, p)$. With certain types of potential function, the p-language leads to difference equations. Because of the great popularity of quantum mechanics the expression of a general problem in difference equations may aid in "the creation of a usable mathematics of the discrete". The usual equation in one dimension is $H(q, p)\psi = E\psi$ where $H(q, p) = (1/2m)p^2 + V(q)$. The equation becomes in the p-language $(1/2m)p^2\psi + V(-hi^{-1}\partial/\partial p)\psi = E\psi$. Limiting potentials $V(q)$ to those which may be expanded in Fourier series, it follows that: $V(-hi^{-1}\partial/\partial p)\psi = \sum_{k=0}^{\infty}[a_k\psi(p+kh) + b_k\psi(p-kh)]$. With some obvious simplification, the general difference equation for this type of one dimensional quantum mechanical problem may be written in the form $\sum_{k=-\infty}^{\infty} C_k\psi(p+kh) = 0$, wherein all C_i are constants except $C_0 = (p^2/2m + a_0 + b_0 - E)$. Some examples and extensions are discussed. (Received March 25, 1943.)

156. A. H. Fox: *Integral representation of the flow of a compressible fluid around a cylinder.* II.

The method outlined in Part I (Bull. Amer. Math. Soc. abstract 49-1-60) is applied for values of $q=0$ and $q=1/2$ to the flow of a perfect gas around a circular obstacle. The nature of the variations in the boundary of the flow are discussed, and the effect of compressibility on the pressure is considered. (Received February 22, 1943.)

157. J. F. Harding and Isaac Opatowski: *An approximate formula for the Legendre elliptic integral of the second kind.*

By means of ultraspherical polynomials $E = \int_0^{\pi/2}(1-k^2\sin^2\phi)^{1/2}d\phi$ is expanded in $E = (1+k')\sum a_n k_1^n$ where $k' = (1-k^2)^{1/2}$, $k_1 = (1-k')/(1+k')$. By a suitable change of the series $\sum a_n k_1^n$ the formula $E = (\pi/4)(1+k')[0.75 + 0.1875k_1^2 + (4-k_1^2)^{-1}] + \epsilon$ is obtained, where $0 \leqq \epsilon \leqq 1 - (61\pi/192) = 0.00189 \cdots$, which is a simpler and a closer approximation than the formulas of G. A. Grünberg (Applied Mathematics and Mechanics vol. 1 (1933) pp. 61–69) or those given by J. Thomae (*Formeln und Sätze aus dem Gebiete der elliptischen Funktionen*, Leipzig, 1905, p. 29) or by W. Laska (Sammlung von Formeln, pp. 501–502). (Received March 26, 1943.)

158. Wilfred Kaplan and Max Dresden: *Topology of the molecular N-body problem.*

The N molecules of a gas are considered as mass-particles exerting forces on each other derived from a potential of the form $ar^{-n} - br^{-m}$, $n > m > 0$, $a > 0$, $b > 0$. The force is thus highly repulsive for small r and weakly attractive for large r. The energy integral: Total Kinetic Energy plus Total Potential Energy = Const. $\equiv C$ is then interpreted as restricting the trajectories of the system, in the corresponding $6N$-dimensional phase space, to a $(6N-1)$-dimensional hypersurface $M(C)$. It is shown that for $C \geqq 0$, $M(C)$ has the topological structure of a $(6N-1)$-sphere, minus certain $(6N-4)$-spheres corresponding to collisions. For $-\delta < C < 0$, where δ is a certain explicitly known function of a, b, n, m (not of N), $M(C)$ is homeomorphic to the topological product of a $(3N-1)$-sphere and a $3N$-sphere, minus certain $(6N-4)$-dimensional loci corresponding to collisions. For $C \leqq -\delta$ the structure of $M(C)$ has yet to be determined. It is planned to correlate the critical values $C=0$ and $C=-\delta$ with the physically known transition temperatures such as the critical temperature. (Received March 23, 1943.)

159. Isaac Opatowski: *An explicit formula for the refractive index in electron optics.*

The refractive index μ is expressed in electron optics (W. Glaser, Zeitschrift für Physik vol. 81 (1933) pp. 647–686) in terms of the electrostatic potential V, the magnetic vector potential $\boldsymbol{A}$ and the unit vector $\mathbf{s}$, which is defined as tangent to the electron trajectory. Since $\mathbf{s}$ is not known a priori and is a function of V and $\boldsymbol{A}$, the elimination of $\mathbf{s}$ from the expression of μ is of advantage. This is done in the paper for a very ample class of fields in which a momentum integral of the equations of motion exists (Bull. Amer. Math. Soc. vol. 46 (1940) p. 887 and Journal of Mathematics and Physics vol. 20 (1941) pp. 418–424). (Received March 26, 1943.)

Geometry

160. Jesse Douglas: *Point transformations and isothermal families of curves.* II.

This paper is a continuation of one with the same title (see Bull. Amer. Math. Soc. abstract 49-1-71). Its new feature is the principal use of synthetic rather than analytic methods. The problem is referred to the investigation of certain properties of a hexagonal web. (Received February 27, 1943.)

161. Jacques Dutka: *Transversality in higher space.*

In this paper, a geometric criterion for transversality developed by Kasner in his paper *Transversality in space of three dimensions* (Trans. Amer. Math. Soc. vol. 30 (1928) pp. 447–452) is generalized for n-dimensional Euclidean space. It is shown here that a necessary and sufficient condition for a given correspondence between a lineal element and a hypersurface element to be a transversality is that a certain induced correlation be a polarity. A principle of transference connecting simple and $(n-1)$-fold integrals in the calculus of variations when they produce equivalent transversalities is established. The result obtained is applied to the theory of infinitesimal contact transformations from which are derived analytic tests equivalent to the above-mentioned geometric criterion. Actual examples of transversalities in addition to the well known condition of orthogonality are also given. (Received March 25, 1943.)

162. Jacques Hadamard: *On fractional iteration and connected questions.*

The author presents some results communicated to him by two younger geometers on fractional iteration and permutable transformations in one variable. This subject is connected with group theory or, more precisely, with Cartan's conception of geodesics in a group-space. (Received March 27, 1943.)