128. D. H. Lehmer: *On Ramanujan's numerical function $\tau(n)$.*

Ramanujan's numerical function $\tau(n)$ may be defined by $\sum \tau(n)x^{n-1} = \{\pi(1-x)^k\}^{24}$. Among several unsolved questions about $\tau(n)$ is the so-called Ramanujan hypothesis to the effect that if p is a prime $|\tau(p)\cdot p^{-11/2}| < 2$, which Ramanujan verified for primes $p < 30$. The present writer, in attempting to disprove this important hypothesis, has examined all primes $p < 300$, as well as $p = 571$, and finds that in all these 47 cases the hypothesis holds true. It nearly fails for $p = 103$ when $\tau(103)\cdot 103^{-11/2} = -1.918\cdots$. In connection with this hypothesis Rankin proved in 1939 that as $x \to \infty$, $x^{-12}\sum_{n \leqq x}\tau(n)^2$ tends to a limit represented by a certain double integral extended over the fundamental region of the full modular group. In this paper a practical method is devised for evaluating this integral whose value is found to be $.0320047918814\cdots$. Various congruence and divisibility properties of $\tau(n)$ are also discussed. For example $\tau(n)$ is composite for $1 < n < 7921$. (Received March 26, 1943.)

129. A. E. Ross: *Positive quaternary quadratic forms representing all integers with at most k exceptions.*

In this paper it is shown that there are a finite number of classes of positive quaternary quadratic forms which represent all integers with at most k exceptions. The determinants of such forms have an upper bound B_k depending on k. This is a generalization of the results of Ramanujan (Proc. Cambridge Philos. Soc. vol. 19 (1917) pp. 11–21), Ross (Proc. Nat. Acad. Sci. U.S.A. vol. 18 (1932) p. 607) and Halmos (Bull. Amer. Math. Soc. vol. 44 (1938) pp. 141–144). Ross gives $B_0 = 112$ for the classic case and Halmos' results imply that $B_1 \geqq 240$ in the classic case. (Received March 26, 1943.)

130. L. I. Wade (National Research Fellow): *Transcendence properties of the Carlitz ψ-function.*

The paper is concerned with quantities transcendental over the field $GF(p^n, x)$. For the Carlitz ψ-function (L. Carlitz, Duke Math. J. vol. 1 (1935) pp. 137–168) and its inverse $\lambda(t)$ the following theorem is proved. If β is algebraic (over $GF(p^n, x)$) and irrational and if $\alpha \neq 0$ is algebraic, then $\psi(\beta\lambda(\alpha))$ is transcendental over $GF(p^n, x)$. In a sense this is an analogue of Hilbert's seventh problem for the transcendence of $\alpha^\beta = e^{\beta \log \alpha}$ over the rational number field. (Received March 26, 1943.)

Analysis

131. L. W. Cohen: *On linear equations in Hilbert space.*

If the rows of the infinite matrix $A = \|a_{ij}\|$ are points in Hilbert space and $a^{i_1\cdots i_m}_{j_1\cdots j_m}$ are the m-rowed determinants with elements in A, it is shown that det $A_{i_1\cdots i_m}B'_{i_1\cdots i_m} = \sum_{j_1\cdots j_m} a^{i_1\cdots i_m}_{j_1\cdots j_m} b^{i_1\cdots i_m}_{j_1\cdots j_m}$ where $A_{i_1\cdots i_m}$, $B_{i_1\cdots i_m}$ are m-rowed minors of A, B respectively. The series is summed over all combinations of integers $j_1, \cdots, j_m$ and converges absolutely. This identity is used to establish sufficient conditions in order that the linear system represented by $Ax = y$ have a solution in Hilbert space for all y in that space. (Received March 24, 1943.)

132. R. J. Duffin: *Some representations for Fourier transforms.*

Let $\phi(x)$ be an arbitrary function and let $f(x)$ and $g(x)$ be defined by the series: $f(cx) = \sum_1^\infty (-1)^n \phi((2n-1)/x)/x$, $g(cx) = \sum_1^\infty (-1)^n \phi(x/(2n-1))/(2n-1)$. Then if

$c=\pi^{1/2}/2^{1/2}$, $g(x)$ is the Fourier sine transform of $f(x)$. The purpose of this paper is to give several conditions for the validity of these formulas. (Received March 26, 1943.)

133. F. A. Ficken: *Note on the existence of scalar products in normed linear spaces.*

In order that a normed linear space Σ with complex scalars may permit the definition of a scalar product, it is necessary and sufficient that, whenever a and b are real scalars and A and B are vectors, $|A|=|B|$ shall imply (α): $|aA+bB|=|bA+aB|$. The necessity of this condition is easily verified. To establish its sufficiency (α) is used to prove, for any vectors A and B, that (β): $|A+B|^2+|A-B|^2=2|A|^2+2|B|^2$. The sufficiency (and necessity) of condition (β) for the existence of a scalar product was established by J. von Neumann and P. Jordan (Ann. of Math. vol. 36 (1935) pp. 719–723). (Received March 29, 1943.)

134. F. A. Ficken: *On two criteria for convexity in a symmetric Banach space.*

The results refer to a Banach space Σ with real scalars and a scalar product. It has been shown by Moskovitz and Dines (Duke Math. J. vol. 5 (1939) pp. 520–534; Bull. Amer. Math. Soc. vol. 46 (1940) pp. 482–489) that: If a set E is closed and has interior points, then E is convex if and only if E has a supporting plane at each point of a set which is everywhere dense in its frontier E'. In their work a supporting plane is required to contain a point of E, and hence can exist only at points of E' which belong to E. Dropping this requirement, their results are used to obtain slightly stronger conclusions without assuming E to be closed. Another criterion, established by Jessen for Euclidean n-space (cf. Math. Rev. vol. 2 (1941), p. 261) requires the presence, for each point of the complement of E, of a unique nearest point on the closure of E. By generalizing the familiar geometric transformation of inversion, this criterion is established for the convexity of a compact set, or of any set if Σ has the property that each bounded subset is compact. (Received March 29, 1943.)

135. P. R. Halmos: *Approximation theories for measure preserving transformations.*

Three topologies (called neighborhood, uniform, and metric) are introduced into the set G of all (1, 1) measure preserving transformations of a finite measure space onto itself. The first two of these are the analogues of the strong and the uniform topologies, respectively, of the set of bounded operators on Hilbert space and the third is defined by the distance function $d(S, T)=m\{x|Sx\neq Tx\}$. It is proved that the uniform and the metric topologies are identical; in both the neighborhood and the metric topologies G becomes a complete topological group. The central results in both cases assert that arbitrary measure preserving transformations may be approximated by transformations with comparatively simple properties (for example, by transformations of finite period). It follows from these approximation theorems that the adage according to which "in general a measure preserving transformation is ergodic" is true in the neighborhood topology (that is, the set of ergodic transformations is a residual G_δ) and false in the metric topology (that is, the set of ergodic transformations is nowhere dense). (Received March 2, 1943.)

136. R. W. Hamming: *The asymptotic location of the roots of exponential sums having polynomial exponents.*

This paper considers the asymptotic location of the zeros of $f(z)=\sum_{i=1}^{n}P_i(z)$ exp $Q_i(z)$, where $P_i(z)$ and $Q_i(z)$ are polynomials in z. An inequality concerning certain regions covering the zeros is also given. (Received March 1, 1943.)

137. R. G. Helsel and Tibor Radó: *The transformation of double integrals.*

Radó and Reichelderfer have established, for certain wide classes of continuous transformations in the plane, extremely general topological transformation formulas for double integrals. The purpose of the present paper is to show that their results imply all of the results on the transformation of double integrals in the literature of which the authors are aware. Particular attention is given to the work that W. H. Young has done on the transformation of double integrals. The paper also contains new results on approximation by integral means. (Received February 19, 1943.)

138. Einar Hille and Gabor Szegö: *Remarks on ergodic theorems.*

Ergodic theorems are considered from the point of view of classical theory of summability. In particular, Abelian and Tauberian theorems give relations between the existence of (A)-lim T^n and (C, k)-lim T^n in a given topology. A detailed study is made of the transformation $T=T_\alpha=I-U_\alpha$ where $U_\alpha f$ is the fractional integral of f of order α. Here the resolvent has an isolated essential singular point at $\lambda=1$, but (A)-lim $T_\alpha^n=0$ for $0<\alpha<2$ in the strong (but not in the uniform) topology of $L_1(0, 1)$ as well as almost everywhere. For $0<\alpha<1$ even lim $T_\alpha^n=0$ and for $\alpha=1$, (C, k)-lim $T_1^n=0$ for $k>1/2$. (Received February 27, 1943.)

139. H. K. Hughes: *On a theorem of Newsom.*

A theorem proved by Newsom (Amer. J. Math. vol. 60 (1938) pp. 561–572) furnishes an expression from which it is often possible to obtain the asymptotic expansions for large values of $|z|$ of the integral function $f(z)=\sum_{n=0}^{\infty}g(n)z^n$ (radius of convergence $=\infty$), the series being regarded as given. It is assumed that the function $g(w)$, where $w=x+iy$, is single-valued and analytic in the finite w plane, and is less in absolute value than a constant multiple of $e^{k\pi}|y|$, k being a positive integer. The present paper considers the situation when $g(w)$ has a singularity in the finite plane, while the inequality remains satisfied for sufficiently large values of $|w|$. The result is that a certain loop integral must be subtracted from the expression obtained by Newsom. A method of obtaining the asymptotic expansion of the loop integral is indicated. (Received March 25, 1943.)

140. H. D. Huskey: *Contributions to the problem of Geöcze.*

Consider a continuous surface S: $z=f(x, y)$ where $f(x, y)$ is continuous on the unit square I: $0\leqq x\leqq 1$, $0\leqq y\leqq 1$. The area of such a surface is given by $A(S)=$greatest lower bound lim inf $A(P_n)$ where P_n: $z=p_n(x, y)$, $p_n(x, y)\rightarrow f(x, y)$ uniformly on I, and the greatest lower bound is taken with respect to all such sequences of polyhedra (not necessarily inscribed). Let $A^*(S)$ denote the value obtained in the preceding definition if the polyhedra are required to be inscribed. The problem of Geöcze asks: When is $A(S)=A^*(S)$? The author has used integral means and results on convergence in area (Radó and Reichelderfer, *Convergence in length and convergence in area*, Duke Math. J. vol. 9 (1942) pp. 527–565) to obtain two different proofs that $A(S)=A^*(S)$ if $f(x, y)$ is absolutely continuous in the sense of Tonelli. (Received February 19, 1943.)

141. D. H. Hyers: *A note on the Michal differential in Banach spaces.*

In this note an example is given which shows that the Michal M-differential (Proc. Nat. Acad. Sci. U.S.A. vol. 24 (1938) pp. 340–342 and Bull. Amer. Math. Soc. vol. 45 (1939) pp. 529–563) is more general than the Fréchet differential in Banach spaces. (Received March 27, 1943.)

142. R. E. Lane: *The values of continued fractions and their computation.*

This paper gives a method of computing the differences between the values of successive approximants to a continued fraction and shows how it is often possible to find the maximum possible absolute value of the difference between the value of the nth approximant and the value of the continued fraction. The principal result, the two-circle theorem, gives a necessary and sufficient condition on z for a certain linear fractional transformation, $v=t_z(u)$, to carry $|u-c|\leqq|c|$ into some portion of $|v-d|\leqq|d|$. The Worpitzky circle, the Paydon-Scott-Wall parabolas, and various other element-regions for continued fractions arise as special cases. Among the other applications of the two-circle theorem is a simple proof that if the elements of a continued fraction lie in a certain parabola, the approximants lie in the inner loop of a certain limacon. A generalization of this result is given. (Received March 27, 1943.)

143. L. H. Loomis: *Spherical volume as a coordinate-free basis for Lebesgue measure theory.*

The formula for the volume of a sphere is deduced in metric spaces having certain homogeneity properties in common with Euclidean spaces. The fundamental properties of spherical volume are established, and the whole of classical measure theory which does not concern a specific dimension number is then immediately available. The development of Lebesgue measure theory is thus freed from the background of a coordinate representation as the nth power (in the sense of Cartesian product) of the real number system. (Received March 15, 1943.)

144. Glynn Owens: *A boundary value problem for an ordinary nonlinear differential equation of the second order.*

The uniqueness and existence in the infinite region $I[0\leqq x<\infty]$ of the solution of the boundary value problem (A): $d^2y/dx^2-\lambda f(x, y)y=q(x, y)$, $y(0)=0$, $\lim_{x\to\infty} y(x)=0$, is to be demonstrated in this paper. A fundamental assumption will be that $f(x,y)$ is positively and uniformly bounded away from zero, and it is necessary to assume that the parameter λ is positive and sufficiently great. In the first section of the paper the uniqueness and existence of the solution of a certain linear equation is considered. In addition certain bounds for the solution are derived which are valid respectively in the region I and in a suitable neighborhood of infinity. The principal result, namely the treatment of problem (A), is dealt with in section two. There, by linearizing the problem (A), subjecting the coefficients of (A) to certain conditions, such as Lipschitz requirements and the existence of certain integrals taken over the region I, it is proved, with the aid of the results of section one, that the method of successive approximations used gives a sequence of functions which converge uniformly in I to the desired solution of (A). (Received March 24, 1943.)

145. George Pólya: *Inequalities for the area of the ellipsoid.*

Let $E=E(a, b, c)$ denote the area of the ellipsoid with semiaxes a, b, c. Let λ, μ, ν satisfy the conditions $\lambda \geqq \mu \geqq \nu \geqq 0$, $\lambda+\mu+\nu=2$. Define $P_{\lambda\mu\nu}=P_{\lambda\mu\nu}(a, b, c)=4\pi\sum a^\lambda b^\mu c^\nu/6$; the sum has 6 terms, obtained from $a^\lambda b^\mu c^\nu$ by performing all possible permutations of a, b, c. Consider linear combinations $P=kP_{\lambda\mu\nu}+k'P_{\lambda'\mu'\nu'}+\cdots$, assume that $k>0$, $k'>0, \cdots$ and $k+k'+\cdots=1$; the number of the terms is finite, $\geqq 1$. Now P is just as E, a continuous function of a, b, c for $a\geqq 0$, $b\geqq 0$, $c\geqq 0$, positive for $a>0$, $b>0$, $c>0$, homogeneous of degree 2, symmetric in a, b, c, and it has the same value as E for $a=b=c$ (for spheres). The problem is to find all linear combinations P "comparable" with E (see G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities*, p. 5). A complete solution is given an essential part of which can be stated thus: The inequalities $E>P_{1,1,0}$, $E<(P_{2,0,0}+P_{1,1,0})/2$ hold for $a>0$, $b>0$, $c>0$ and are the best of their kind. The linear combination $(64P_{1,1,0}-2P_{2,0,0}-27P_{2/3,2/3,2/3})/35$ is not a P, because it has negative coefficients; but it gives a remarkably good numerical approximation to E if the ellipsoid is not very different from a sphere. (Received March 26, 1943.)

146. Robert Schatten: *On certain properties of crossnorms.*

For two Banach spaces E_1, E_2, a Banach space $E_1\otimes E_2$ is constructed with a crossnorm N. With $E_1\otimes E_2$, there is considered the associate Banach space $\overline{E}_1\otimes\overline{E}_2$ with a norm $\overline{N}$, associate with N. Similarly $\overline{\overline{N}}$ is defined in $\overline{\overline{E}}_1\otimes\overline{\overline{E}}_2$. N is minimal if $\overline{N}^*=\overline{N}$ implies $N^*\geqq N$. N is reflexive if $\overline{\overline{N}}=N$. N has an "associate property" if for a certain N^*, $\overline{N}^*=N$. It is shown that minimal and reflexive norms and those having an associate property are identical. The existence of a least crossnorm is proved, and that the associate with every crossnorm is also a crossnorm. It is also shown that a uniformly convex crossnorm sets up the relation $\overline{E_1\otimes E_2}=\overline{E}_1\otimes\overline{E}_2$, if and only if it is minimal. Let k denote a natural number. The values of a crossnorm for all expressions of rank at most k do not necessarily determine the crossnorm. Finally "semi-self-associate" crossnorms are constructed. In particular, when E_1, E_2 denote two Hilbert spaces, for every natural k, minimal crossnorms S_k are constructed, such that $S_k\not\equiv S$, and $S_k=\overline{S}_k=S$ for all expressions of rank at most k, where S denotes the self-associate crossnorm for Hilbert spaces constructed by F. J. Murray and J. von Neumann (Ann. of Math. (2) vol. 37 (1936) pp. 118–125). (Received February 16, 1943.)

147. W. S. Snyder: *A note on the interiority of real functions.* Preliminary report.

Let F be a continuous real-valued function defined on a space X. Let A be the set of points of X where F has a local maximum and let B denote the set of points of X where F has a local minimum. If X is locally connected then the function (transformation) F is interior at $p\in X$ if and only if $p\in X-(A+B)$. If X has a countable basis then $F(A+B)$ is countable. These results together constitute an extension of a theorem of Whyburn (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 942–945). (Received March 25, 1943.)

148. Otto Szász: *On some trigonometric summability methods and Gibbs phenomenon.*

In this paper certain summability methods of trigonometric type are discussed in relation to Cesàro summability. Applied to Fourier series they lead to new formulae for the determination of the jump at a point, and to a generalized Gibbs phenomenon. In particular, the Fourier series of an integrable function $f(t)$ presents a Gibbs phenomenon at $\theta=0$, whenever for some $j\neq 0$, $(1/\theta)\int_0^\theta |f(t)-j/2|dt\to 0$ as $\theta\to 0$. (Received March 1, 1943.)

149. A. E. Taylor: *Banach spaces of functions representable by Cauchy's integral formula.*

Let $I[\phi; z]$ denote Cauchy's integral $(2\pi i)^{-1}\int\phi(t)(t-z)^{-1}dt$, the integration extended over $|t|=1$ in the counterclockwise sense. Let E^p be the class of functions $f(z)$ representable when $|z|<1$ by $f(z)=I[\phi; z]$, where $\phi(t)$ is an arbitrary function of class L^p on $|t|=1$. Let G^p denote the class of functions representable in the same way when $|z|>1$. Let H^p denote the class of functions $f(z)$ analytic when $|z|<1$, and such that the mean value of $|f(re^{i\theta})|^p$ on $|z|=r$ is bounded when $r<1$. It is assumed that $p>1$. The author proves that E^p and H^p are identical, and that $f(z)$ is in G^p if and only if it is analytic when $|z|>1$, zero at infinity, and such that the mean value of $|f(re^{i\theta})|^p$ on $|z|=r$ is bounded when $r>1$. Any function $F(t)$ of class L^p on $|t|=1$ is representable uniquely in the form $f(t)+g(t)$, almost everywhere on $|t|=1$, where $f\in E^p$ and $g\in G^p$. $I[F; z]$ is equal to $f(z)$ if $|z|<1$, and to $-g(z)$ if $|z|>1$. E^p and G^p are, in a natural sense, complex Banach spaces equivalent to complementary closed linear manifolds M and N, respectively, in L^p. The form of linear functionals on E^p is determined, and it is proved that $(E^p)^*$ is isomorphic with E^q, where $p^{-1}+q^{-1}=1$. A sequence $\{f_n(z)\}$ in E^p converges weakly to $f(z)$ if and only if $f_n(z)\to f(z)$, $|z|<1$, and the mean value of $|f_n(re^{i\theta})|^p$ on $|z|=r$ is bounded uniformly as to n and r, $r<1$. (Received February 23, 1943.)

150. W. J. Thron: *Twin convergence regions for continued fractions.*

Two regions B_0 and B_1 in the complex number plane are called twin convergence regions for the continued fraction $1+K(b_n^2/1)$ if the conditions $b_{2n}\in B_0$ and $b_{2n-1}\in B_1$ (for all $n\geqq 1$) insure its convergence. It is shown that the regions $z\in B_0$, if $|z|\leqq M$; $z\in B_1$, if $|z-i|\geqq M+\epsilon$ and $|z+i|\geqq M+\epsilon$, are twin convergence regions for every arbitrary small $\epsilon>0$ and every positive $M<1$. Another family of twin convergence regions is formed by sets B_0 and B_1, if both sets are closed and bounded and if $z=x+iy\in B_0$, if $ax-b<y<ax+b$; and $z\in B_1$, if $ax-1+b<y<ax+1-b$. The number a here can be an arbitrary real number and b any number $0<b<1$. Concerning the bestness of these results the following theorem has been proved. Two regions B_0 and B_1 can be twin convergence regions only if for every $z\in B_0$ the four points $z+i$, $z-i$, $-z+i$, $-z-i$ are not interior points of B_1. These results contain and improve certain earlier results by Leighton and Wall (Amer. J. Math. vol. 38 (1936)), Paydon and Wall (Duke Math. J. vol. 9 (1942)) and Leighton and Thron (Duke Math. J. vol. 9 (1942)). (Received March 1, 1943.)

151. S. M. Ulam: *Theory of the operation of product of sets.* II. Preliminary report.

A projective algebra is a Boolean algebra of sets contained in a direct product E^n of a set E with itself and closed with respect to the operations of projection of a set into the component sets E (considered as a subset of E^n) and the direct product of n sets of the algebra contained in E. A systematic investigation of structures of projective algebras and the properties of projective isomorphisms is undertaken. The cases of infinite projective algebras generated from a finite number of elements by the Boolean operations and the operation of direct product and projection, and the question of existence of finite bases in the above sense for countable algebras are studied. Particular Boolean algebras studied in connection with these ideas include: the algebra of all sets of integers modulo finite sets, the algebra of sets of integers modulo the sets of density 0, the algebra of finite sums of dyadic intervals on (0, 1), the algebra of Borel sets modulo sets of measure 0. (Received March 25, 1943.)

152. F. A. Valentine: *A Lipschitz condition preserving extension for a vector function.*

The function $f(x)$ mapping a set S in the n-dimensional Euclidean space E_n into a set S' in E_n, and satisfying the condition $|f(x_1)-f(x_2)| \leqq K|x_1-x_2|$ on S, can be extended to any set $T \supset S$ so as to preserve the Lipschitz condition. This is a generalization of a previous result given by the author for the plane (see Bull. Amer. Math. Soc. vol. 49 (1943) p. 100). This extension is a consequence of the following fact: Consider in E_n a set of hyperspheres which have a point in common. Move these hyperpsheres to new positions in E_n, subject to the condition that the distance between any pair of centers is not increased. Then all the hyperspheres in the new positions will still have a point in common. This is first proved for $n+1$ hyperspheres. A theorem of Helly is used to proceed from a set containing $n+1$ hyperspheres to an arbitrary set. Corresponding theorems are obtained for the surface of an n-dimensional hyperhemisphere and for a Hilbert space. (Received March 17, 1943.)

153. H. S. Wall and Marion D. Wetzel: *Contributions to the analytic theory of J-fractions.* Preliminary report.

The authors develop the theory of the class of J-fractions $1/(b_1+z)-a_1^2/(b_2+z)-a_2^2(b_3+z)-\cdots$, $a_p \neq 0$, all of whose approximants $f_p(z)$ satisfy the condition $\mathfrak{I}[f_p(z)]<0$ for $\mathfrak{I}(z)>0$. These J-fractions are completely characterized by the condition: $\sum_{r=1}^{p}\mathfrak{I}(b_r)x_r^2-\sum_{r=1}^{p-1}\mathfrak{I}(a_r)x_r x_{r+1} \geqq 0$, $p=1, 2, 3, \cdots$, for all real x_r. Hence they include the classical J-fractions ($\mathfrak{I}(b_r)=\mathfrak{I}(a_r)=0$) and also those discussed in a recent paper of Hellinger and Wall (Ann. of Math. vol. 44 (1943) pp. 103–127) where $\mathfrak{I}(b_r) \geqq 0$, $\mathfrak{I}(a_r)=0$. It is shown that all the poles of $f_p(z)$ lie in the lower half-plane $\mathfrak{I}(z) \leqq 0$ and there exists a nondecreasing function $\phi_p(u)$, $0 \leqq \phi_p(u) \leqq 1$, such that $f_p(z)=\int_{-\infty}^{\infty} d\phi_p(u)/(z+u)$; therefore the sequence $\{f_p(z)\}$ is uniformly bounded for $\mathfrak{I}(z) \geqq \delta > 0$ and contains an infinite subsequence converging uniformly on the interior of the upper half-plane to an integral of the same form. The older theories, including asymptotic properties and the moment problem, are extended to this general class of J-fractions. (Received March 26, 1943.)

154. Alexander Weinstein: *Differential equations with general boundary conditions.*

The following problem, discussed in this paper, is a typical example of a new kind of differential eigenvalue problems. Let $g_1, g_2, \cdots, g_m$ be a basis of a linear manifold L_m of functions defined on the boundary C of a domain S. It is required to find the eigenvalues of the partial differential equation $\Delta u+\lambda u=0$ with the boundary conditions: u is orthogonal to all elements of L_m and du/dn is an element of L_m. Other examples of the same kind have been introduced by the author in previous papers. The corresponding problems for ordinary differential equations are connected with the classical theory. (Received March 23, 1943.)