ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of the issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

125. A. T. Brauer: *On the non-existence of odd perfect numbers of form* $p^{\alpha}q_1^2q_2^2 \cdots q_{t-1}^2q_t^4$.

If an odd perfect number exists, it must have the form $n = p^{\alpha}q_1^{2\beta_1}q_2^{2\beta_2} \cdots q_t^{2\beta_t}$ where $p, q_1, q_2, \cdots, q_t$ are primes and $p \equiv \alpha \equiv 1 \pmod 4$. This was proved by Euler. Sylvester obtained estimates for t, in particular $t \geqq 4$, and $t \geqq 7$ if $n \not\equiv 0 \pmod 3$. Recently, it was shown by R. Steuerwald (Sitzungsberichte der Bayerischen Akademie der Wissenschaften 1937) that the case $\beta_1 = \beta_2 = \cdots = \beta_t = 1$ is impossible, and by H. J. Kanold (J. Reine Angew. Math. vol. 183 (1941)) that the same is true for $\beta_1 = \beta_2 = \cdots = \beta_t = 2$. Moreover Kanold proved that n is not perfect if the greatest common divisor d of $2\beta_1+1, 2\beta_2+1, \cdots, 2\beta_t+1$ is divisible by 9, 15, 21, or 33, and some similar results. All these results deal with the case $d > 1$. In this paper it is proved that no odd number of form $p^{\alpha}q_1^2q_2^2 \cdots q_{t-1}^2q_t^4$ exists; here $d=1$. For the proof some theorems of T. Nagell on Diophantine equations are used (Norsk Matematisk Forenings Skrifter 1921). (Received March 24, 1943.)

126. R. P. Dilworth: *Lattices with unique complements.*

For some time an outstanding problem in lattice theory has been the following: Is every lattice having unique complements a Boolean algebra? It is shown here that the statement is *not* true. Indeed the following counter theorem is proved: Every lattice is a sublattice of a lattice with unique complements. (Received March 20, 1943.)

127. J. E. Eaton: *A Galois theory for differential fields.*

Let Λ be an algebraically transcendental extension of the partial differential field Γ, and consider all isomorphisms of Λ leaving Γ invariant. These isomorphisms may be grouped into a finite number of disjoint sets $\{m_i\} = \mathfrak{M}$ which, with a suitably defined multiplication, is a multigroup called the Galois multigroup of Λ over Γ. There is a 1-1 correspondence between the subfields of Λ containing Γ and the submultigroups of $\mathfrak{M}$ such that if $\Delta \rightleftarrows \mathfrak{H}$, then $\mathfrak{H}$ is the Galois multigroup of Λ over Δ and $\mathfrak{M}//\mathfrak{H}$ (the multiplicative system of the double coset decomposition of $\mathfrak{M}$ with respect to $\mathfrak{H}$) is the Galois multigroup of Δ over Γ. These results are an extension of the work of H. W. Raudenbush (*Hypertranscendental extensions of partial differential fields*, Bull. Amer. Math. Soc. vol. 40 (1934) pp. 714–720) and E. R. Kolchin (*Extensions of differential fields*, I, Ann. of Math. vol. 43 (1942) pp. 724–729). (Received March 25, 1943.)

128. D. H. Lehmer: *On Ramanujan's numerical function $\tau(n)$.*

Ramanujan's numerical function $\tau(n)$ may be defined by $\sum \tau(n)x^{n-1} = \{\pi(1-x)^k\}^{24}$. Among several unsolved questions about $\tau(n)$ is the so-called Ramanujan hypothesis to the effect that if p is a prime $|\tau(p) \cdot p^{-11/2}| < 2$, which Ramanujan verified for primes $p < 30$. The present writer, in attempting to disprove this important hypothesis, has examined all primes $p < 300$, as well as $p = 571$, and finds that in all these 47 cases the hypothesis holds true. It nearly fails for $p = 103$ when $\tau(103) \cdot 103^{-11/2} = -1.918 \cdots$. In connection with this hypothesis Rankin proved in 1939 that as $x \to \infty$, $x^{-12}\sum_{n \leqq x} \tau(n)^2$ tends to a limit represented by a certain double integral extended over the fundamental region of the full modular group. In this paper a practical method is devised for evaluating this integral whose value is found to be $.0320047918814 \cdots$. Various congruence and divisibility properties of $\tau(n)$ are also discussed. For example $\tau(n)$ is composite for $1 < n < 7921$. (Received March 26, 1943.)

129. A. E. Ross: *Positive quaternary quadratic forms representing all integers with at most k exceptions.*

In this paper it is shown that there are a finite number of classes of positive quaternary quadratic forms which represent all integers with at most k exceptions. The determinants of such forms have an upper bound B_k depending on k. This is a generalization of the results of Ramanujan (Proc. Cambridge Philos. Soc. vol. 19 (1917) pp. 11–21), Ross (Proc. Nat. Acad. Sci. U.S.A. vol. 18 (1932) p. 607) and Halmos (Bull. Amer. Math. Soc. vol. 44 (1938) pp. 141–144). Ross gives $B_0 = 112$ for the classic case and Halmos' results imply that $B_1 \geqq 240$ in the classic case. (Received March 26, 1943.)

130. L. I. Wade (National Research Fellow): *Transcendence properties of the Carlitz ψ-function.*

The paper is concerned with quantities transcendental over the field $GF(p^n, x)$. For the Carlitz ψ-function (L. Carlitz, Duke Math. J. vol. 1 (1935) pp. 137–168) and its inverse $\lambda(t)$ the following theorem is proved. If β is algebraic (over $GF(p^n, x)$) and irrational and if $\alpha \neq 0$ is algebraic, then $\psi(\beta\lambda(\alpha))$ is transcendental over $GF(p^n, x)$. In a sense this is an analogue of Hilbert's seventh problem for the transcendence of $\alpha^\beta = e^{\beta \log \alpha}$ over the rational number field. (Received March 26, 1943.)

Analysis

131. L. W. Cohen: *On linear equations in Hilbert space.*

If the rows of the infinite matrix $A = \|a_{ij}\|$ are points in Hilbert space and $a_{j_1 \cdots j_m}^{i_1 \cdots i_m}$ are the m-rowed determinants with elements in A, it is shown that det $A_{i_1 \cdots i_m} B'_{i_1 \cdots i_m} = \sum_{j_1 \cdots j_m} a_{j_1 \cdots j_m}^{i_1 \cdots i_m} b_{j_1 \cdots j_m}^{i_1 \cdots i_m}$ where $A_{i_1 \cdots i_m}$, $B_{i_1 \cdots i_m}$ are m-rowed minors of A, B respectively. The series is summed over all combinations of integers $j_1, \cdots, j_m$ and converges absolutely. This identity is used to establish sufficient conditions in order that the linear system represented by $Ax = y$ have a solution in Hilbert space for all y in that space. (Received March 24, 1943.)

132. R. J. Duffin: *Some representations for Fourier transforms.*

Let $\phi(x)$ be an arbitrary function and let $f(x)$ and $g(x)$ be defined by the series: $f(cx) = \sum_1^\infty (-1)^n \phi((2n-1)/x)/x$, $g(cx) = \sum_1^\infty (-1)^n \phi(x/(2n-1))/(2n-1)$. Then if